

## Quantum Logic 2023-24

### Exercises, Module 1

#### Assignment 1.

1.1. Show that cut, weakening, contraction and permutation are admissible in the sequent calculus for intuitionistic logic.

(The first lecture was relatively short because some students were still missing, so I gave only one problem.)

#### Assignment 2.

2.1. Show that  $\lambda xy.xy$  and  $\lambda yx.yx$  are  $\alpha$ -equivalent.

2.2. Prove that if  $t$  and  $t'$  are  $\alpha$ -equivalent, then

- (a) The set of free variables in  $t$  coincides with the set of free variables in  $t'$ .
- (b)  $t\{y/x\}$  and  $t'\{y/x\}$  are  $\alpha$ -equivalent for any variables  $x$  and  $y$ .
- (c)  $t$  and  $t\{y/x\}$  are  $\alpha$ -equivalent for any variables  $x$  and  $y$ .

2.3. Show that  $(\lambda x.xx)(\lambda x.xx)$  is not a term in the simply-typed  $\lambda$ -calculus.

#### Assignment 3.

3.1. Prove the following holds for the category Set of sets and functions.

- (i) An arrow is a mono iff it is an injection.
- (ii) An arrow is an epi iff it is a surjection.
- (iii) An arrow is an isomorphism iff it is a bijection.

3.2. Show that an initial and a terminal objects are unique up to isomorphisms.

3.3. Show that in Set cartesian products are categorical products.

#### Assignment 4.

4.1. Prove the following.

- (i)  $\langle f, g \rangle h = \langle fh, gh \rangle$
- (ii)  $(p \times q) \langle f, g \rangle = \langle pf, qg \rangle$ .

4.2. Explain the following.

- (i) An initial object in  $C^{\text{op}}$  is a terminal object in  $C$ .
- (ii) A product in  $C^{\text{op}}$  is a coproduct in  $C$ .

#### Assignment 5.

5.1. Prove that the category Cat of small categories and functors has a terminal object and products.

5.2. Prove that, for any categories  $C$  and  $D$ , the category  $[C, D]$  of functors  $C \rightarrow D$  and natural transformations is well-defined.

5.3. Prove the following.

- (i) Exponents define a functor  $C^{\text{op}} \times C \rightarrow C$ .
- (ii) A right adjoint to  $_ \times B$  is equivalent to exponents.

#### Assignment 6.

6.1. Prove that the natural deduction system and the sequent calculus for intuitionistic logic are equivalent.

(Because the students had an essay and exams in that period, and also because this problem is relatively heavy, I assigned just one problem that week.)