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JOHANNES KEPLER
UNIVERSITÄT LINZ

An introduction to measurement-based quantum computation (MBQC)

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Department for Quantum Information and Computation at Kepler (QUICK)

October 2026

Class outline

Introducing information processing with measurements…

1. Core concepts and ideas
2. One-way quantum computing (1WQC)
3. Fusion-based quantum computation (FBQC)
4. Pauli-based computation

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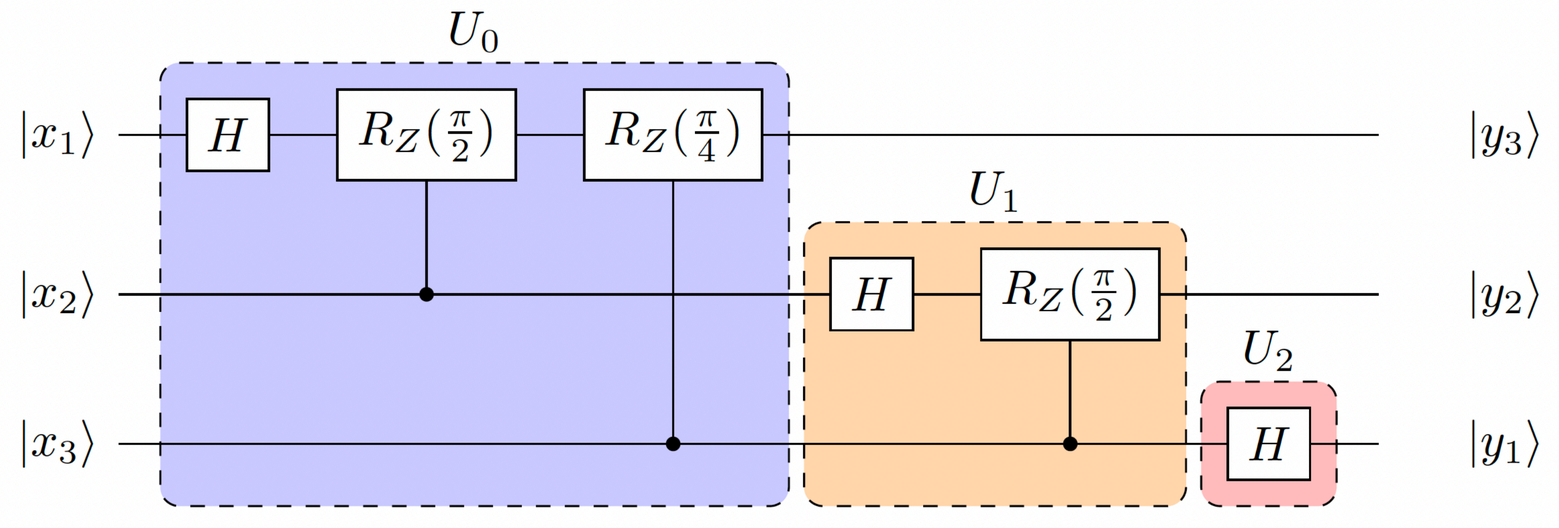
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Quantum circuits...



What if we use measurements instead?

Letter | Published: 25 November 1999

Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations

Daniel Gottesman & Isaac L. Chuang

Nature **402**, 390–393 (1999) | Cite this article

15k Accesses | **62** Altmetric | Metrics

Abstract

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Quantum teleportation is a universal computational primitive

Daniel Gottesman[1,2] * *and* Isaac L. Chuang[3] †

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Single-qubit operations

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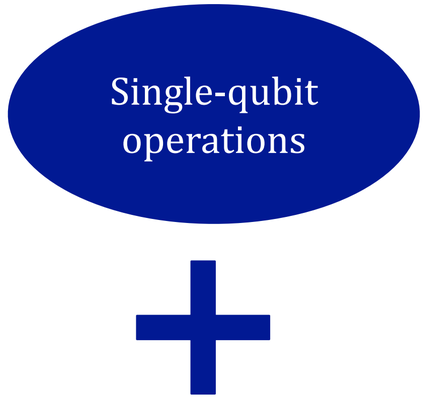
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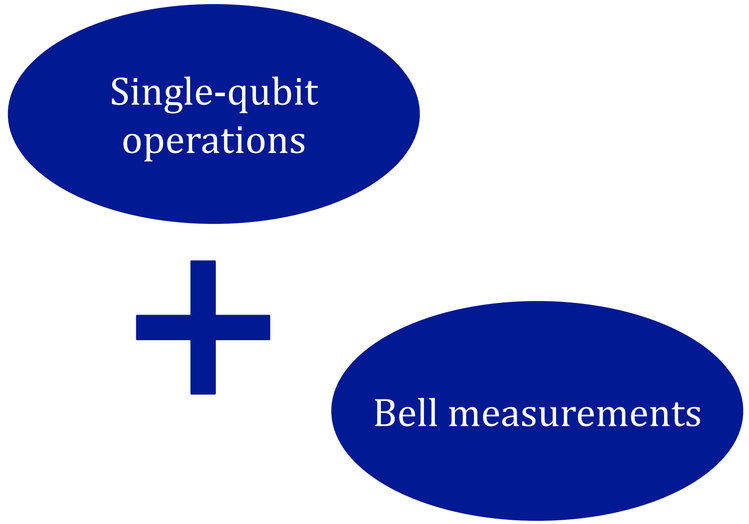
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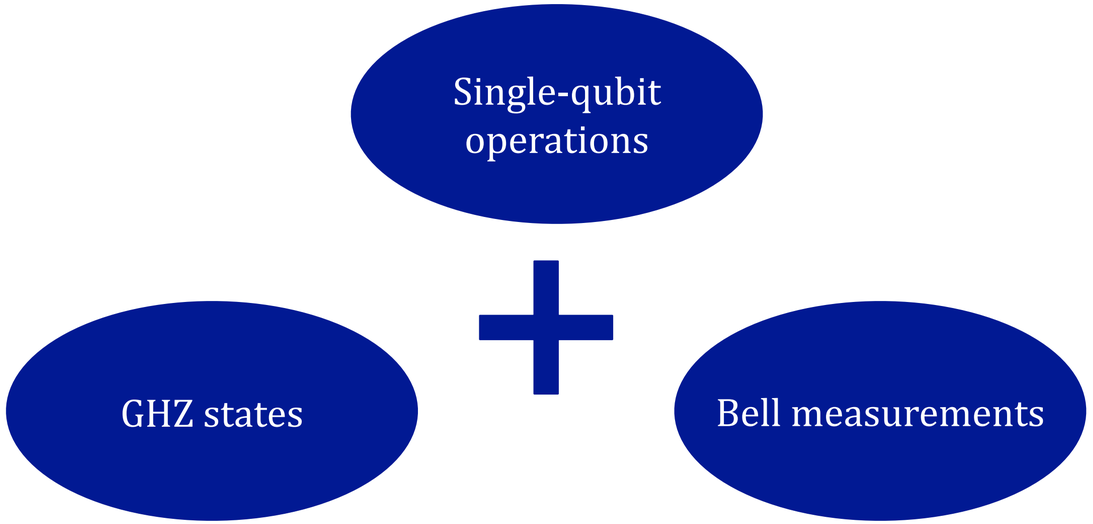
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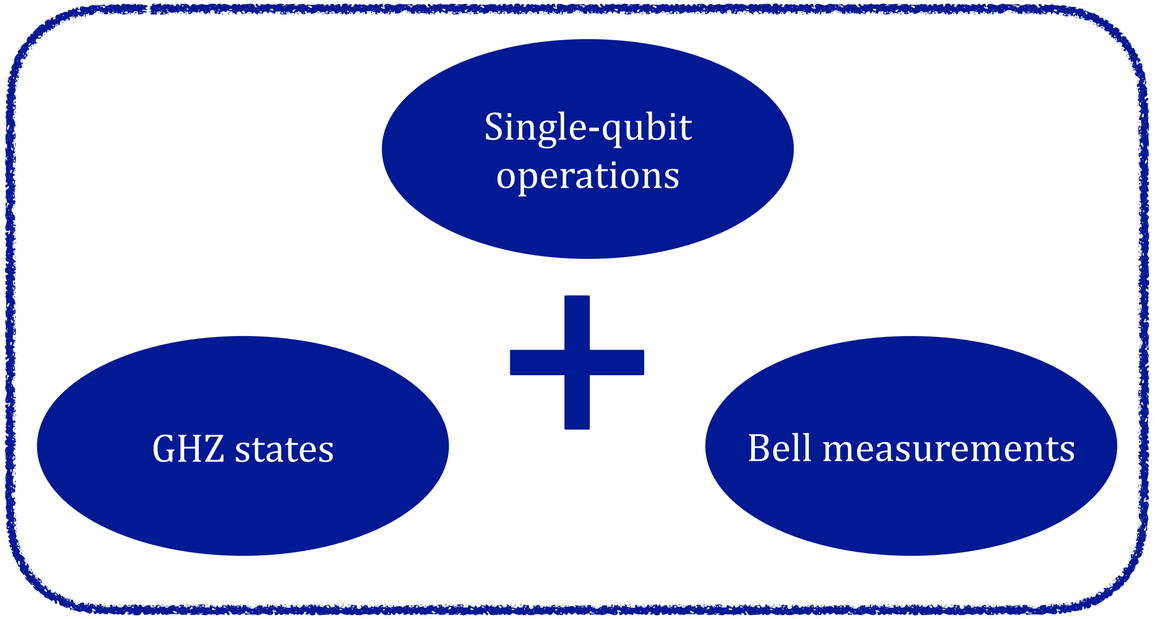
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UNIVERSAL QUANTUM COMPUTER

[Gottesman and Chuang, Nature **402**, 390 (1999)]

But, before we move on…

… what is a ***universal*** quantum computer?

Strict Universality

vs

Computational Universality

Strict Universality

vs

Computational Universality

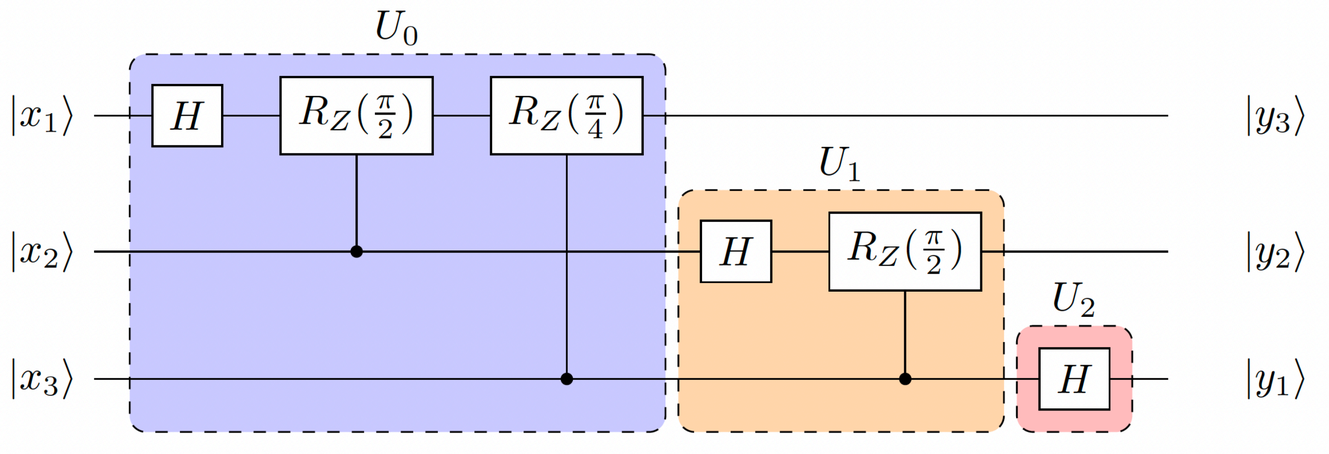
Exact Universality

vs

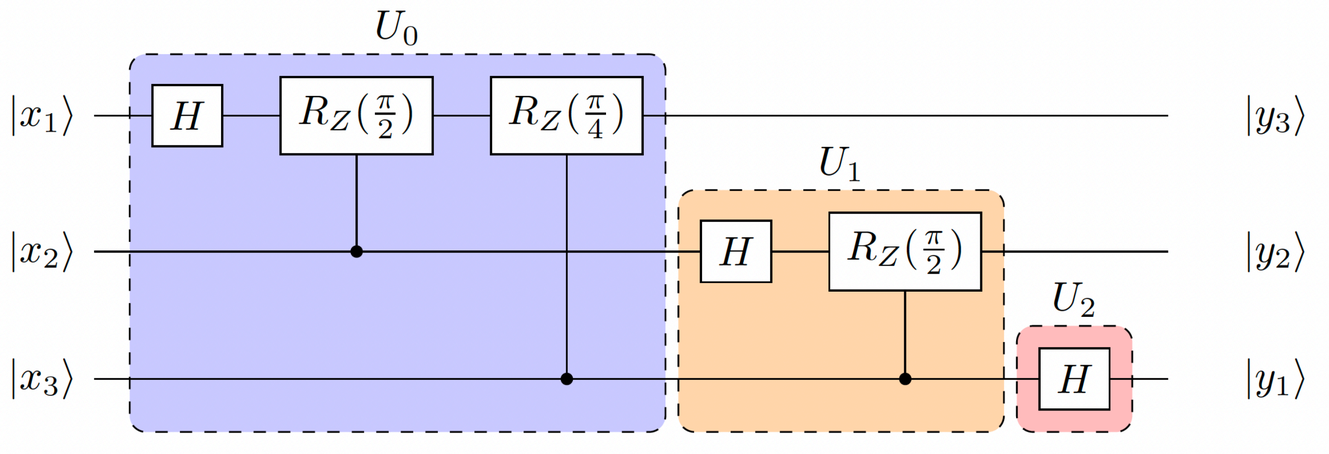
Approximate Universality

[Aharonov, arXiv:quant-ph/0301040 (2003)] or [van den Nest et al., New Journal of Physics **9**, 204 (2007)]

Quantum circuits...

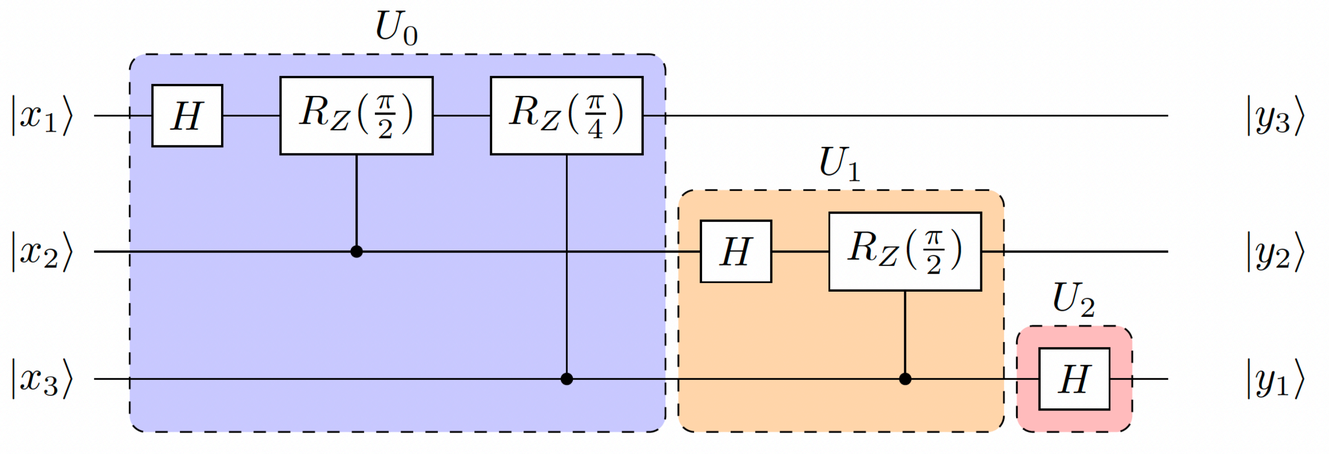


Quantum circuits…



Universal gate sets

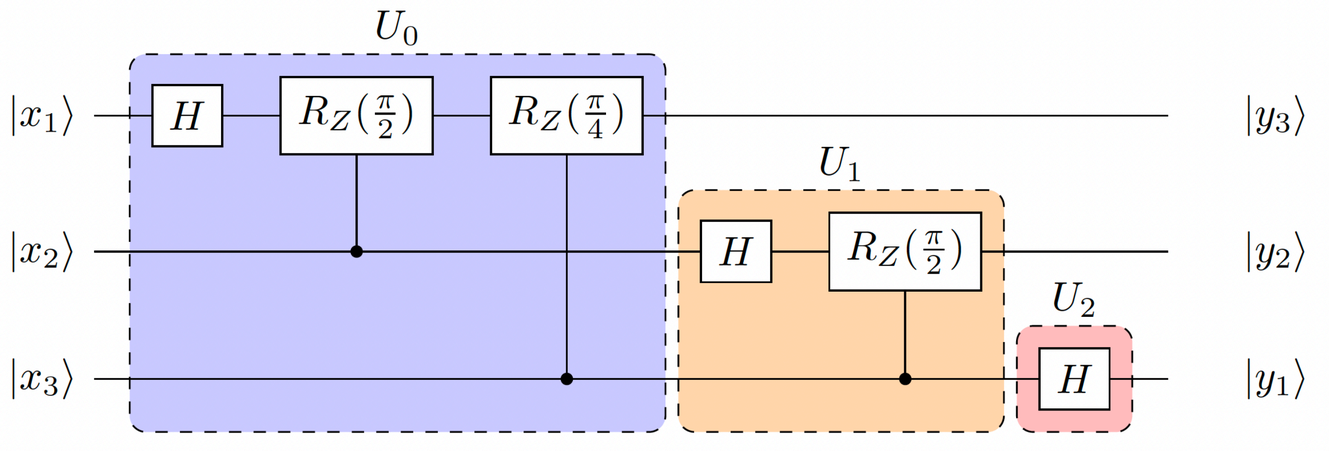
Quantum circuits…



Universal gate sets

- Clifford unitaries + T gate

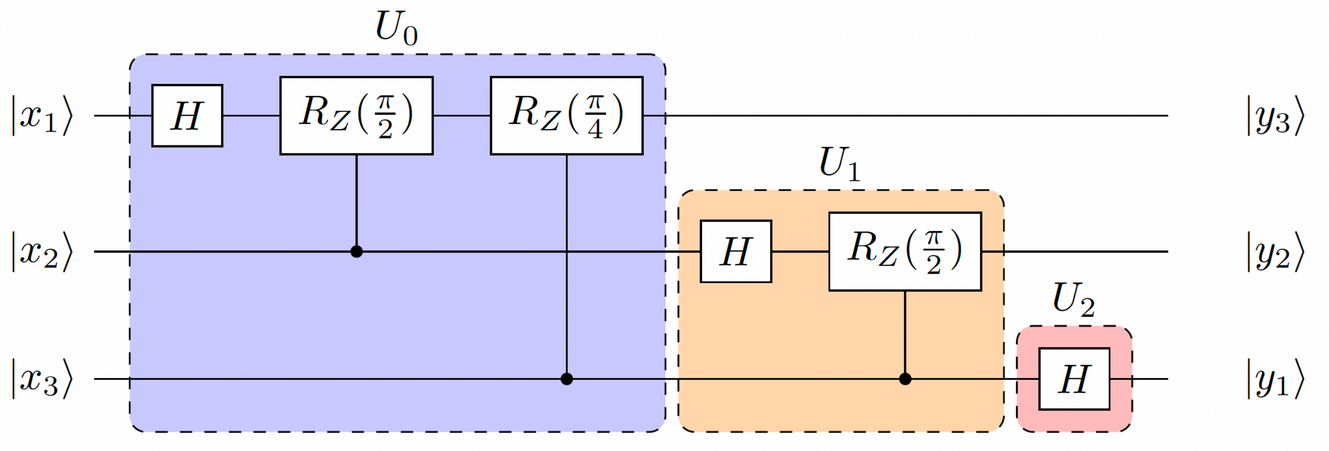
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- Hadamard gate + Toffoli gate

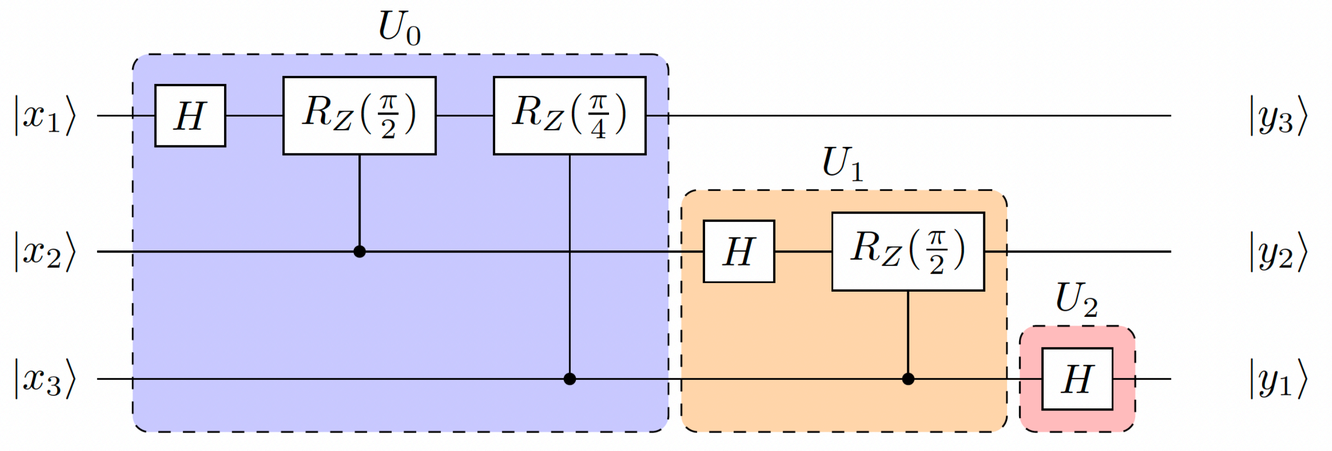
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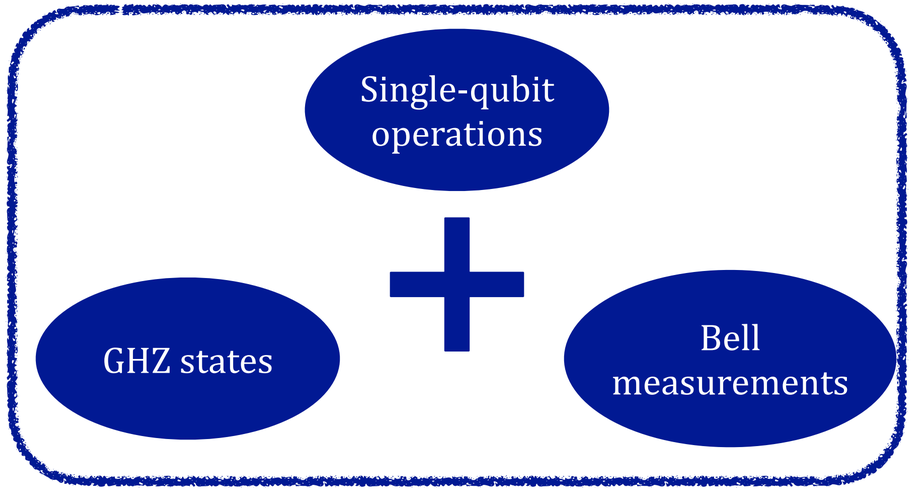
- Clifford unitaries + T gate
- Hadamard gate + Toffoli gate
- Single-qubit rotations + CNOT

Quantum circuits…

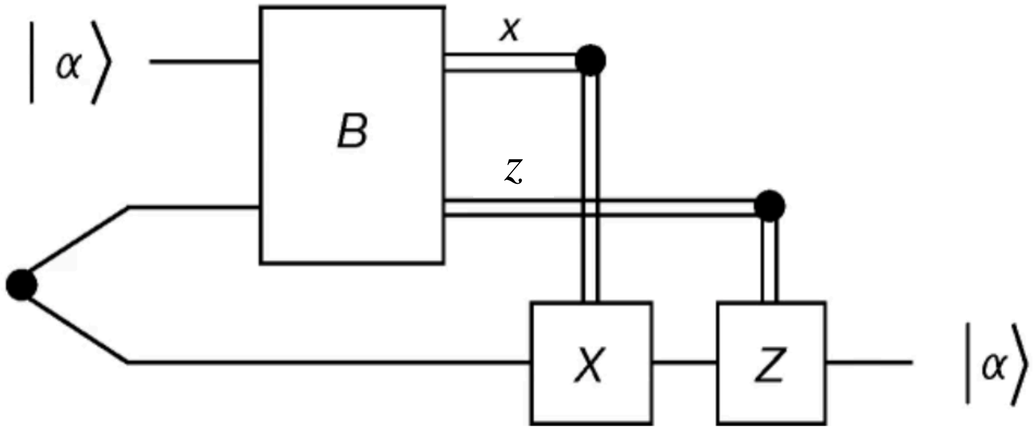


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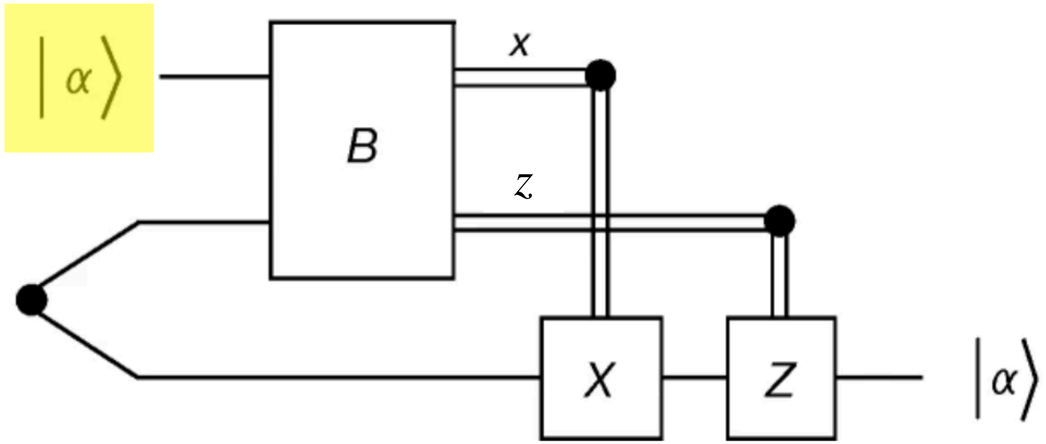


Quantum teleportation



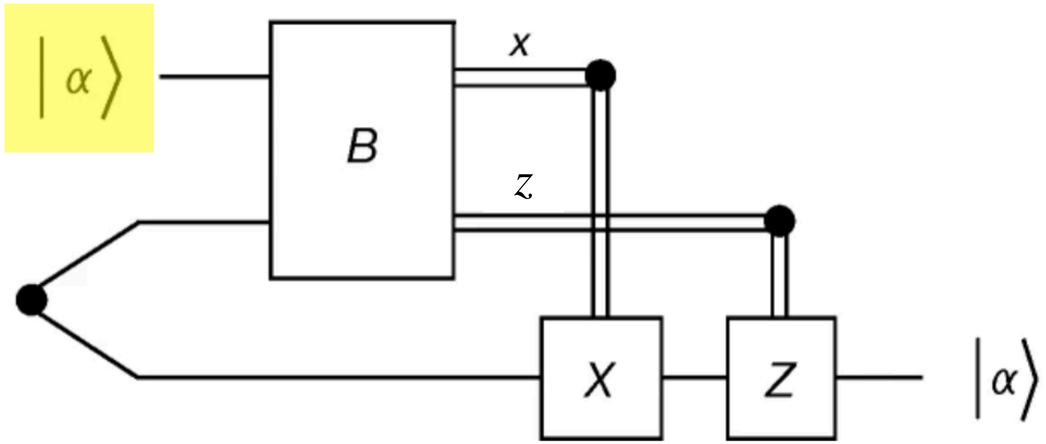
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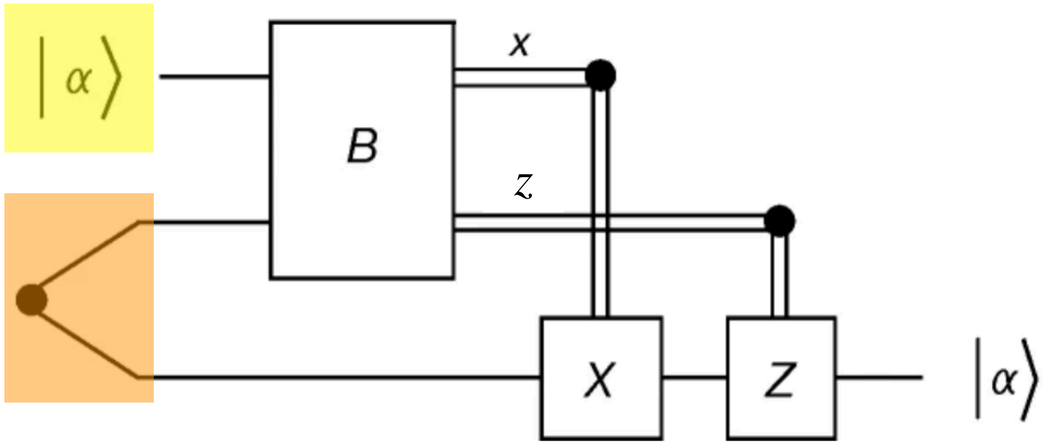
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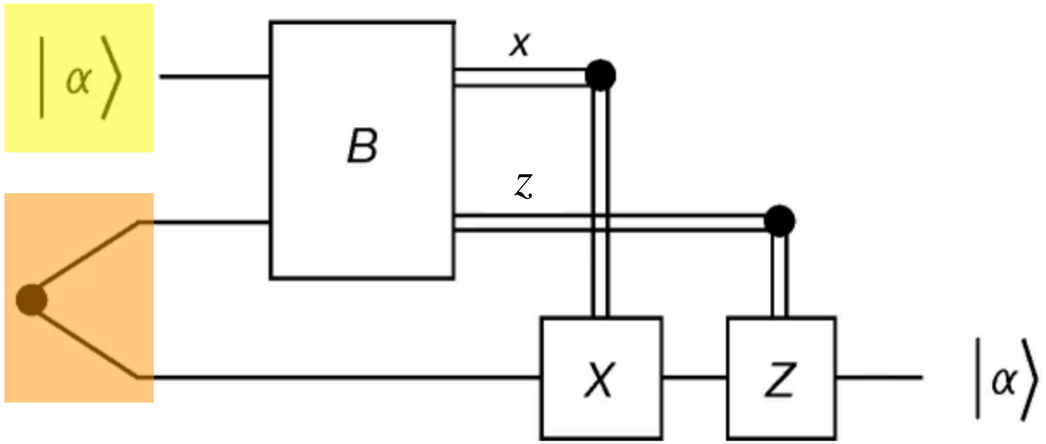
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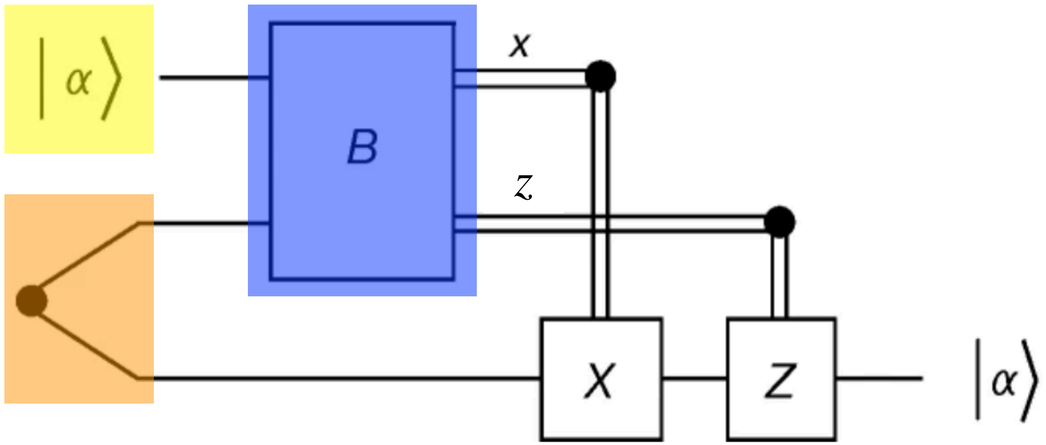


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$$|\Psi_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

[Gottesman and Chuang, Nature **402**, 390 (1999)]

Quantum teleportation

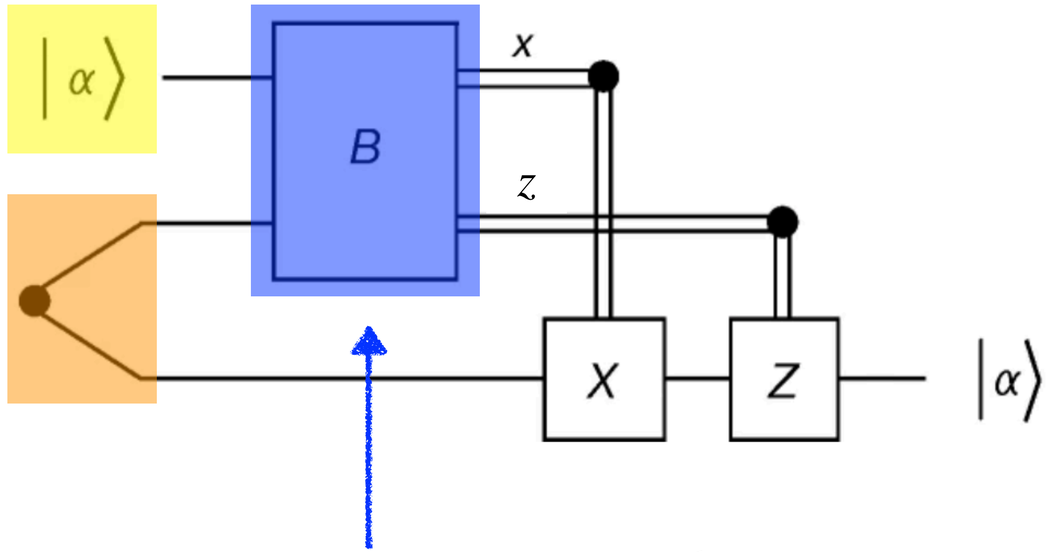


$$|\alpha\rangle = a\,|0\rangle + b\,|1\rangle$$

$$|\Psi_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

[Gottesman and Chuang, Nature **402**, 390 (1999)]

Quantum teleportation



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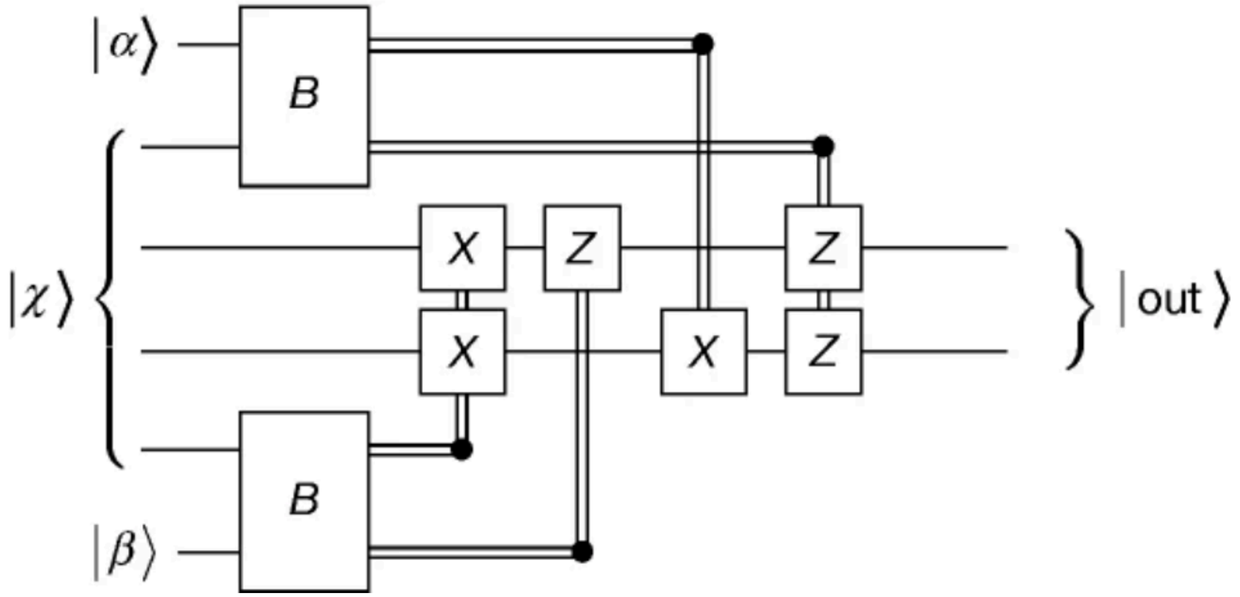
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Bell measurement: $\left\{ |\Psi_{00}\rangle\ , |\Psi_{01}\rangle, |\Psi_{10}\rangle, |\Psi_{11}\rangle \right\}$

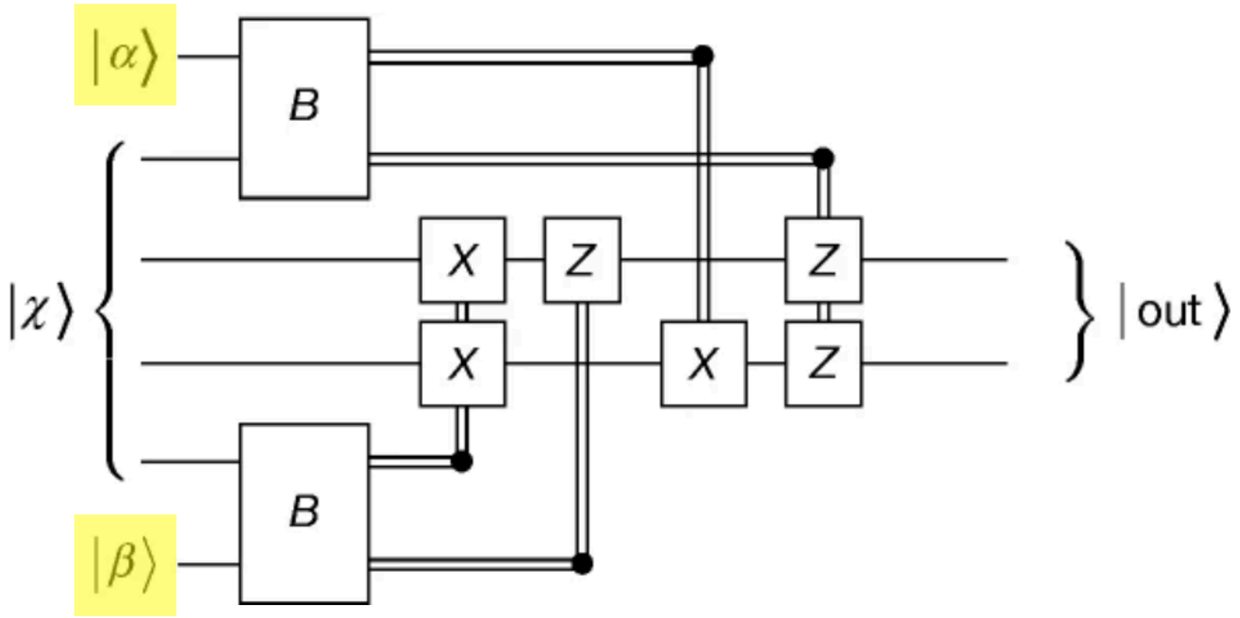
$$|\Psi_{xz}\rangle = \frac{|0x\rangle + (-1)^z |1\bar{x}\rangle}{\sqrt{2}}$$

[Gottesman and Chuang, Nature **402**, 390 (1999)]

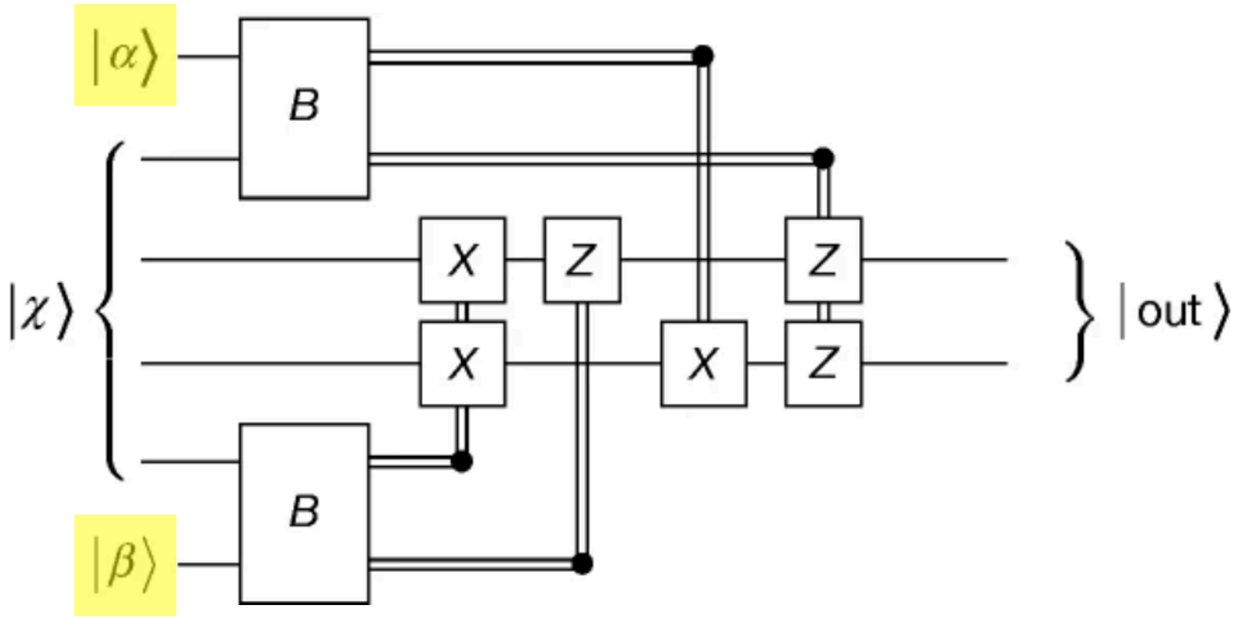
Proof step 1: Implementing the CNOT gate



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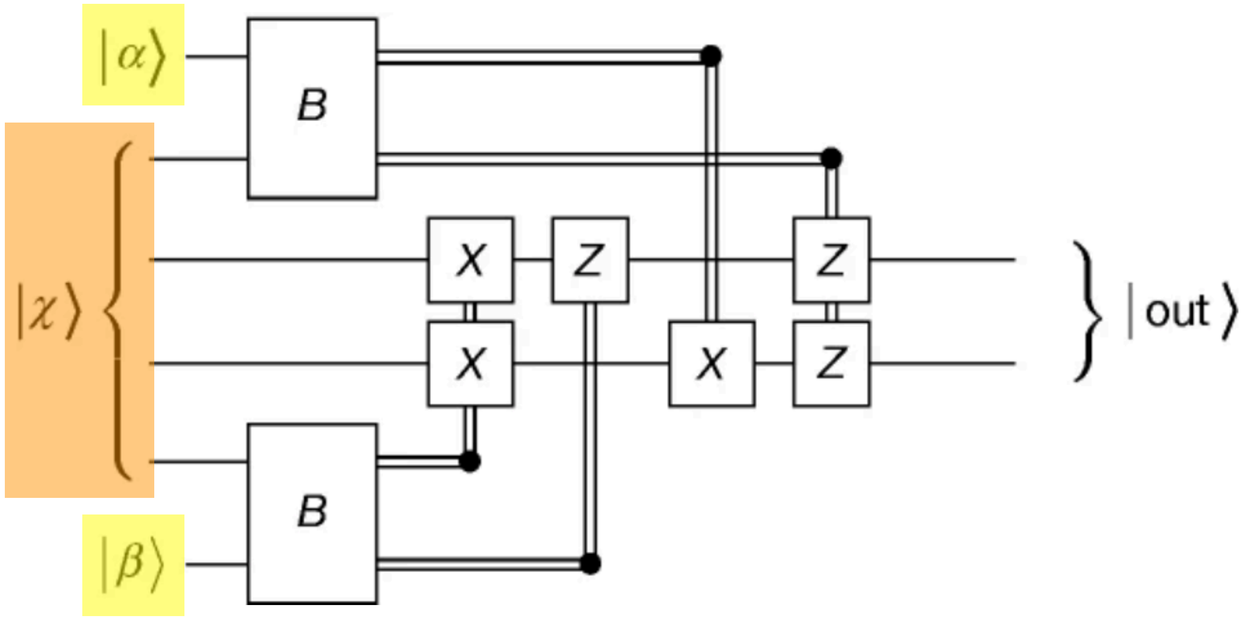
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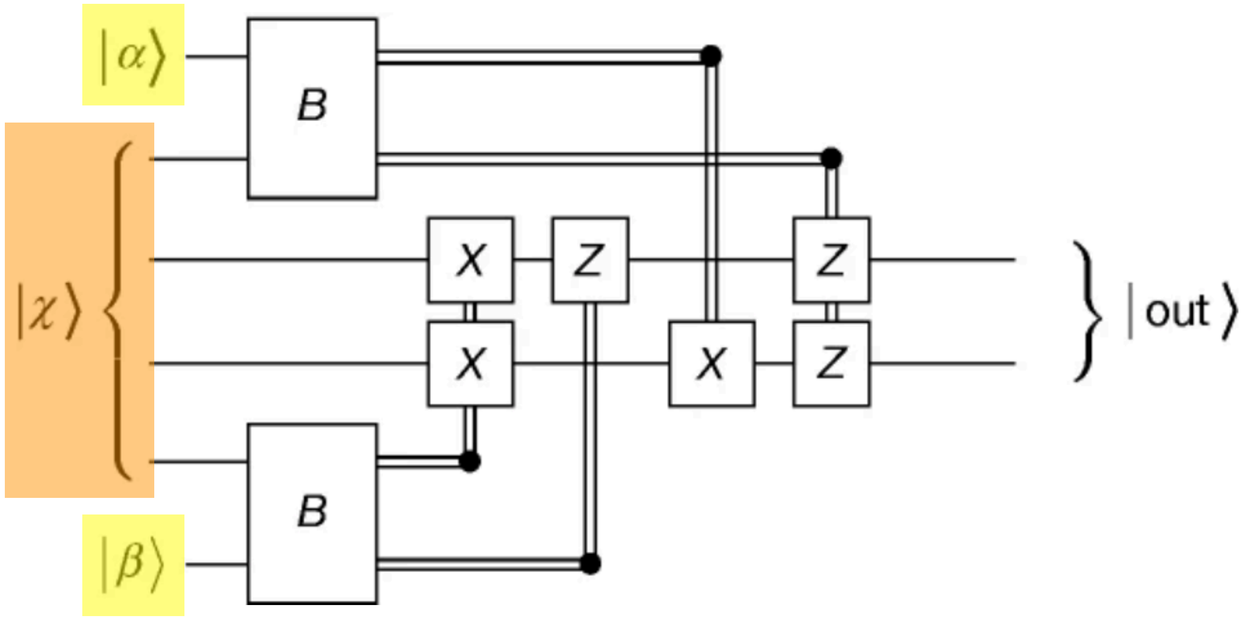


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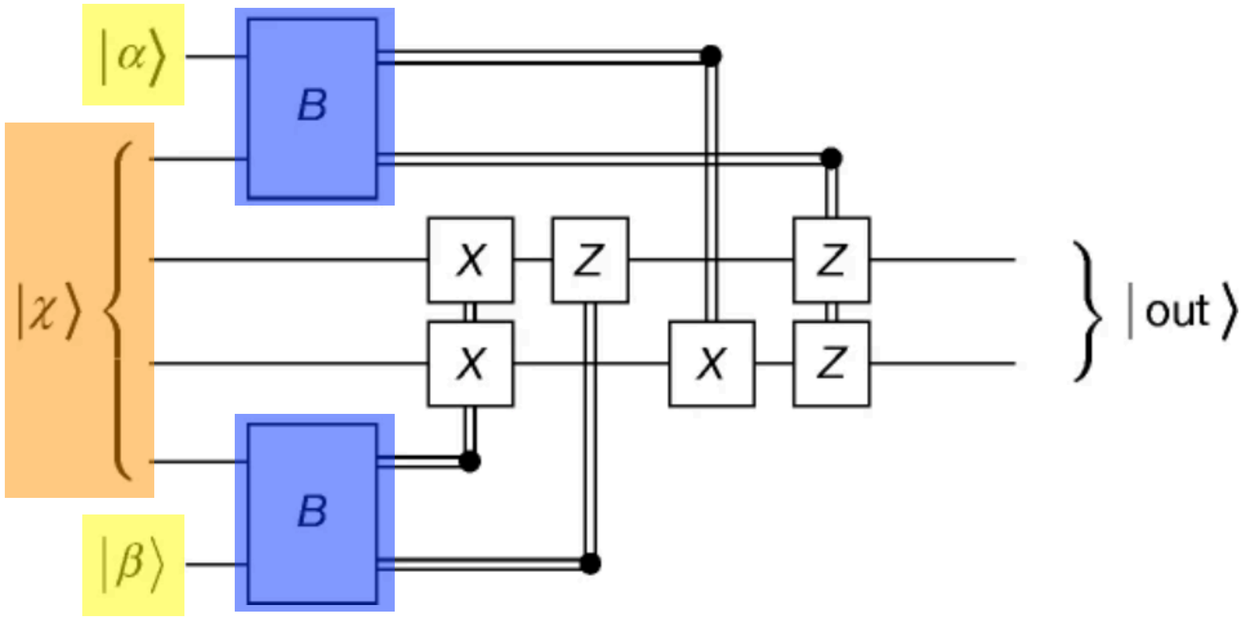
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$$|\chi\rangle = \frac{\left(|00\rangle + |11\rangle\right)|00\rangle + \left(|01\rangle + |10\rangle\right)|11\rangle}{2}$$

[Gottesman and Chuang, Nature **402**, 390 (1999)]

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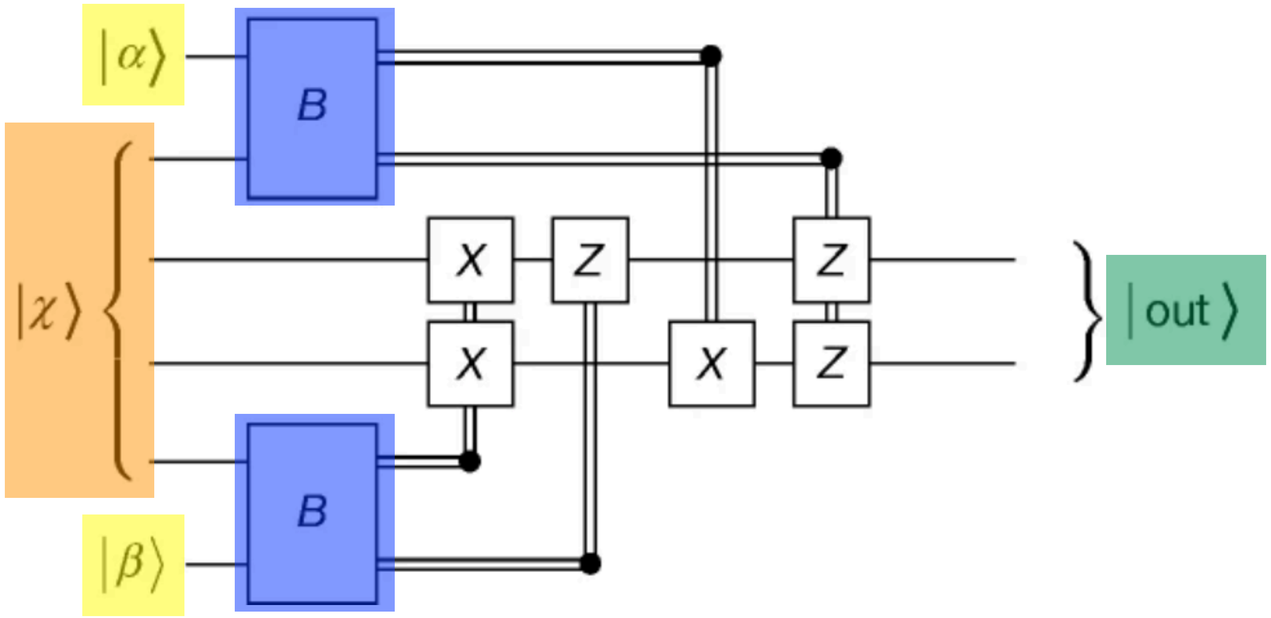
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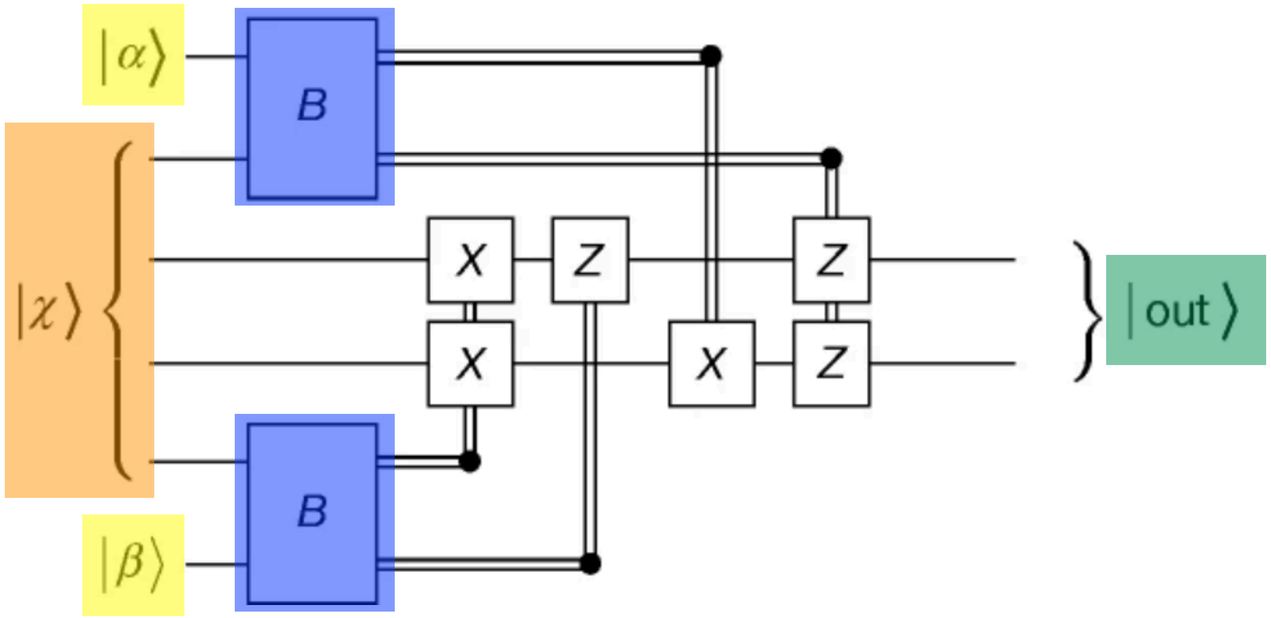
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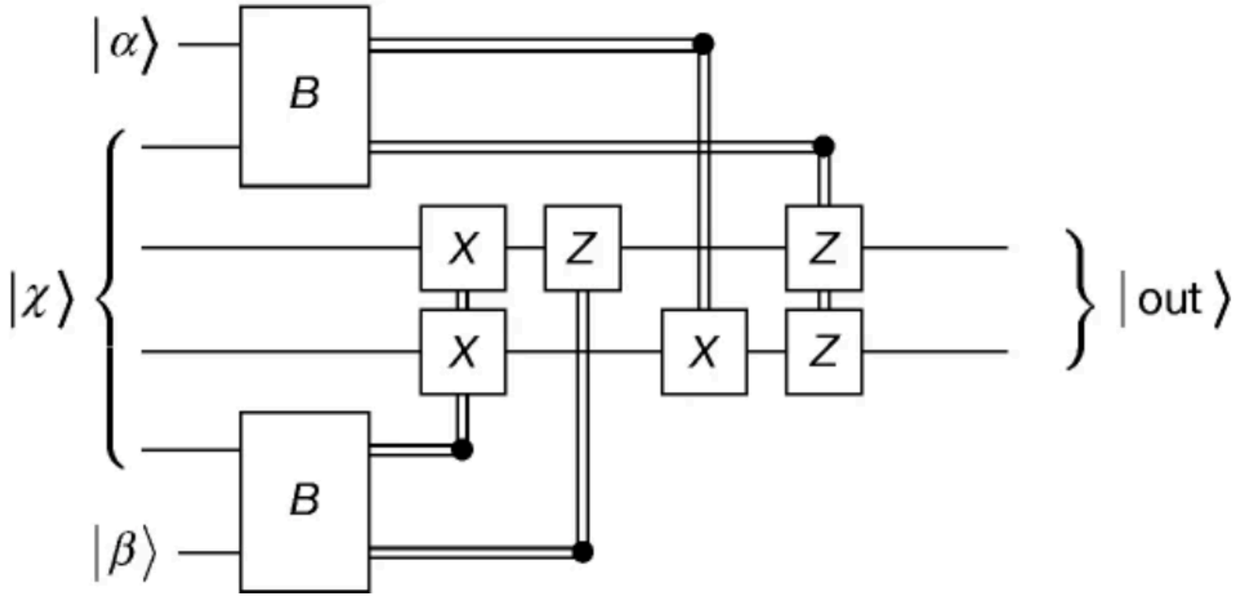
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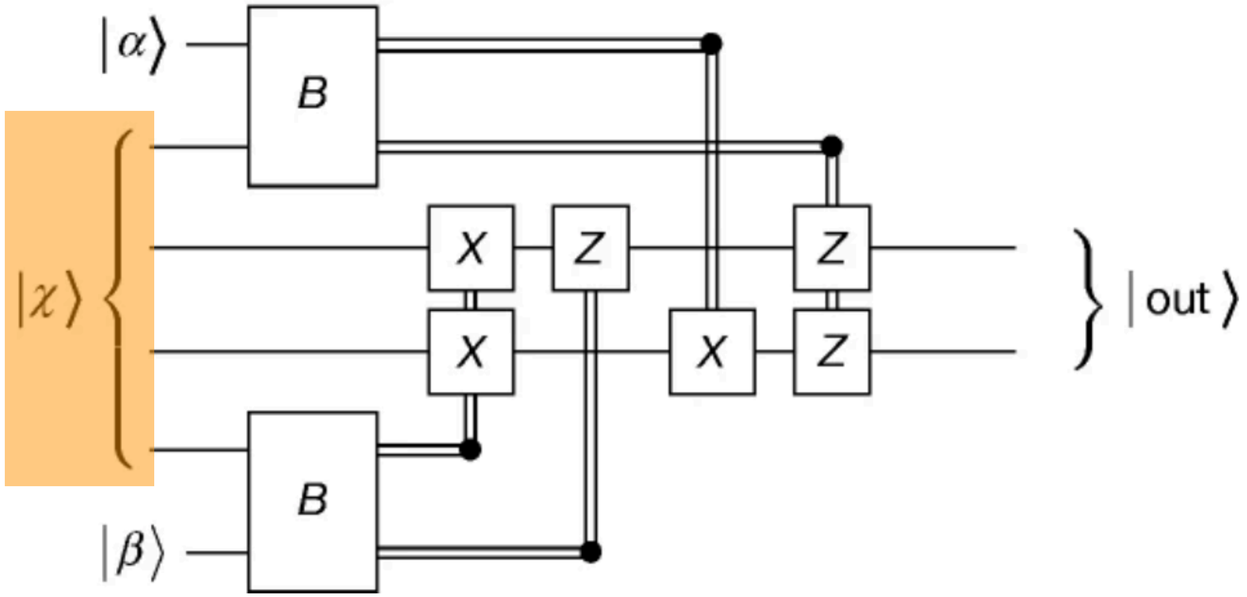
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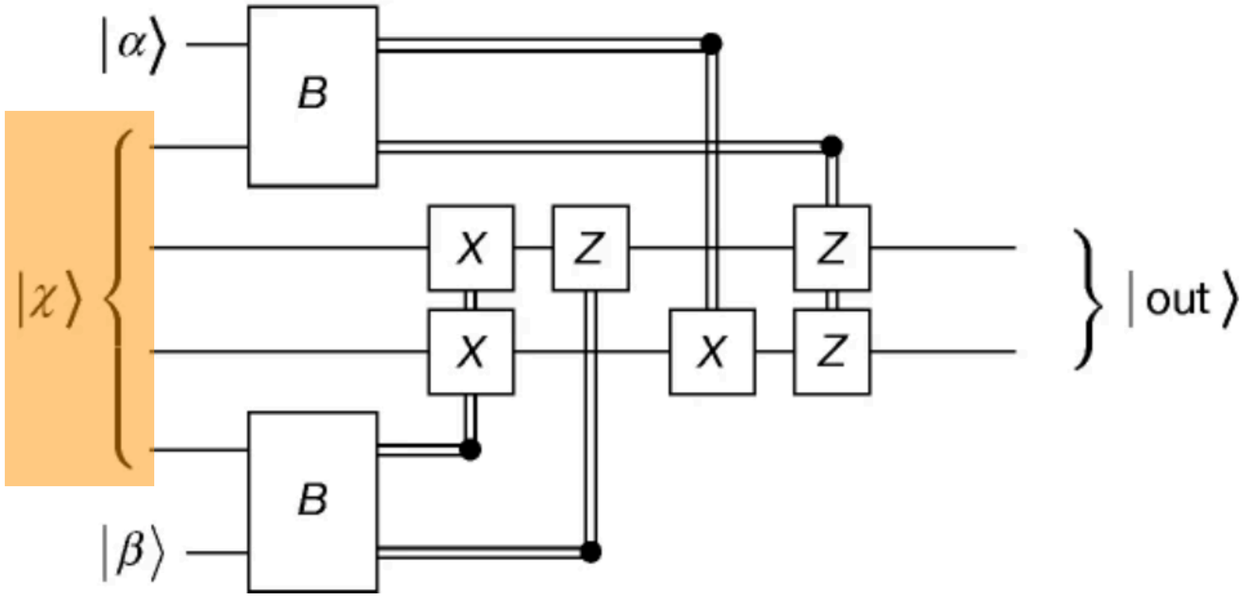
$$|\text{out}\rangle = \text{CNOT}\,|\beta\rangle\,|\alpha\rangle$$

[Gottesman and Chuang, Nature **402**, 390 (1999)]



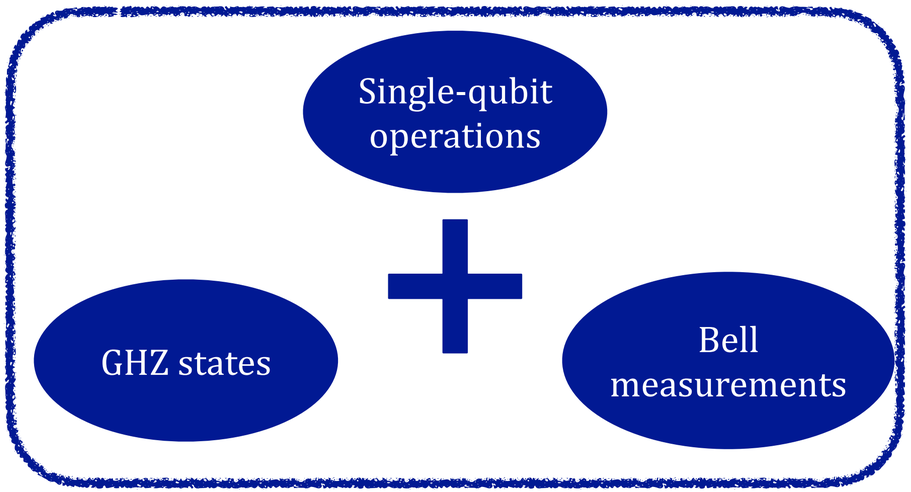
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$|\alpha\rangle$
B
$|\chi\rangle$
X
Z
Z
X
X
Z
B
$|\beta\rangle$
$|\text{out}\rangle$

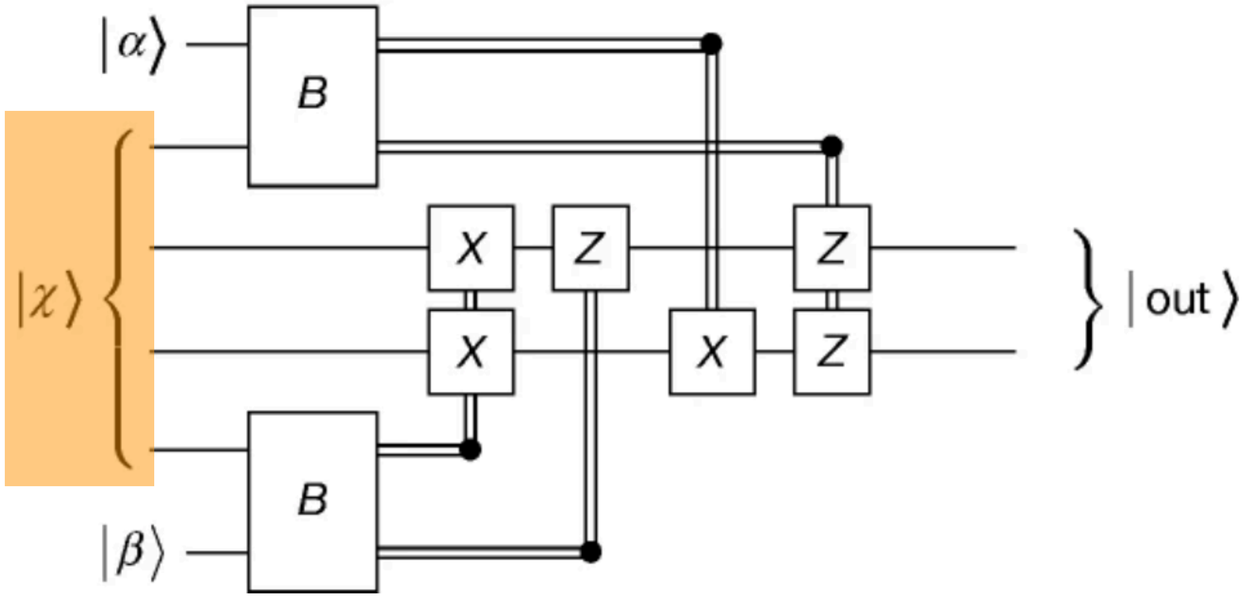


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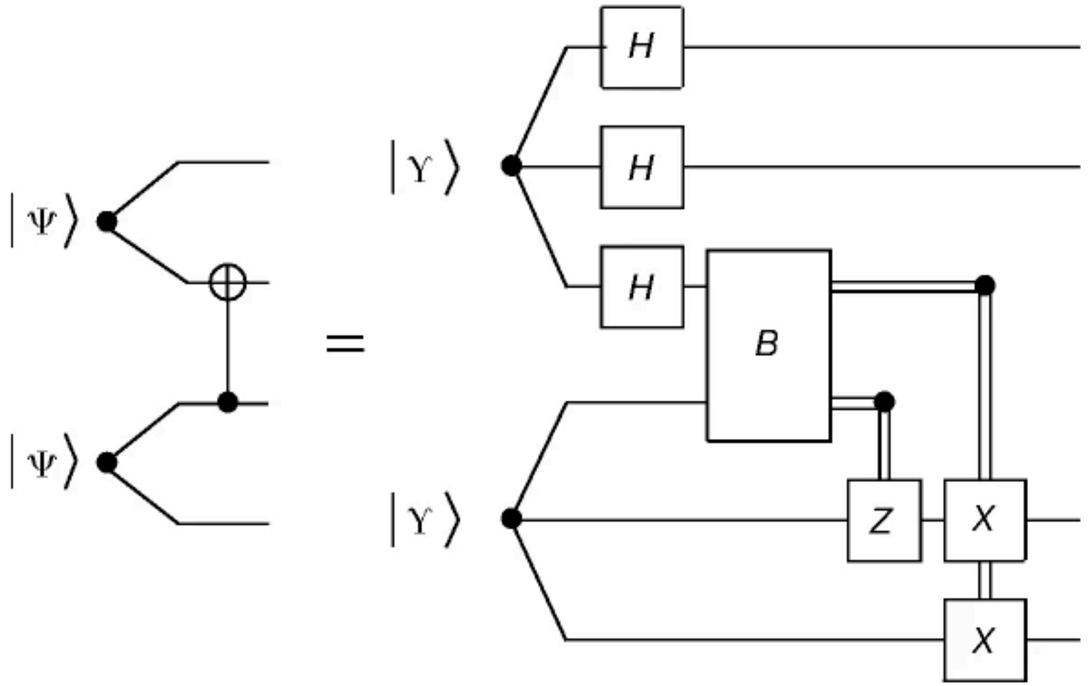


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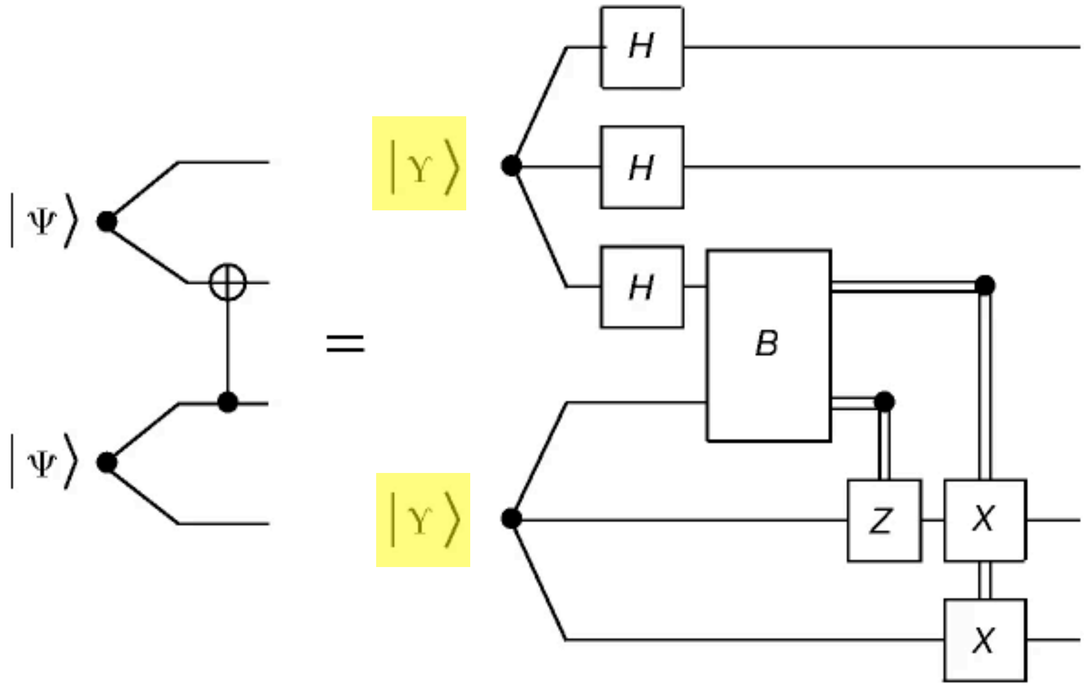
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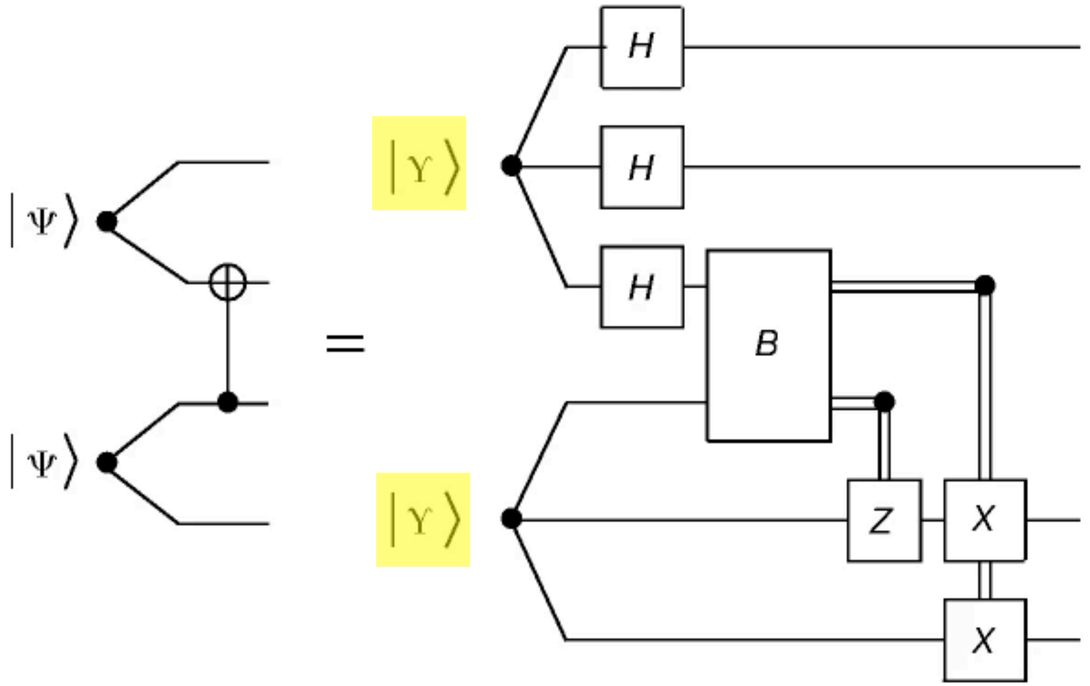
Proof step 2: Generating the $|\chi\rangle$ state



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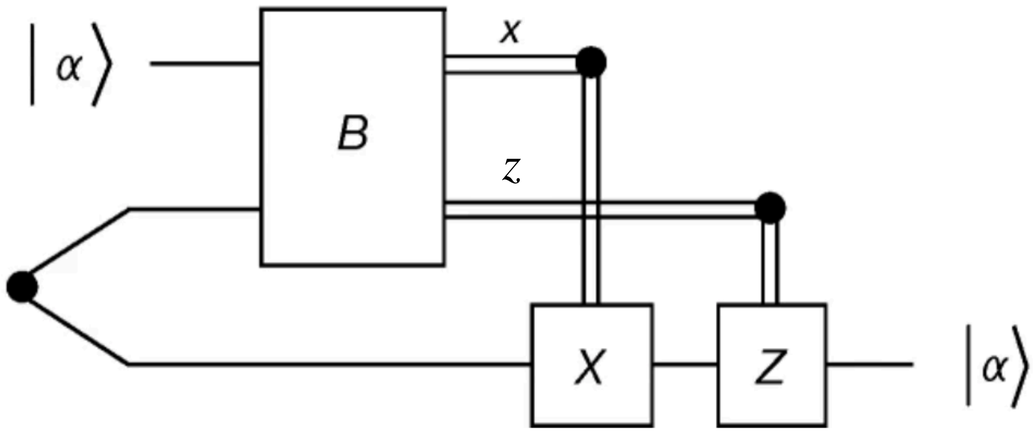
Proof step 2: Generating the $|\chi\rangle$ state



Greenberger-Horne-Zeilinger (GHZ) state: $|\Upsilon\rangle := |\mathrm{GHZ}\rangle = \dfrac{|000\rangle + |111\rangle}{\sqrt{2}}$

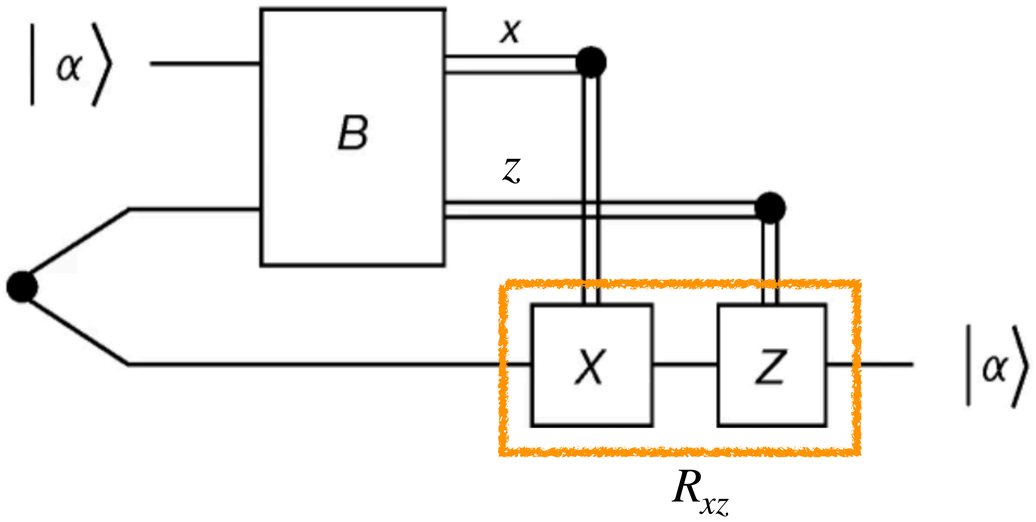
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Proof step 3: Gate teleportation



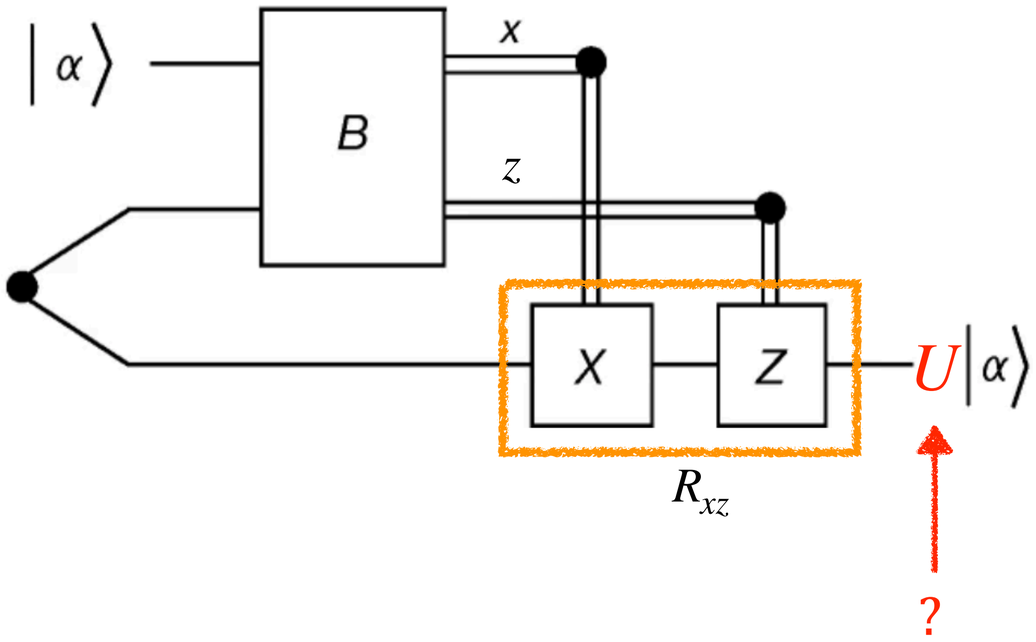
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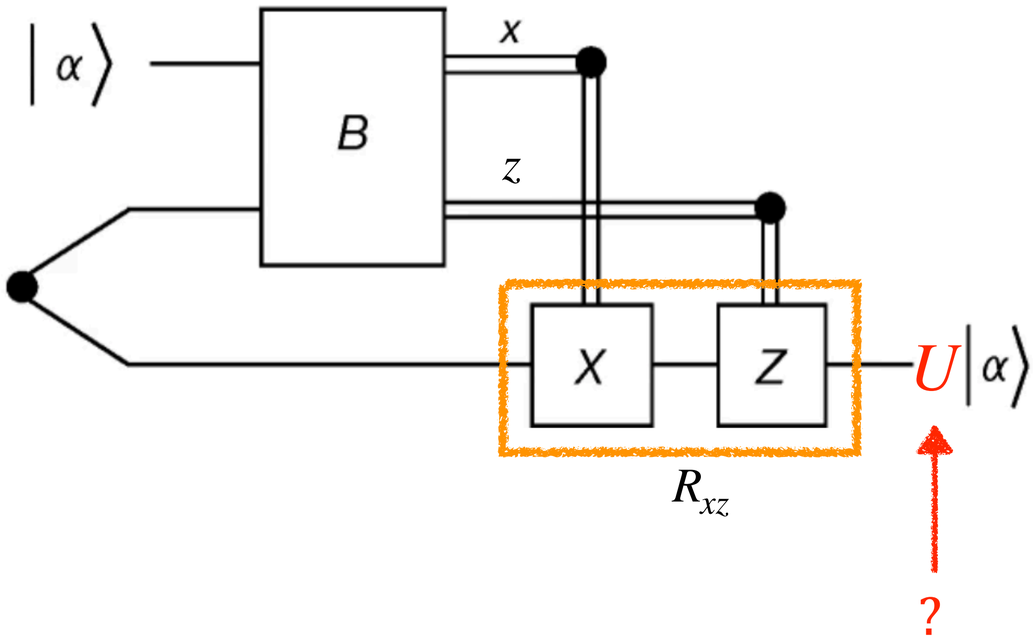


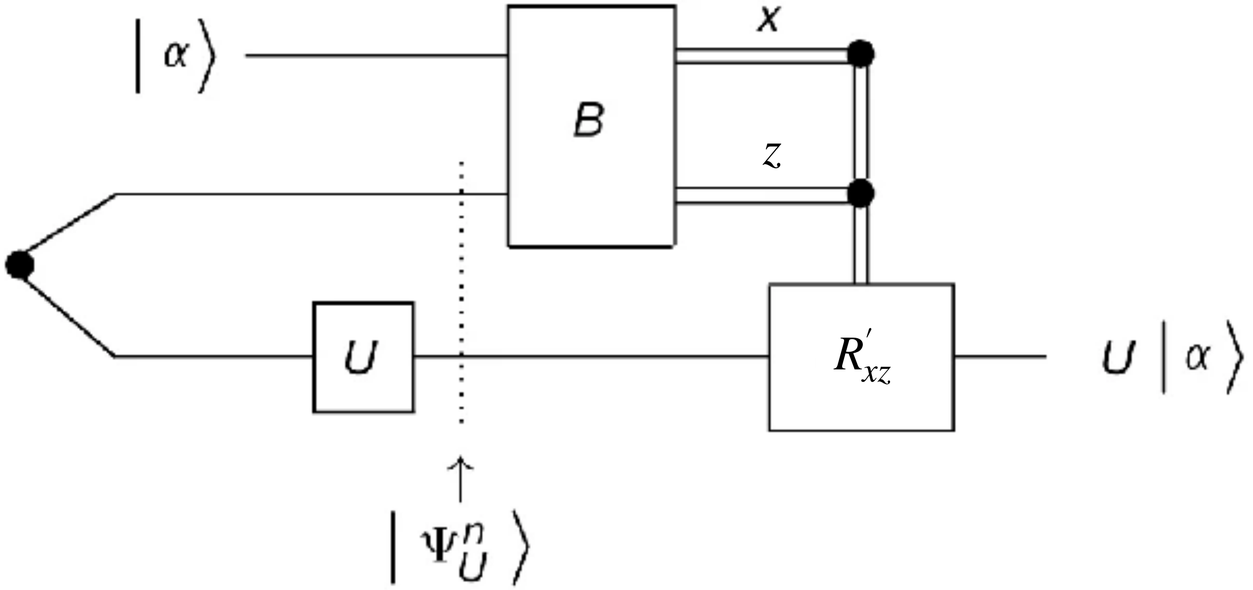
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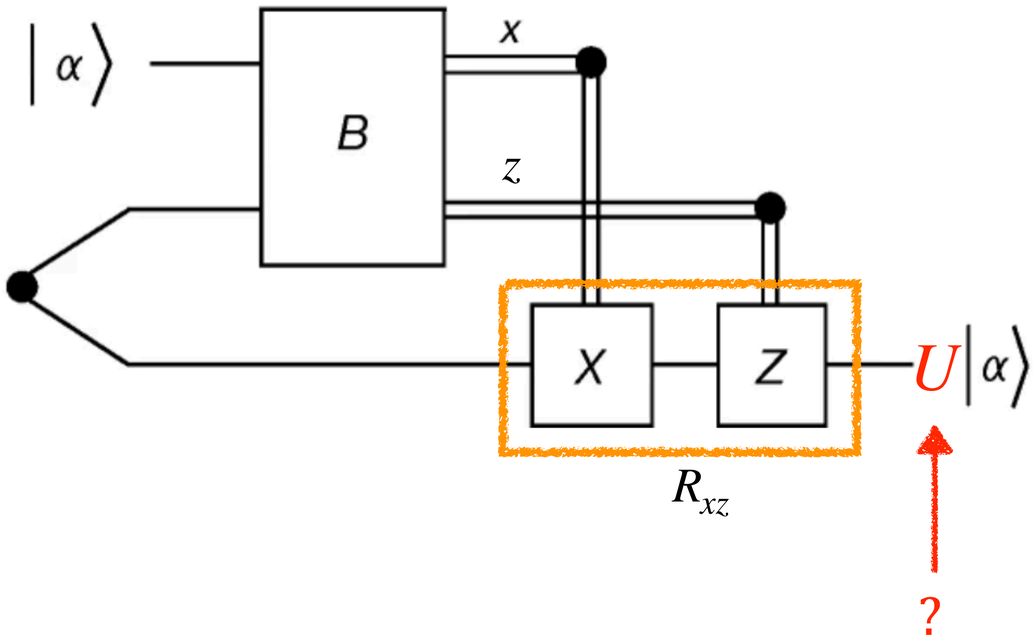
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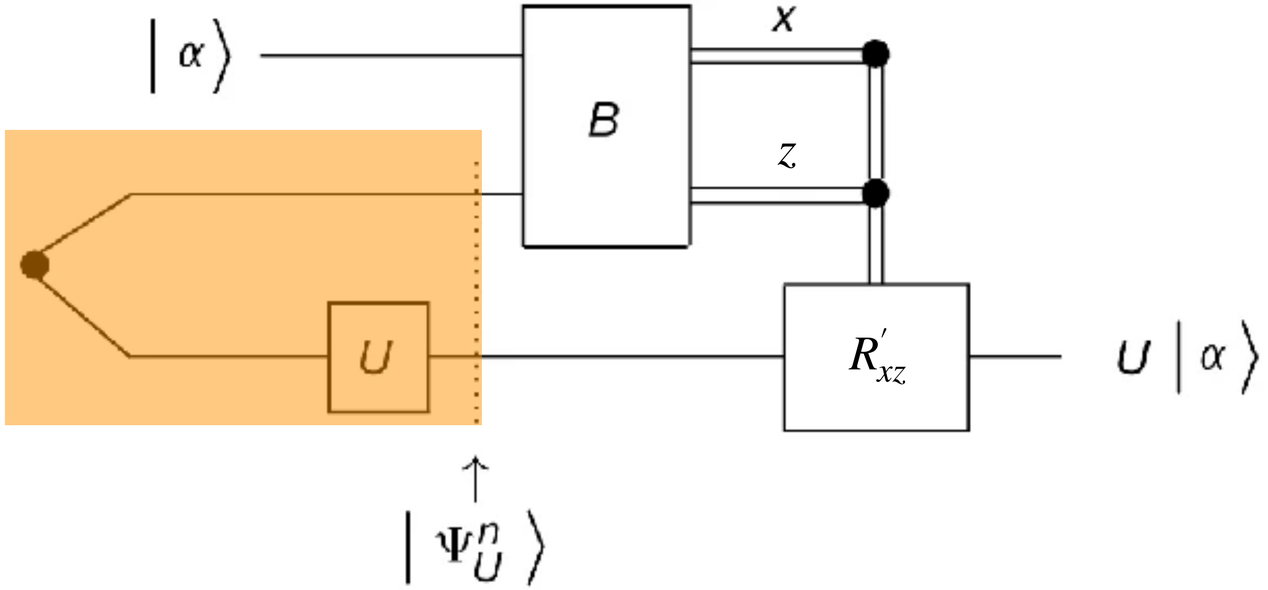




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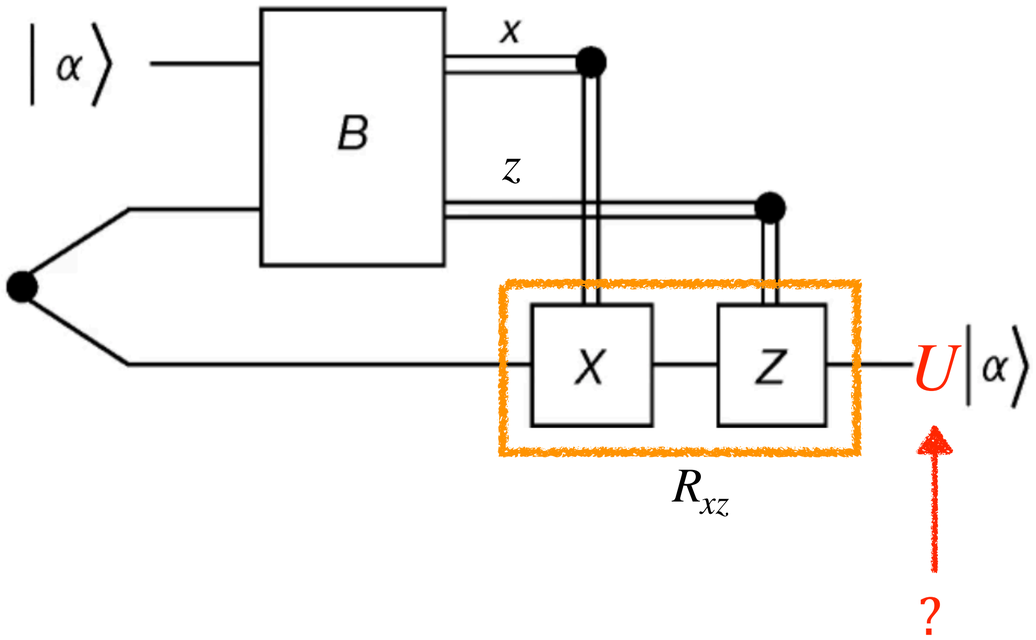
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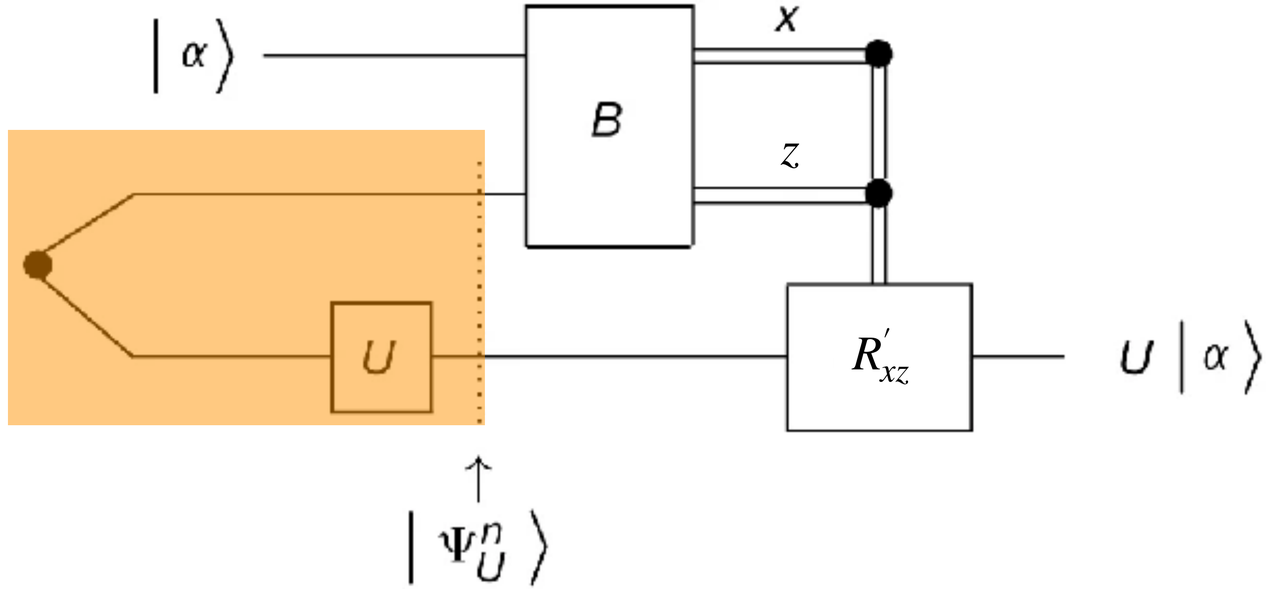




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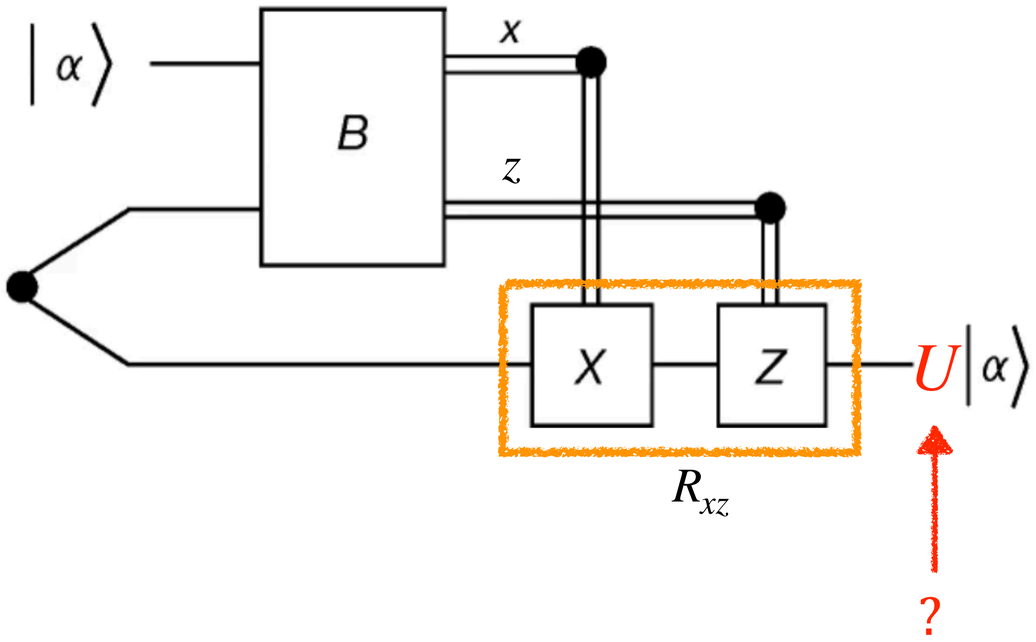


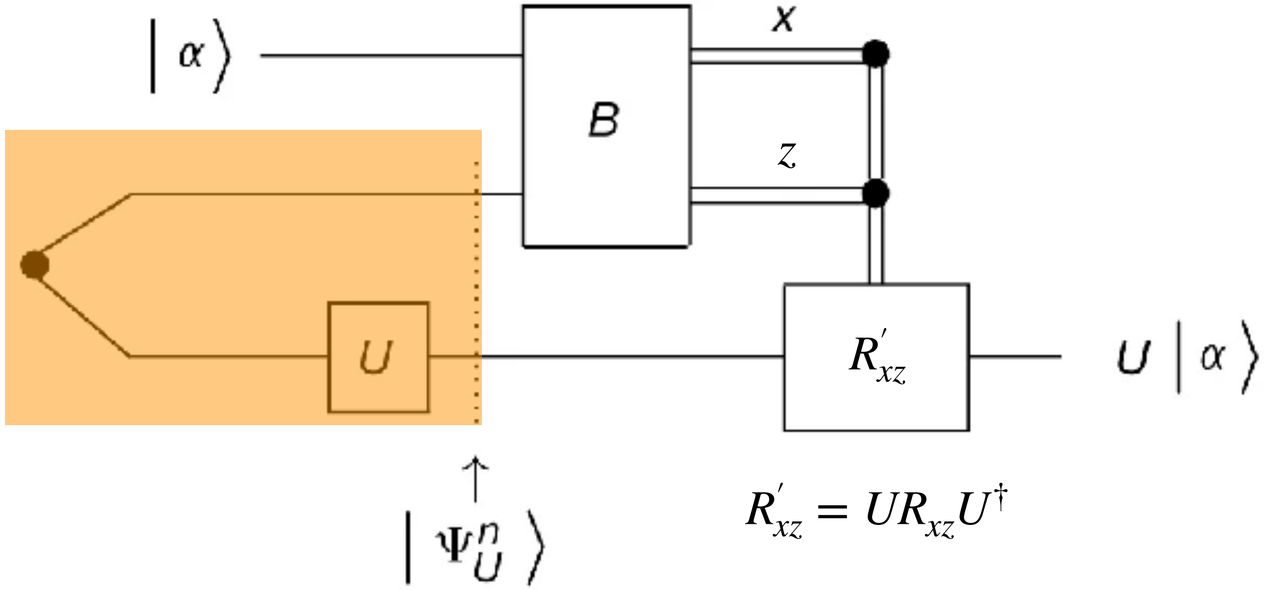


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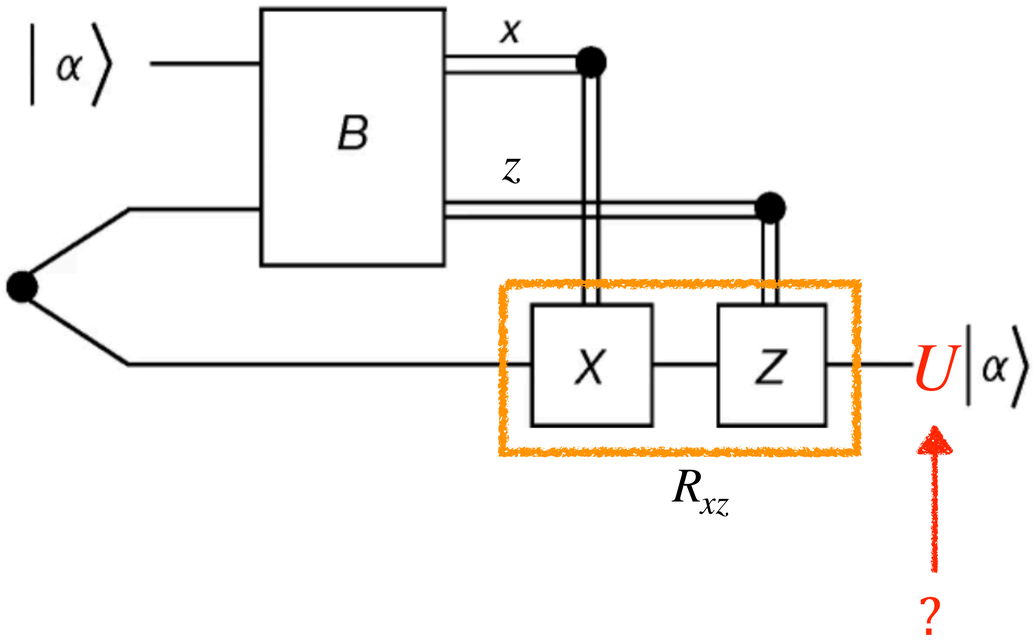


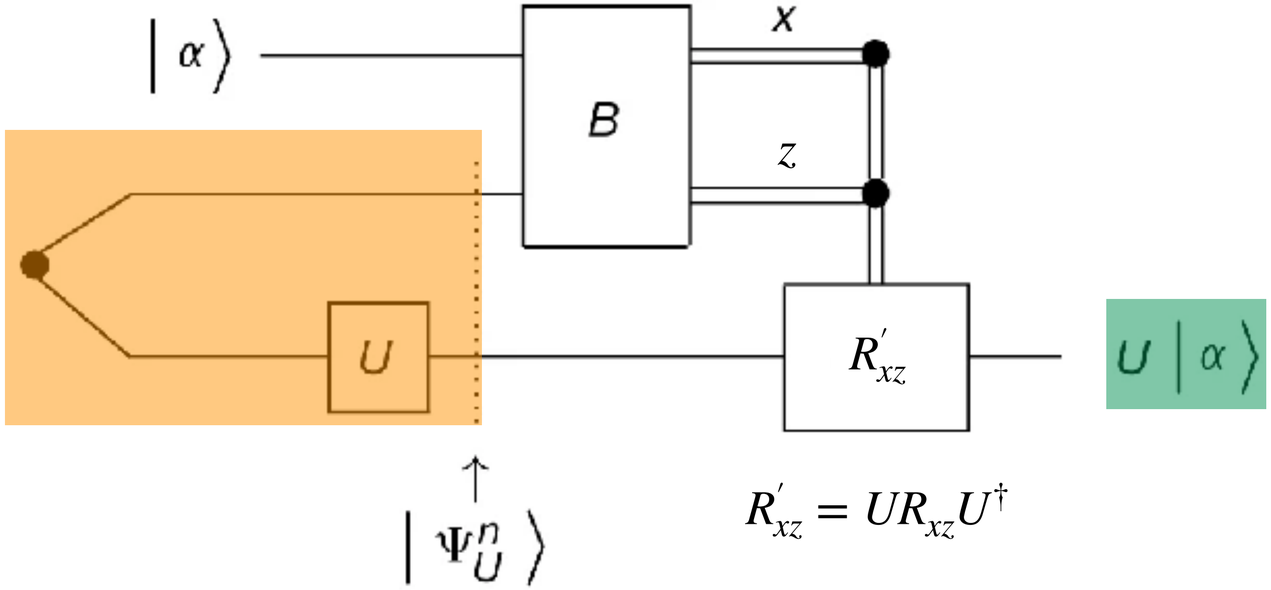


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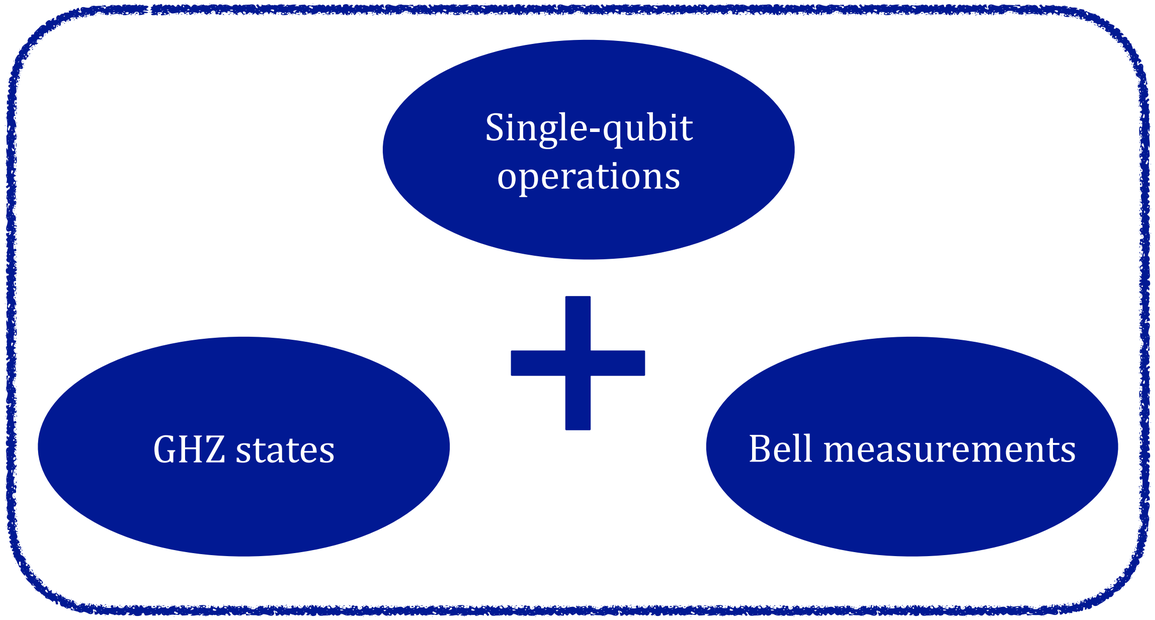
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Physics Letters A

Volume 308, Issues 2–3, 24 February 2003, Pages 96-100



Quantum computation by measurement and quantum memory

Michael A. Nielsen

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Abstract

What resources are universal for quantum computation? In the standard model of a quantum computer, a computation consists of a sequence of unitary gates acting coherently on the qubits making up the computer. This requirement for coherent unitary dynamical operations is widely believed to be the critical element of quantum computation. Here we show that a very different model involving only projective measurements and quantum memory is also universal for quantum computation. In particular, *no coherent unitary dynamics are involved in the computation.*

International Journal of Quantum Information | Vol. 02, No. 01, pp. 33-43 (2004)

QUANTUM COMPUTATION BY MEASUREMENTS

DEBBIE W. LEUNG

https://doi.org/10.1142/S0219749904000055 | **Cited by:** 84 **(Source: Crossref)**

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Abstract

We first consider various methods for the indirect implementation of unitary gates. We apply these methods to rederive the universality of 4-qubit measurements based on a scheme much simpler than Nielsen's original construction [quant-ph/0108020]. Then, we prove the universality of simple discrete sets of 2-qubit measurements, again using a scheme simplifying the initial construction [quant-ph/0111122]. Finally, we show how to use a single 4-qubit measurement to achieve universal quantum computation, and outline a proof for the universality of almost all maximally entangling 4-qubit measurements.

International Journal of Quantum Information | Vol. 03, No. 01, pp. 219-223 (2005) | Special issue: Proceedings...

STATE TRANSFER INSTEAD OF TELEPORTATION IN MEASUREMENT-BASED QUANTUM COMPUTATION

SIMON PERDRIX

https://doi.org/10.1142/S0219749905000785 | **Cited by:** 23 **(Source: Crossref)**

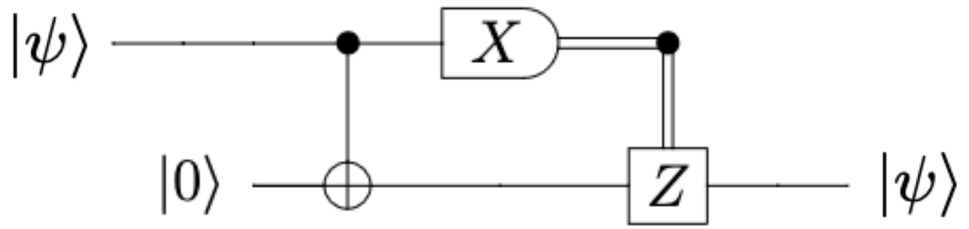
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Abstract

Quantum measurement is universal for quantum computation. The model of quantum computation introduced by Nielsen and further developed by Leung relies on a generalized form of teleportation. In order to simulate any n-qubit unitary transformation with this model, four auxiliary qubits are required. Moreover Leung exhibited a universal family of observables composed of one one-qubit measurement and four two-qubit measurements. We introduce a model of quantum computation via measurements only, relying on *state transfer*: state transfer only retains the part of teleportation which is necessary for computation. In order to simulate any n-qubit unitary transformation with this new model, only one auxiliary qubit is required. Moreover we exhibit a universal family of observables composed of three one-qubit measurements and only one two-qubit measurement. This model improves those of Nielsen and Leung in terms of both the number of auxiliary qubits and the number of two-qubit measurements required for quantum universality. In both cases, the minimal amounts of necessary resources are now reached: one auxiliary qubit (because measurement is destructive) and one two-qubit measurement (for creating entanglement).

1-bit Z-teleportation:



This will be useful later…

[Zhou, Leung, and Chuang, arXiv:quant-ph/0002039 (2000).]

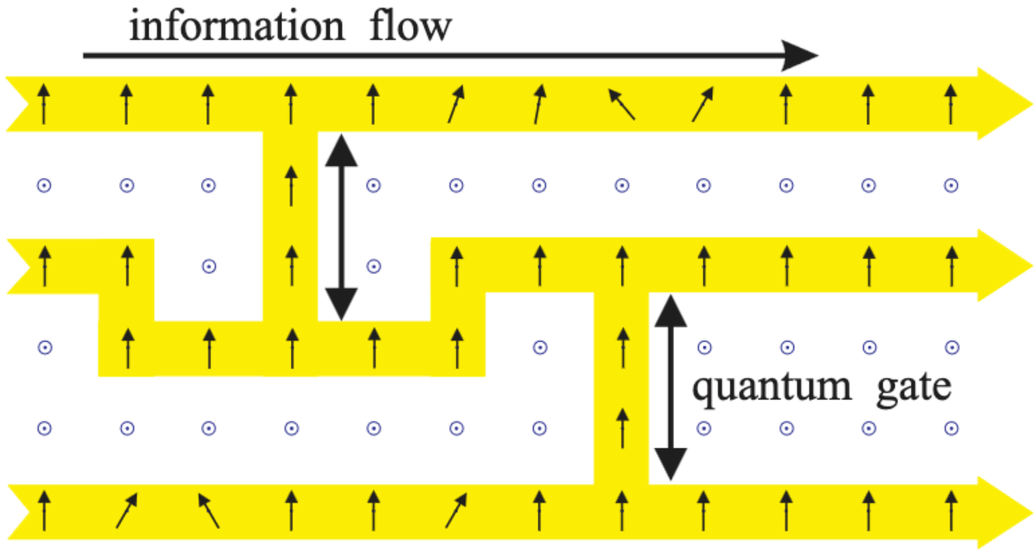
VOLUME 86, NUMBER 22 PHYSICAL REVIEW LETTERS 28 MAY 2001

A One-Way Quantum Computer

Robert Raussendorf and Hans J. Briegel
Theoretische Physik, Ludwig-Maximilians-Universität München, Germany
(Received 25 October 2000)

We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

DOI: 10.1103/PhysRevLett.86.5188 PACS numbers: 03.67.Lx, 03.65.Ud



[Raussendorf and Briegel, Phys. Rev. Lett. **86**, 5188 (2001)]

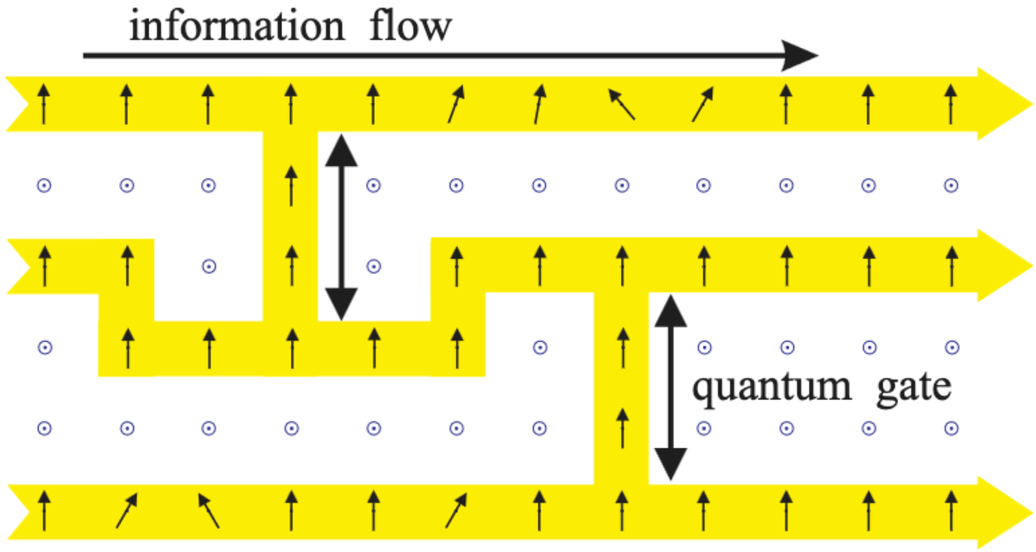
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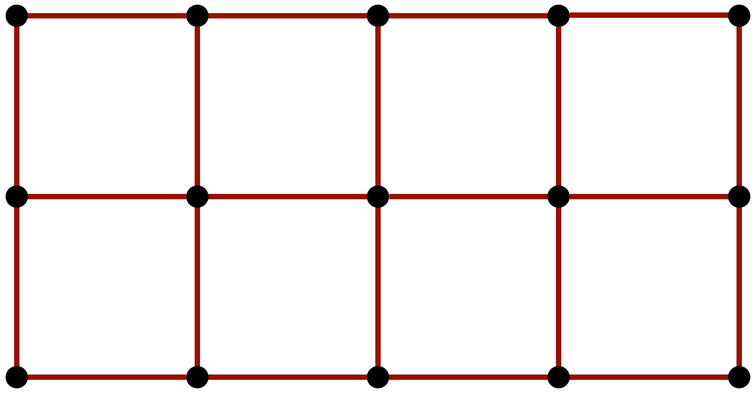
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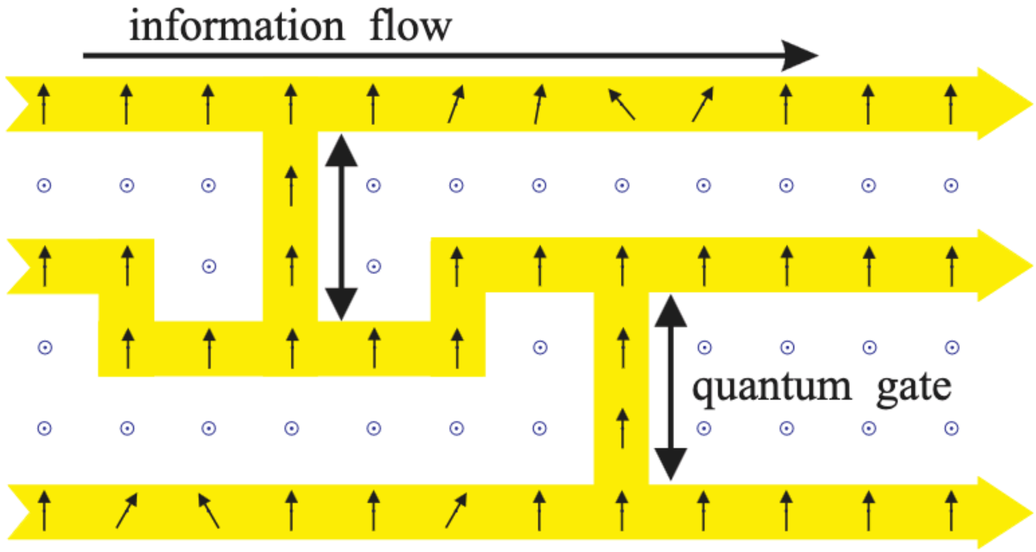
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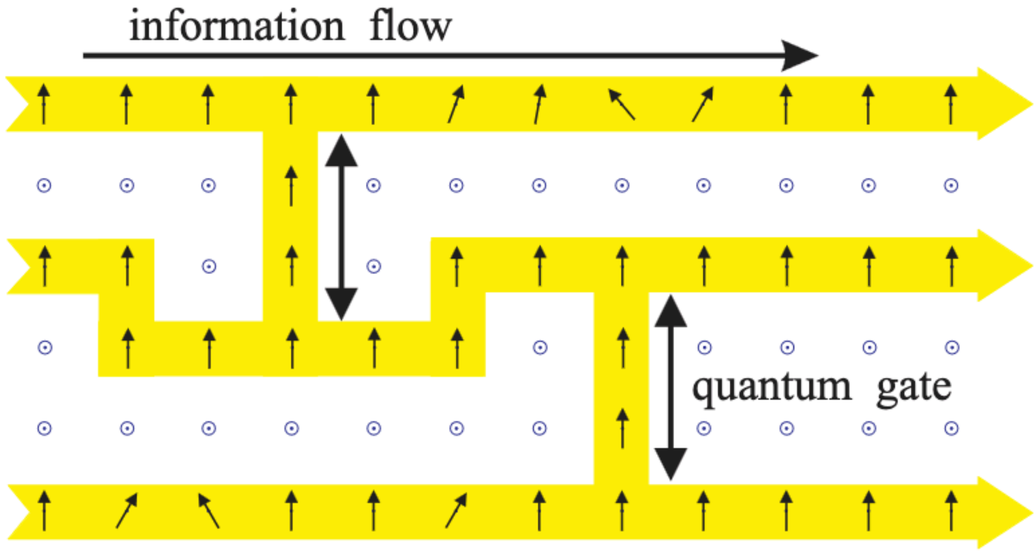
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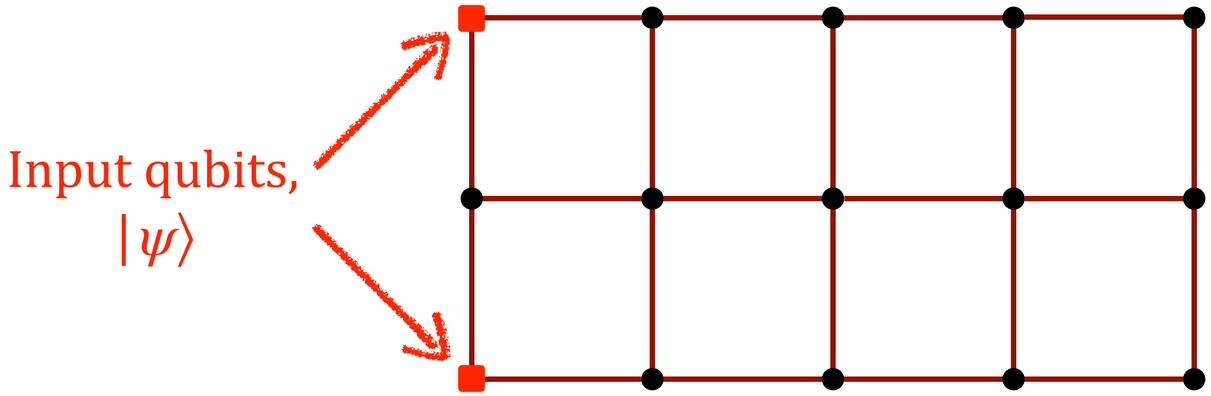
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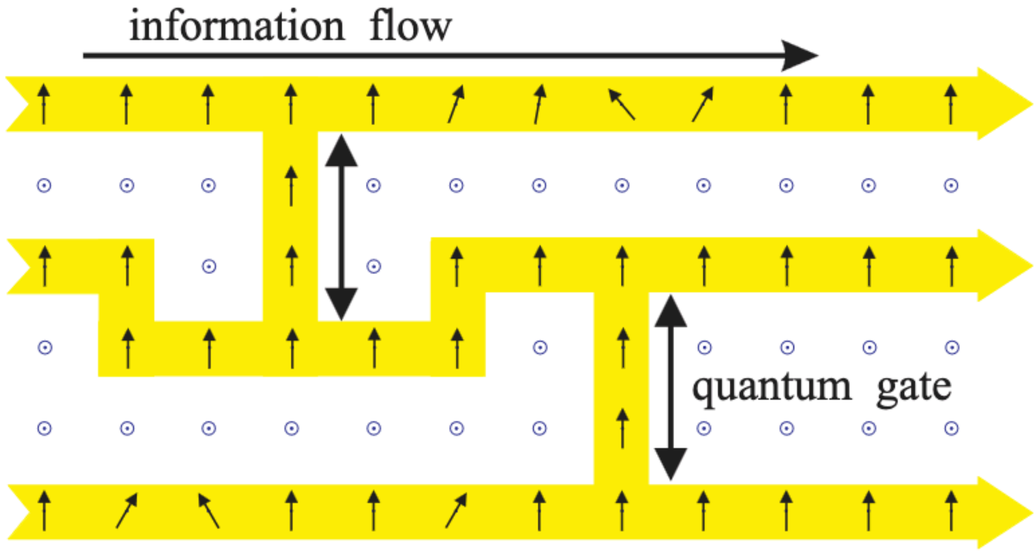
VOLUME 86, NUMBER 22 PHYSICAL REVIEW LETTERS 28 MAY 2001

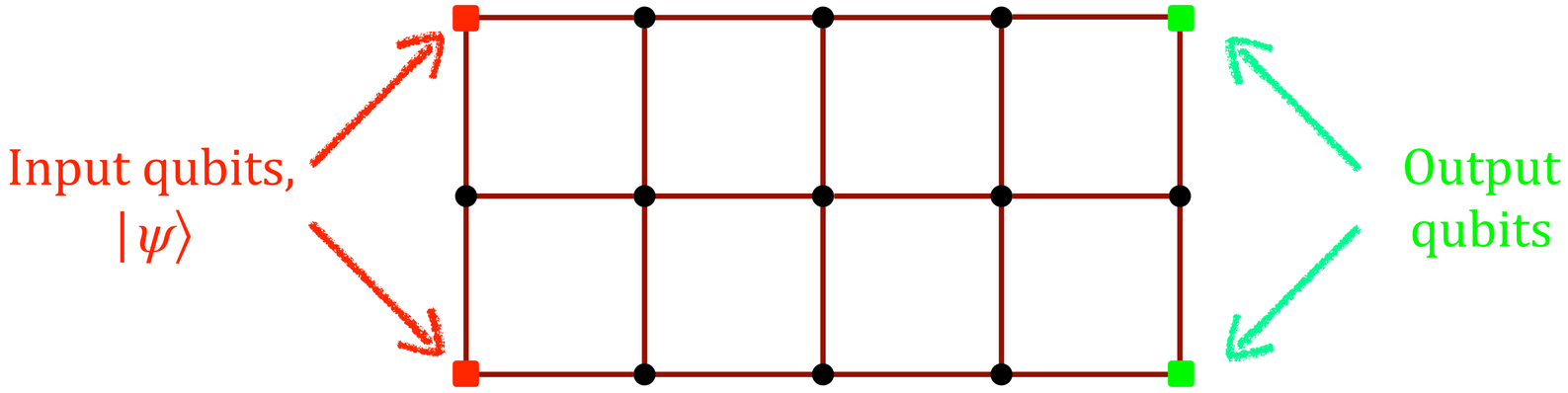
A One-Way Quantum Computer

Robert Raussendorf and Hans J. Briegel
Theoretische Physik, Ludwig-Maximilians-Universität München, Germany
(Received 25 October 2000)

We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

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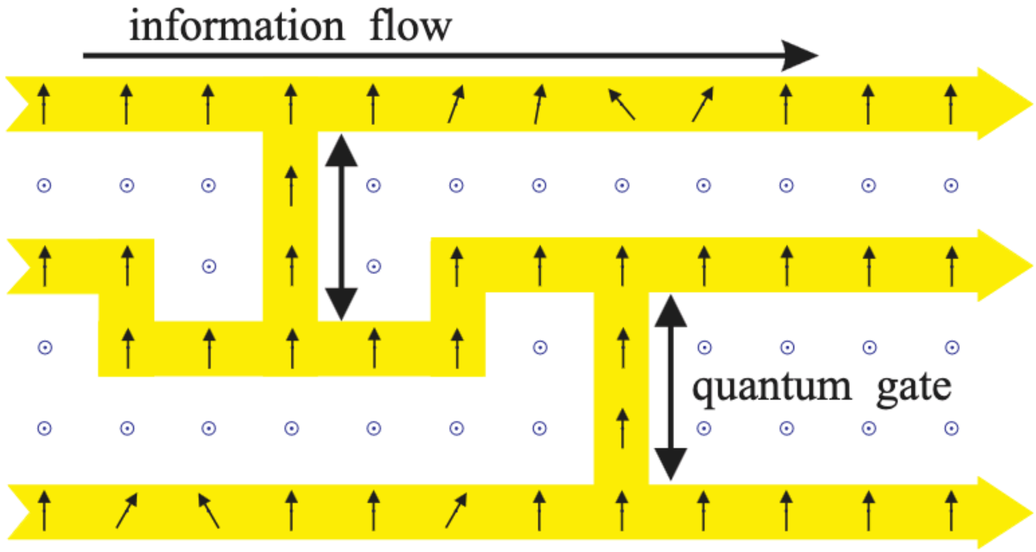
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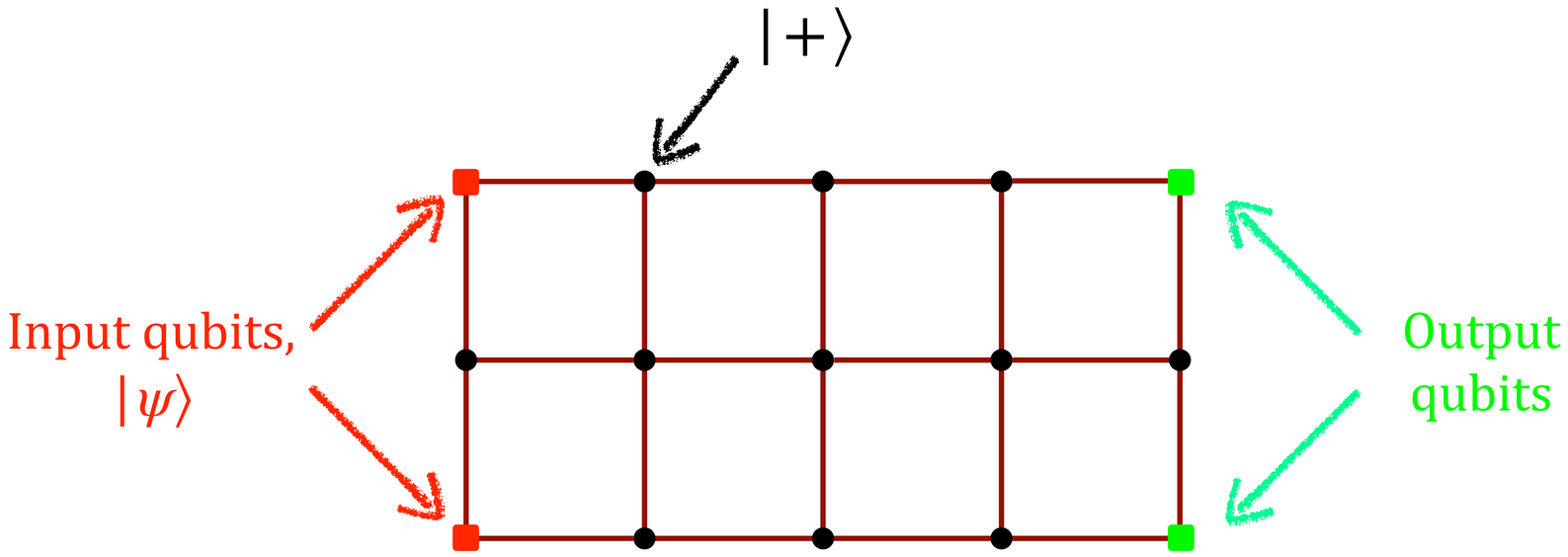
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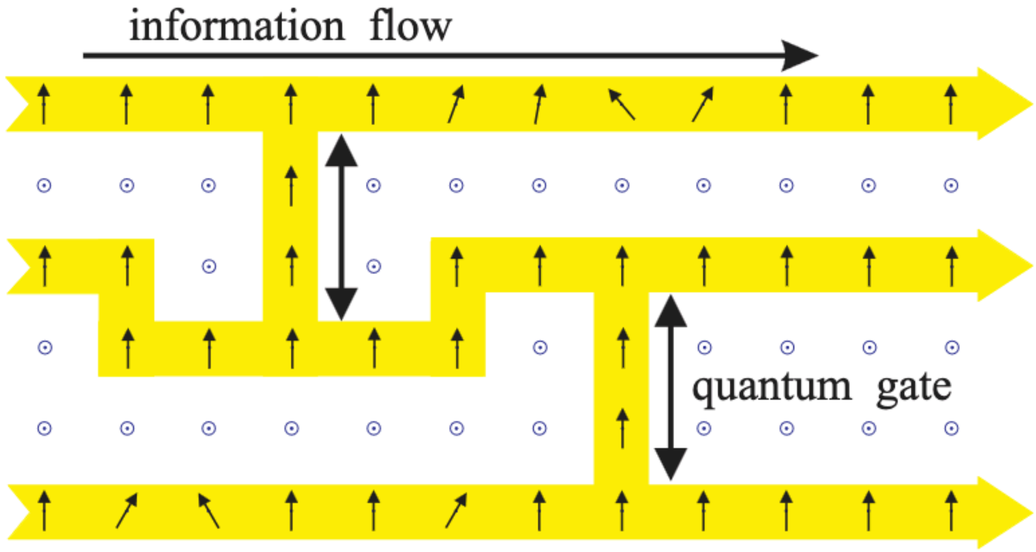
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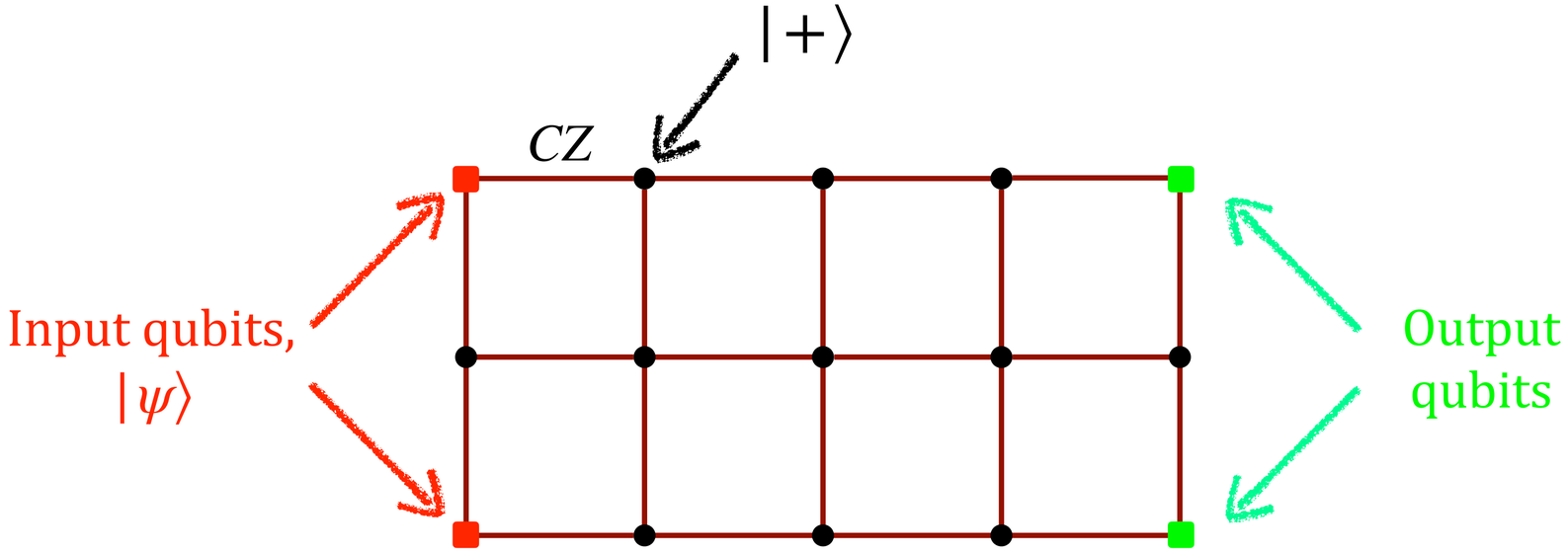
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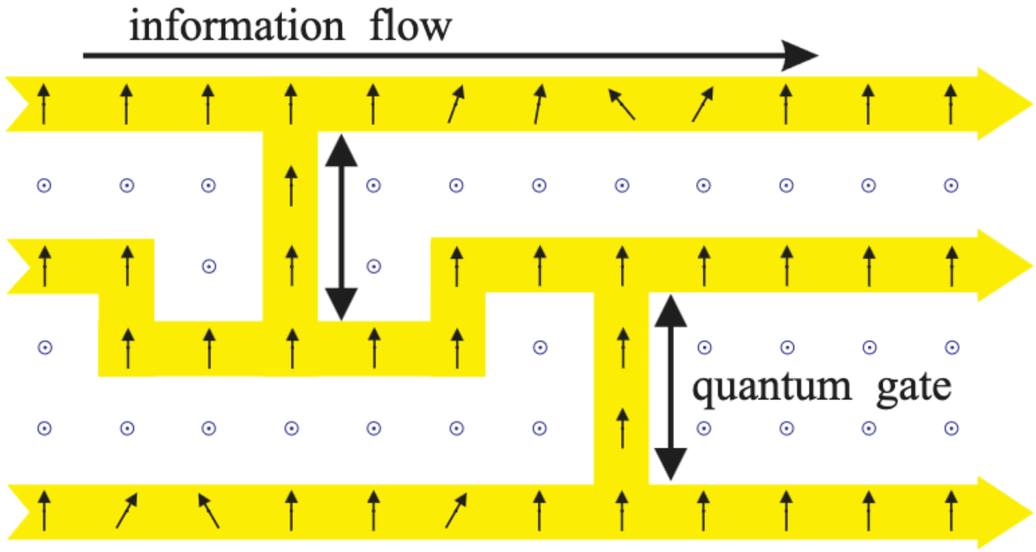
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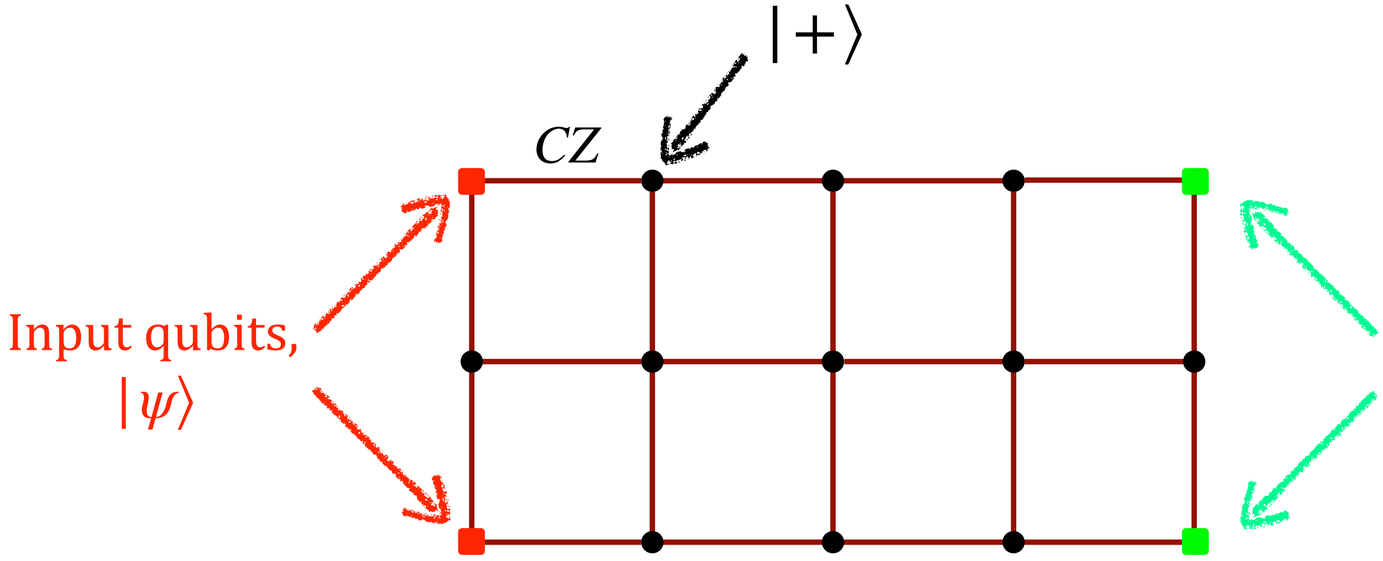
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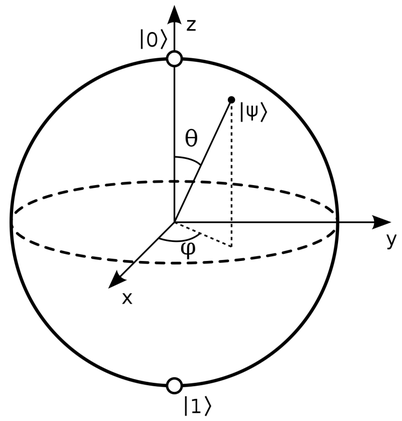
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Measurements

- in Z direction
- in X direction
- in X-Y plane

[Raussendorf and Briegel, Phys. Rev. Lett. **86**, 5188 (2001)]

But, but…

… measurements are non-deterministic!

But, but…

… measurements are non-deterministic!

Can I still compute deterministically?

Of course!

And we already saw a hint of how…

Clifford gates

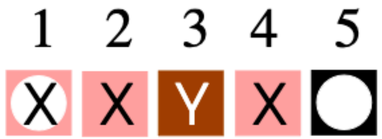
Clifford gates



Hadamard-gate

Clifford gates

Hadamard-gate

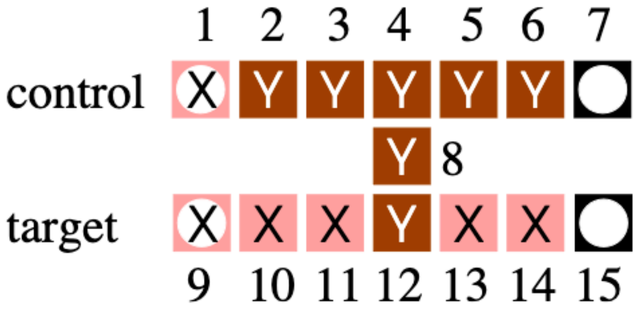


$\pi/2$-phase gate

Clifford gates

Hadamard-gate

$\pi/2$-phase gate



CNOT-gate

Clifford gates

Hadamard-gate

$\pi/2$-phase gate

CNOT-gate

Clifford gates

Other gates

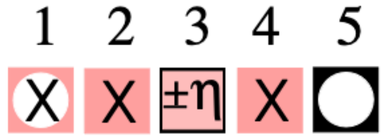
Clifford gates

Hadamard-gate

$\pi/2$-phase gate

CNOT-gate

Other gates



z-rotation

Clifford gates

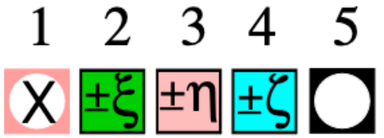
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Other gates

z-rotation



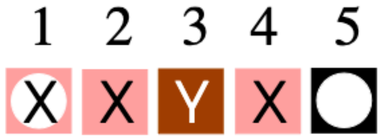
general rotation

[Raussendorf, Browne, and Briegel, Phys. Rev. A **68**, 022312 (2003)]

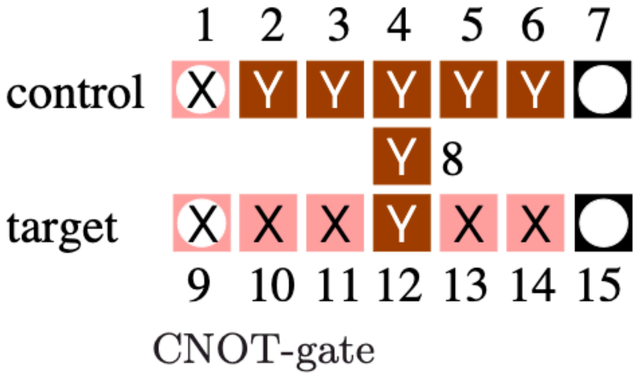
Clifford gates



Hadamard-gate

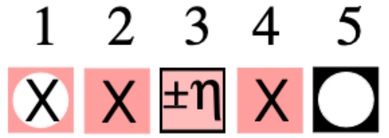


$\pi/2$-phase gate

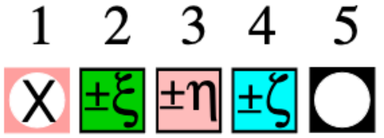


CNOT-gate

Other gates



z-rotation



general rotation

Exercise

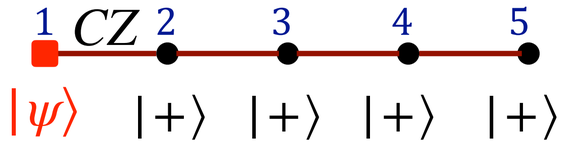


Hadamard-gate

Exercise

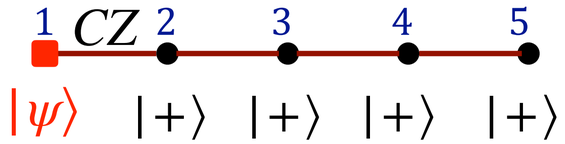


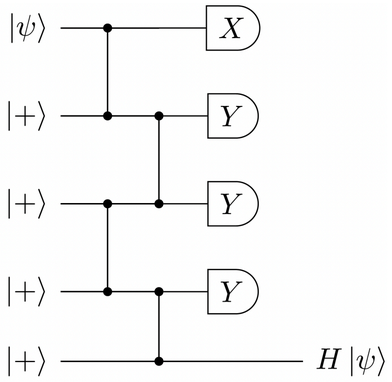
Hadamard-gate



Exercise

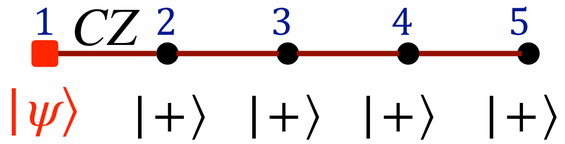


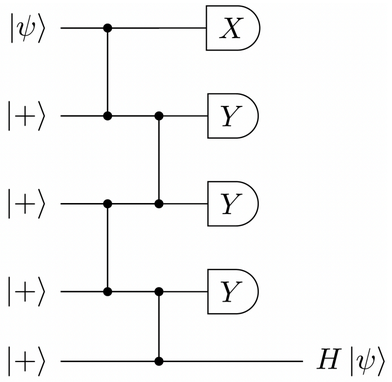




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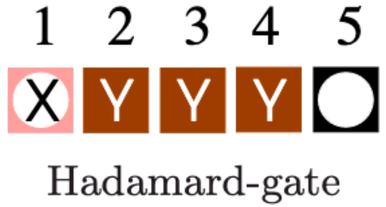


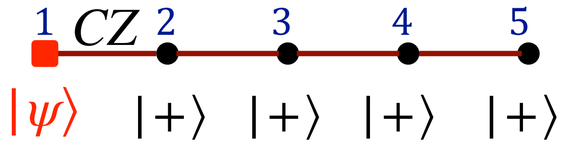


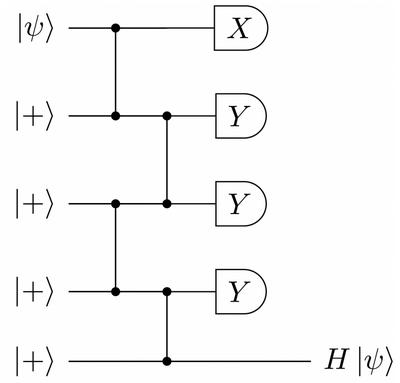


$$X_2 \longrightarrow$$
$$X_3 \longrightarrow$$
$$X_4 \longrightarrow$$
$$X_5 \longrightarrow$$
$$\bar{X} : X_1 \longrightarrow$$
$$\bar{Z} : Z_1 \longrightarrow$$

Exercise







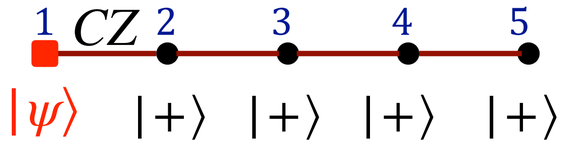
CZs

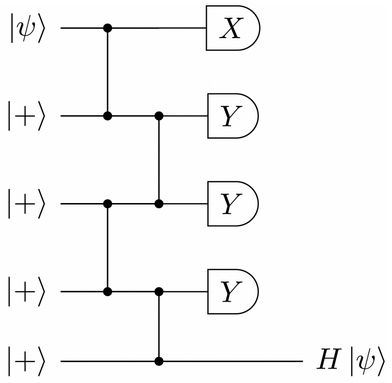
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Exercise



Hadamard-gate





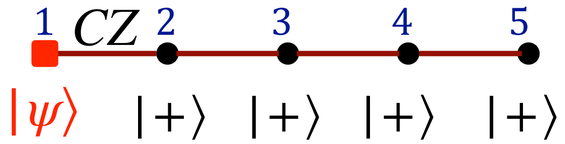
CZs

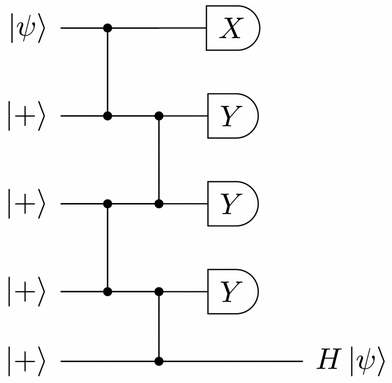
$$X_2 \longrightarrow Z_1X_2Z_3 \longrightarrow$$
$$X_3 \longrightarrow Z_2X_3Z_4 \longrightarrow$$
$$X_4 \longrightarrow Z_3X_4Z_5 \longrightarrow$$
$$X_5 \longrightarrow Z_4X_5 \longrightarrow$$
$$\bar{X} : X_1 \longrightarrow X_1Z_2 \longrightarrow$$
$$\bar{Z} : Z_1 \longrightarrow Z_1 \longrightarrow$$

Exercise



Hadamard-gate



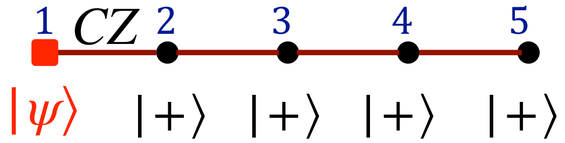


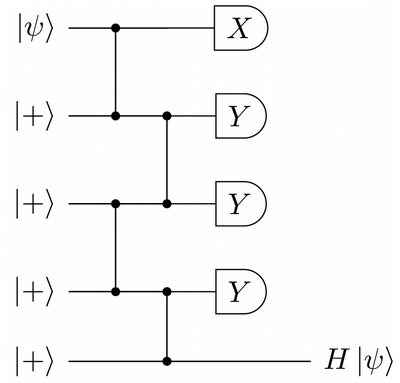
$$
\begin{aligned}
& \xrightarrow{CZs} \xrightarrow{\Pi_{X_1}} \\
X_2 &\longrightarrow Z_1X_2Z_3 \longrightarrow \\
X_3 &\longrightarrow Z_2X_3Z_4 \longrightarrow \\
X_4 &\longrightarrow Z_3X_4Z_5 \longrightarrow \\
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\bar{Z} : Z_1 &\longrightarrow Z_1 \longrightarrow
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Exercise



Hadamard-gate

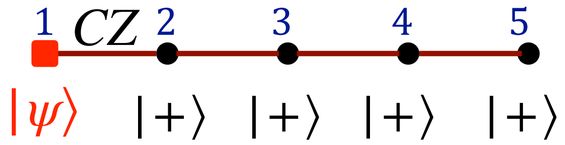


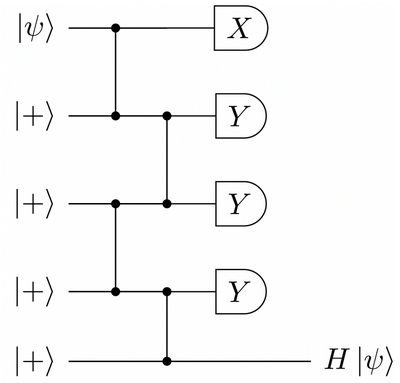


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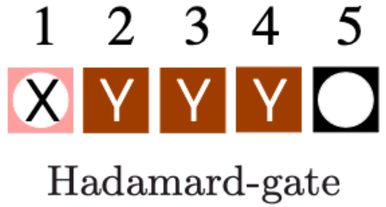


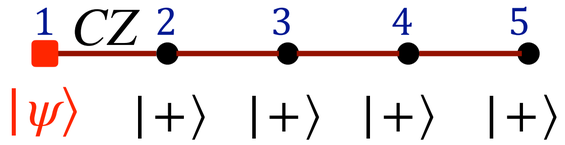


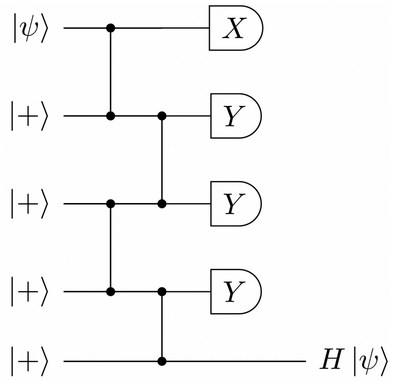


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Exercise

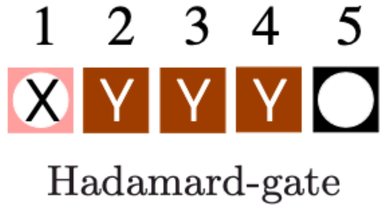


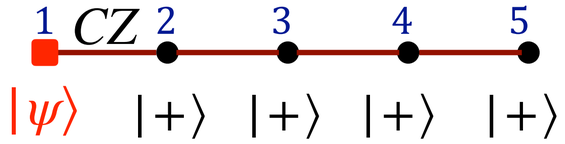


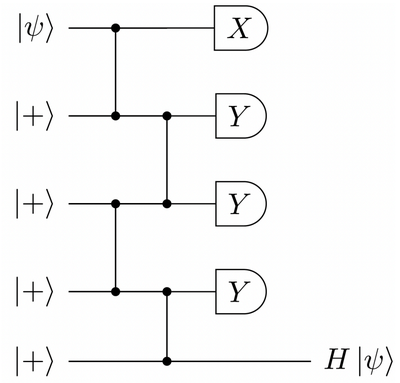


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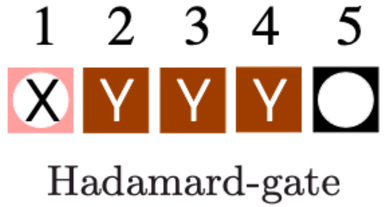


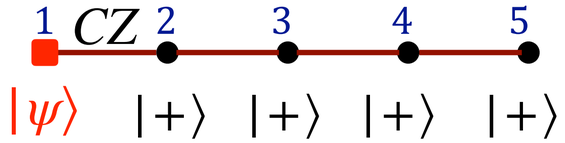


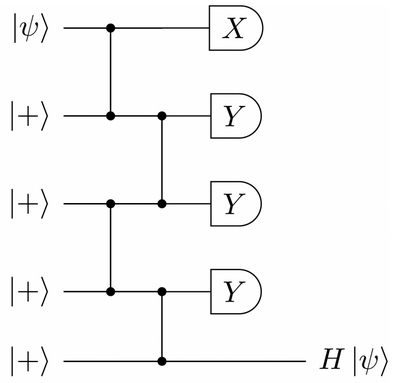


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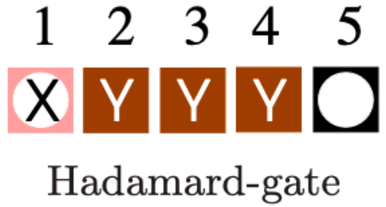


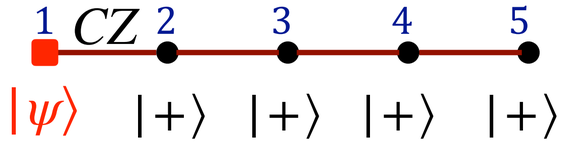


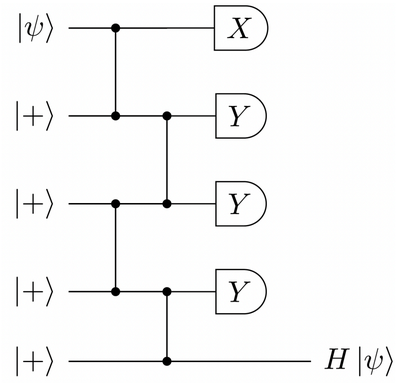


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\begin{aligned}
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X_2 &\longrightarrow \boxed{Z_1X_2Z_3} \longrightarrow (-1)^{m_1}X_1 \longrightarrow \\
X_3 &\longrightarrow Z_2X_3Z_4 \longrightarrow Z_2X_3Z_4 \longrightarrow \\
X_4 &\longrightarrow Z_3X_4Z_5 \longrightarrow Z_3X_4Z_5 \longrightarrow \\
X_5 &\longrightarrow Z_4X_5 \longrightarrow Z_4X_5 \longrightarrow \\
\bar{X} : X_1 &\longrightarrow X_1Z_2 \longrightarrow \\
\bar{Z} : Z_1 &\longrightarrow Z_1 \longrightarrow
\end{aligned}
$$

Exercise





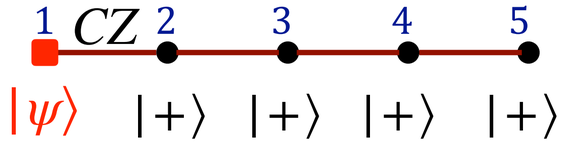


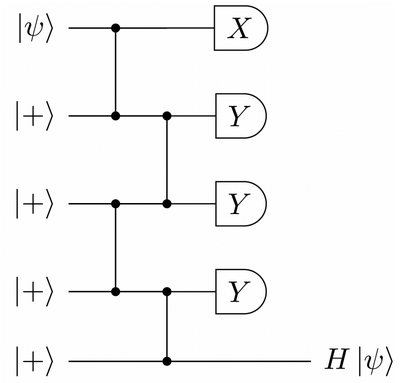
$$
\begin{array}{lccccc}
 & \xrightarrow{CZs} & & \xrightarrow{\Pi_{X_1}} & & \\
X_2 & \longrightarrow & \boxed{Z_1X_2Z_3} & \longrightarrow & (-1)^{m_1}X_1 & \longrightarrow \\
X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & & \\
\bar{Z}: Z_1 & \longrightarrow & Z_1 & \longrightarrow & &
\end{array}
$$

Exercise



Hadamard-gate



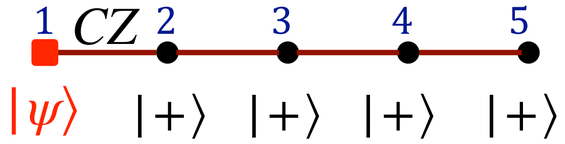


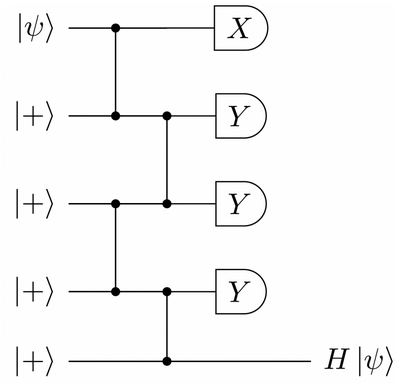
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X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow \\
\bar{X}: X_1 & \longrightarrow & X_1Z_2 & \longrightarrow & (-1)^{m_1}Z_2 & \longrightarrow \\
\bar{Z}: Z_1 & \longrightarrow & Z_1 & \longrightarrow & &
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Exercise



Hadamard-gate



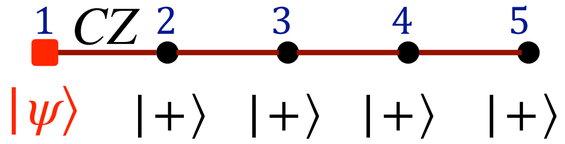


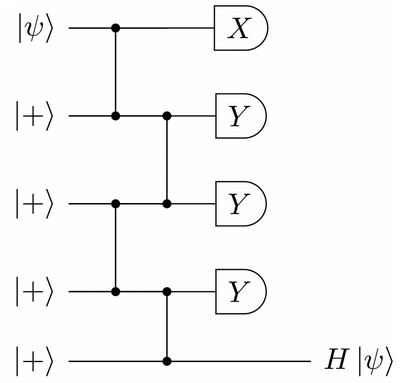
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X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 \longrightarrow \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 \longrightarrow \\
\bar{X}: X_1 & \longrightarrow & X_1Z_2 & \longrightarrow & (-1)^{m_1}Z_2 \longrightarrow \\
\bar{Z}: Z_1 & \longrightarrow & Z_1 & \longrightarrow &
\end{array}
$$

Exercise



Hadamard-gate



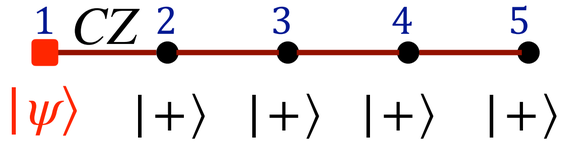


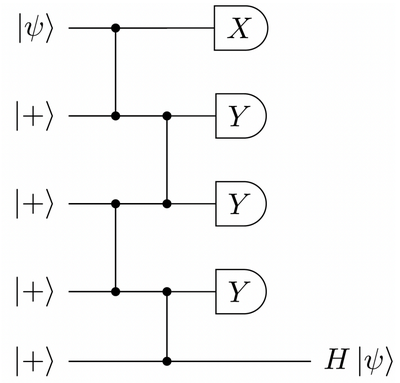
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X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow \\
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\bar{Z}: Z_1 & \longrightarrow & Z_1 & \longrightarrow & X_2Z_3 & \longrightarrow
\end{array}
$$

Exercise



Hadamard-gate



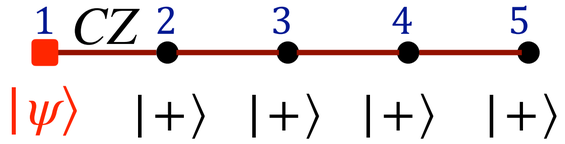


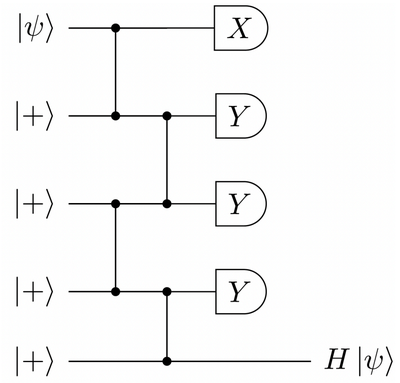
$$
\begin{array}{lccccc}
 & & CZs & & \Pi_{X_1} & & \Pi_{Y_2} \\
X_2 & \longrightarrow & \boxed{Z_1X_2Z_3} & \longrightarrow & (-1)^{m_1}X_1 & \longrightarrow & \\
X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & (-1)^{m_1}Z_2 & \longrightarrow & \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & X_2Z_3 & \longrightarrow &
\end{array}
$$

Exercise



Hadamard-gate



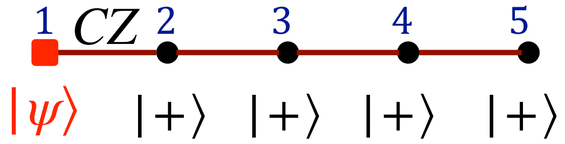


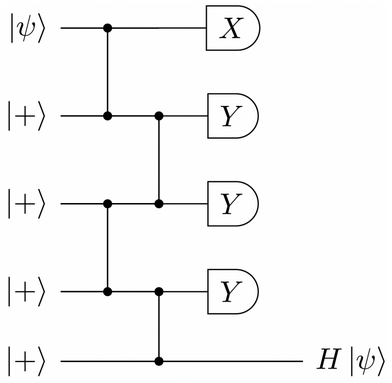
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X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & & \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & & \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & & \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & (-1)^{m_1}Z_2 & \longrightarrow & & \\
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\end{array}
$$

Exercise



Hadamard-gate



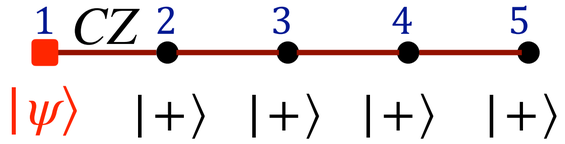


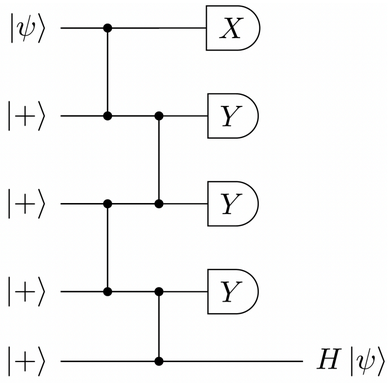
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X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & & \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & & \\
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Exercise



Hadamard-gate



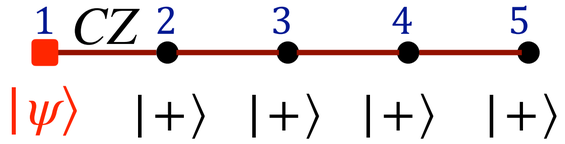


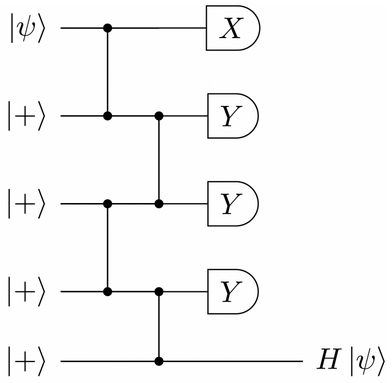
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X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & \boxed{Z_2X_3Z_4} & \longrightarrow & (-1)^{m_2}Y_2 & \longrightarrow \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & & \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & & \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & (-1)^{m_1}Z_2 & \longrightarrow & & \\
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$$

Exercise



Hadamard-gate



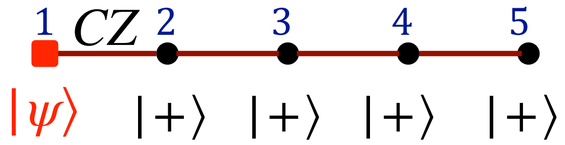


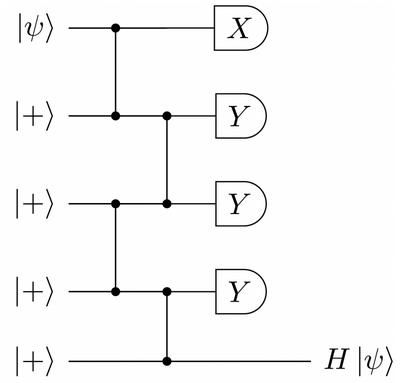
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X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & \boxed{Z_2X_3Z_4} & \longrightarrow & (-1)^{m_2}Y_2 & \longrightarrow \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & & \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & (-1)^{m_1}Z_2 & \longrightarrow & & \\
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$$

Exercise



Hadamard-gate



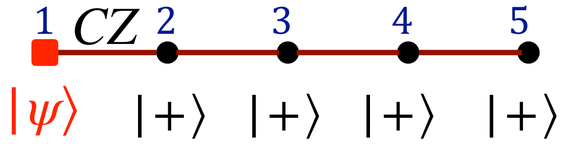


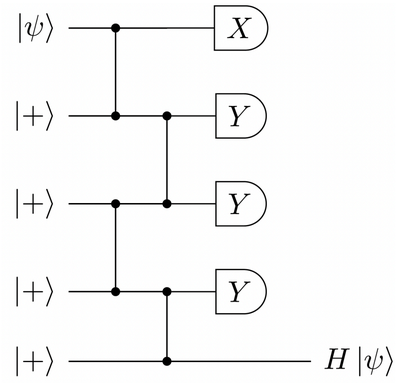
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X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & \boxed{Z_2X_3Z_4} & \longrightarrow & (-1)^{m_2}Y_2 & \longrightarrow \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & (-1)^{m_1}Z_2 & \longrightarrow & & \\
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Exercise



Hadamard-gate



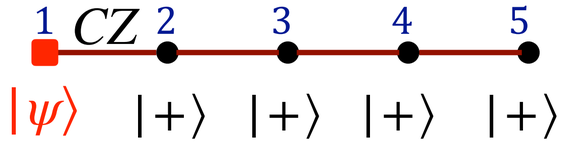


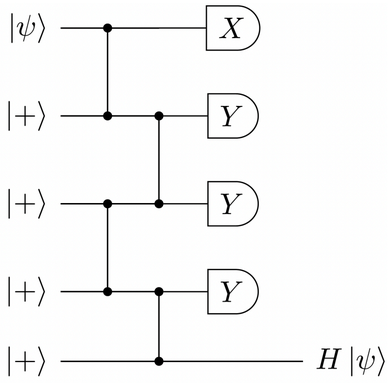
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X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & (-1)^{m_2}Y_2 & \longrightarrow \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow \\
\bar{X}: X_1 & \longrightarrow & X_1Z_2 & \longrightarrow & (-1)^{m_1}Z_2 & \longrightarrow & & \\
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Exercise



Hadamard-gate



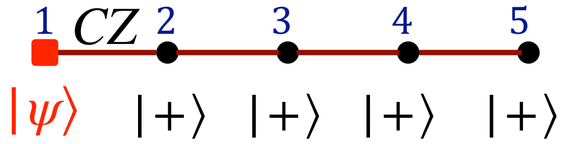


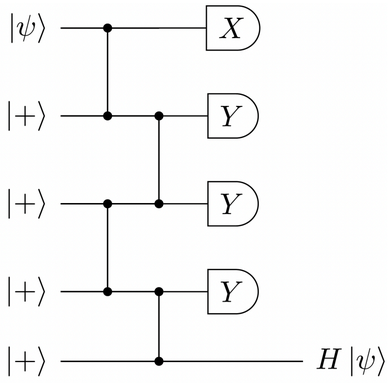
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X_3 & \longrightarrow Z_2X_3Z_4 & \longrightarrow & \boxed{Z_2X_3Z_4} & \longrightarrow & (-1)^{m_2}Y_2 \longrightarrow \\
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X_5 & \longrightarrow Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 \longrightarrow \\
\bar{X}: X_1 & \longrightarrow \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & (-1)^{m_1}X_3Z_4 \longrightarrow \\
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Exercise



Hadamard-gate



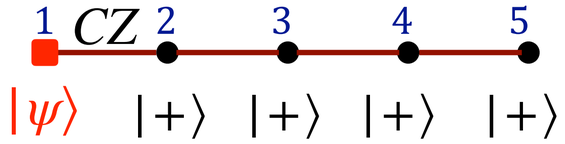


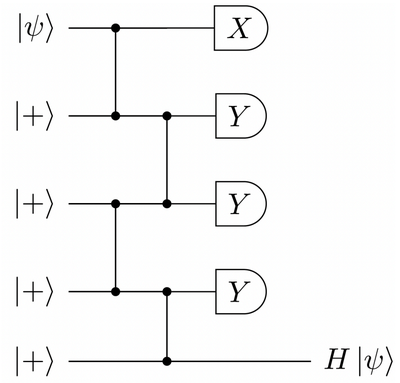
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X_5 & \longrightarrow Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 \longrightarrow \\
\bar{X}: X_1 & \longrightarrow \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & (-1)^{m_1}X_3Z_4 \longrightarrow \\
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\end{array}
$$

Exercise



Hadamard-gate



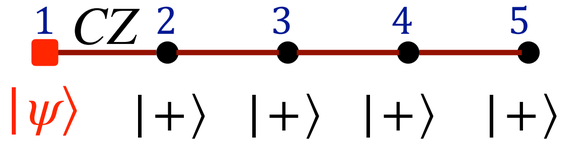


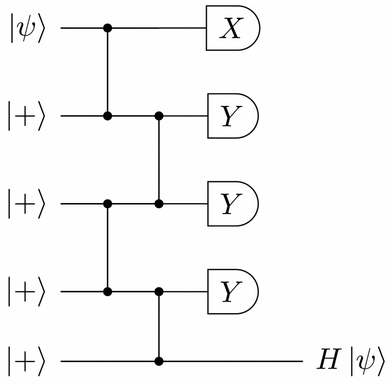
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\end{array}
$$

Exercise



Hadamard-gate



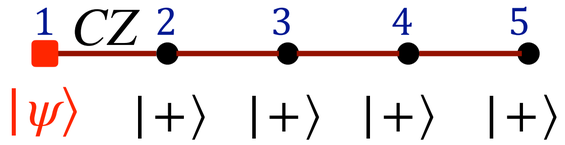


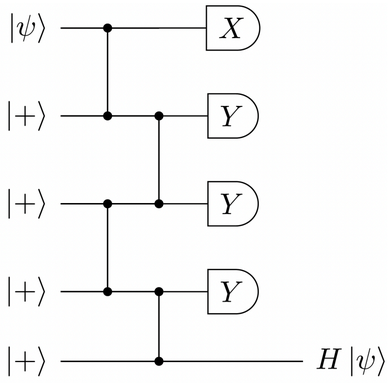
$$
\begin{array}{lccccc}
 & \textit{CZs} & & \Pi_{X_1} & & \Pi_{Y_2} & & \Pi_{Y_3} \\
X_2 & \longrightarrow & \boxed{Z_1X_2Z_3} & \longrightarrow & (-1)^{m_1}X_1 & \longrightarrow & (-1)^{m_1}X_1 & \longrightarrow \\
X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & \boxed{Z_2X_3Z_4} & \longrightarrow & (-1)^{m_2}Y_2 & \longrightarrow \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & (-1)^{m_1}X_3Z_4 & \longrightarrow \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & \boxed{X_2Z_3} & \longrightarrow & (-1)^{m_2}Y_3Z_4 & \longrightarrow
\end{array}
$$

Exercise



Hadamard-gate



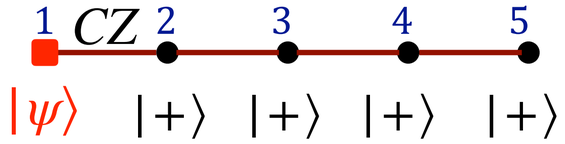


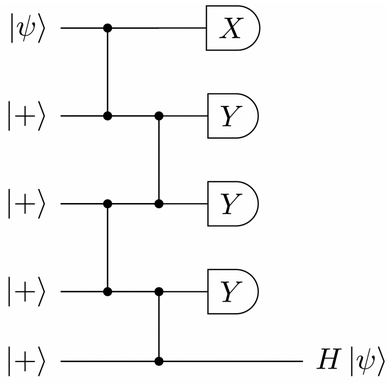
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\begin{array}{lccccc}
 & \xrightarrow{CZs} & & \xrightarrow{\Pi_{X_1}} & & \xrightarrow{\Pi_{Y_2}} & & \xrightarrow{\Pi_{Y_3}} \\
X_2 & \longrightarrow & \boxed{Z_1X_2Z_3} & \longrightarrow & (-1)^{m_1}X_1 & \longrightarrow & (-1)^{m_1}X_1 & \longrightarrow & (-1)^{m_1}X_1 \\
X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & \boxed{Z_2X_3Z_4} & \longrightarrow & (-1)^{m_2}Y_2 & \longrightarrow & (-1)^{m_2}Y_2 \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & (-1)^{m_1}X_3Z_4 & \longrightarrow & \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & \boxed{X_2Z_3} & \longrightarrow & (-1)^{m_2}Y_3Z_4 & \longrightarrow &
\end{array}
$$

Exercise



Hadamard-gate



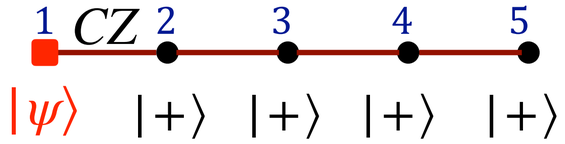


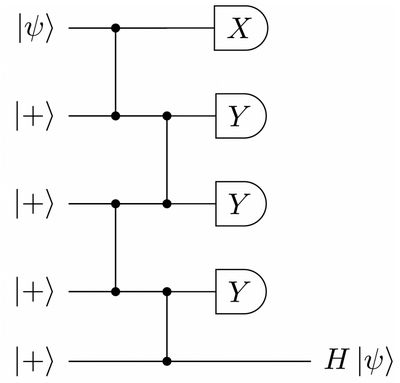
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X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & \boxed{Z_3X_4Z_5} & \longrightarrow & \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & (-1)^{m_1}X_3Z_4 & \longrightarrow & \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & \boxed{X_2Z_3} & \longrightarrow & (-1)^{m_2}Y_3Z_4 & \longrightarrow &
\end{array}
$$

Exercise



Hadamard-gate



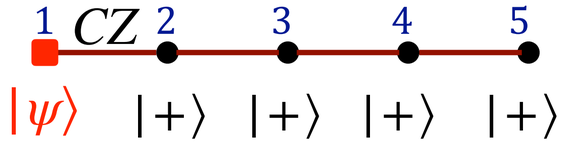


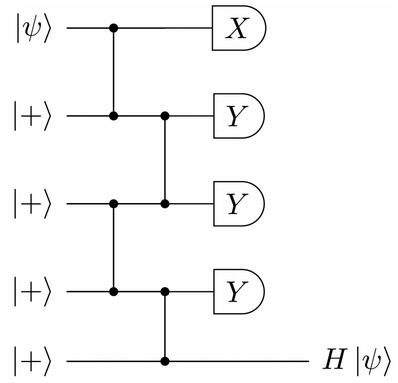
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X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & (-1)^{m_1}X_3Z_4 & \longrightarrow \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & \boxed{X_2Z_3} & \longrightarrow & (-1)^{m_2}Y_3Z_4 & \longrightarrow
\end{array}
$$

Exercise



Hadamard-gate



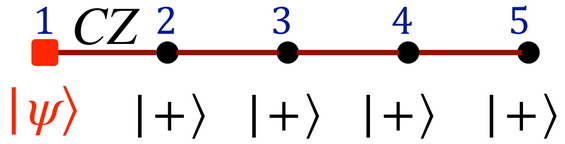


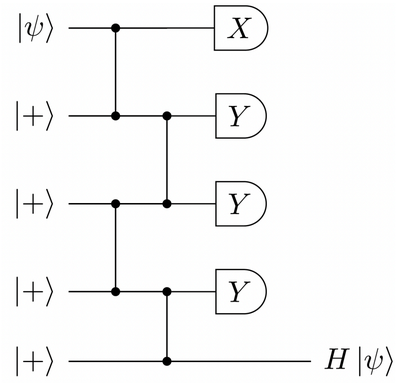
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X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow Z_4X_5 \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & (-1)^{m_1}X_3Z_4 & \longrightarrow \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & \boxed{X_2Z_3} & \longrightarrow & (-1)^{m_2}Y_3Z_4 & \longrightarrow
\end{array}
$$

Exercise



Hadamard-gate



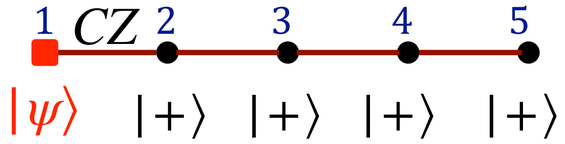


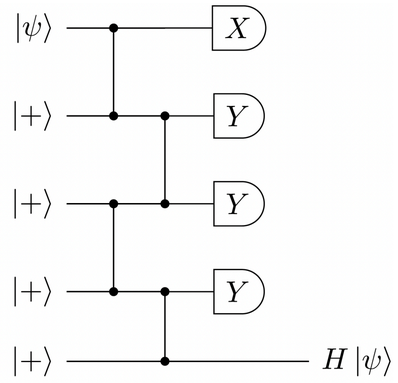
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X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & \boxed{(-1)^{m_1}X_3Z_4} & \longrightarrow & \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & \boxed{X_2Z_3} & \longrightarrow & (-1)^{m_2}Y_3Z_4 & \longrightarrow &
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$$

Exercise



Hadamard-gate



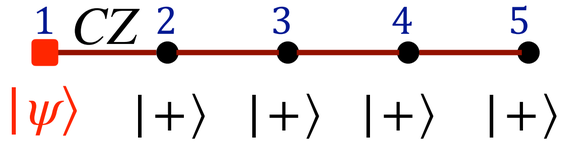


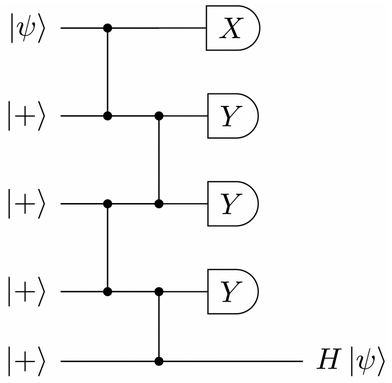
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X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & \boxed{Z_3X_4Z_5} & \longrightarrow & (-1)^{m_3}Y_3 \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & \boxed{(-1)^{m_1}X_3Z_4} & \longrightarrow & (-1)^{m_1+m_3}Y_4Z_5 \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & \boxed{X_2Z_3} & \longrightarrow & (-1)^{m_2}Y_3Z_4 & \longrightarrow &
\end{array}
$$

Exercise



Hadamard-gate



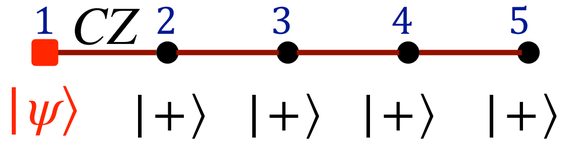


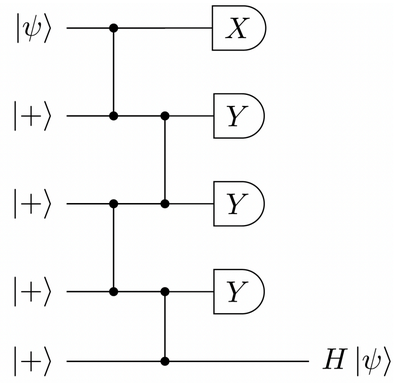
$$
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X_2 & \longrightarrow & Z_1X_2Z_3 & \longrightarrow & (-1)^{m_1}X_1 & \longrightarrow & (-1)^{m_1}X_1 & \longrightarrow & (-1)^{m_1}X_1 \\
X_3 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & Z_2X_3Z_4 & \longrightarrow & (-1)^{m_2}Y_2 & \longrightarrow & (-1)^{m_2}Y_2 \\
X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & (-1)^{m_3}Y_3 \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 \\
\bar{X}: X_1 & \longrightarrow & X_1Z_2 & \longrightarrow & (-1)^{m_1}Z_2 & \longrightarrow & (-1)^{m_1}X_3Z_4 & \longrightarrow & (-1)^{m_1+m_3}Y_4Z_5 \\
\bar{Z}: Z_1 & \longrightarrow & Z_1 & \longrightarrow & X_2Z_3 & \longrightarrow & (-1)^{m_2}Y_3Z_4 & \longrightarrow &
\end{array}
$$

Exercise



Hadamard-gate



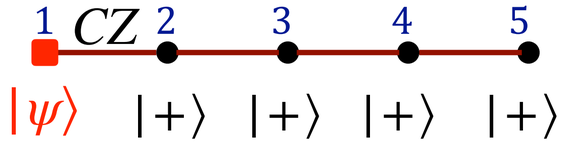


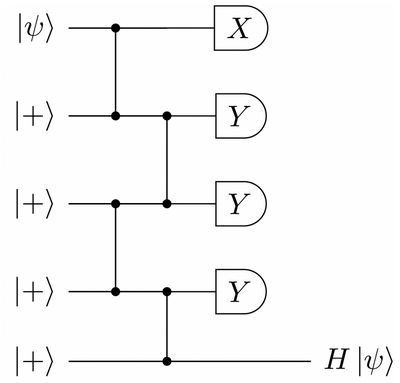
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\begin{array}{lccccc}
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X_4 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & Z_3X_4Z_5 & \longrightarrow & \boxed{Z_3X_4Z_5} & \longrightarrow & (-1)^{m_3}Y_3 \\
X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 & \longrightarrow & Z_4X_5 \\
\bar{X}: X_1 & \longrightarrow & \boxed{X_1Z_2} & \longrightarrow & \boxed{(-1)^{m_1}Z_2} & \longrightarrow & \boxed{(-1)^{m_1}X_3Z_4} & \longrightarrow & (-1)^{m_1+m_3}Y_4Z_5 \\
\bar{Z}: Z_1 & \longrightarrow & \boxed{Z_1} & \longrightarrow & \boxed{X_2Z_3} & \longrightarrow & \boxed{(-1)^{m_2}Y_3Z_4} & \longrightarrow & (-1)^{m_2+m_3}Z_4
\end{array}
$$

Exercise



Hadamard-gate



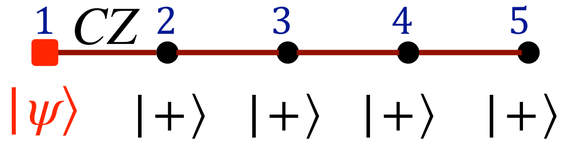


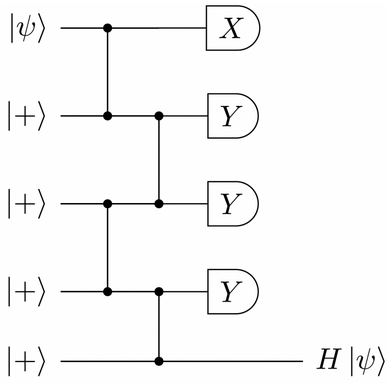
$$(-1)^{m_1} X_1 \longrightarrow$$
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$$(-1)^{m_3} Y_3 \longrightarrow$$
$$Z_4 X_5 \longrightarrow$$
$$\bar{X} : (-1)^{m_1+m_3} Y_4 Z_5 \longrightarrow$$
$$\bar{Z} : (-1)^{m_2+m_3} Z_4 \longrightarrow$$

Exercise



Hadamard-gate



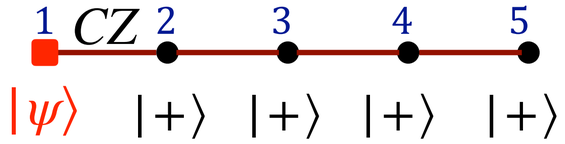


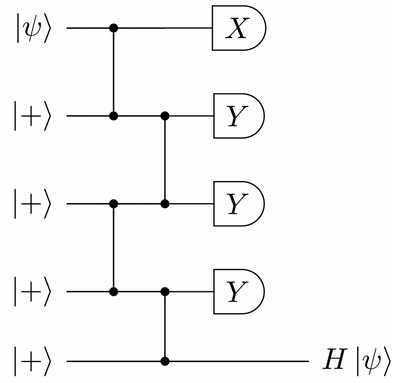
$$
\begin{aligned}
(-1)^{m_1} X_1 &\xrightarrow{\Pi_{Y_4}} \\
(-1)^{m_2} Y_2 &\longrightarrow \\
(-1)^{m_3} Y_3 &\longrightarrow \\
Z_4 X_5 &\longrightarrow \\
\bar{X} : (-1)^{m_1+m_3} Y_4 Z_5 &\longrightarrow \\
\bar{Z} : (-1)^{m_2+m_3} Z_4 &\longrightarrow
\end{aligned}
$$

Exercise



Hadamard-gate



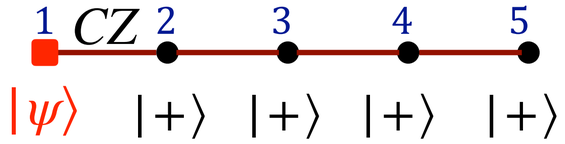


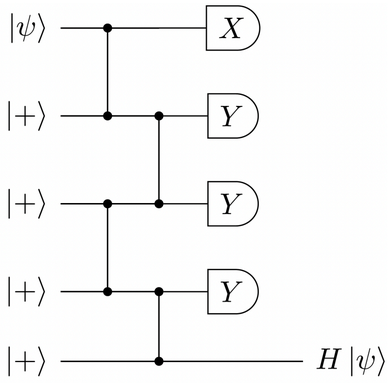
$$(-1)^{m_1}X_1 \xrightarrow{\Pi_{Y_4}} (-1)^{m_1}X_1$$
$$(-1)^{m_2}Y_2 \longrightarrow (-1)^{m_2}Y_2$$
$$(-1)^{m_3}Y_3 \longrightarrow (-1)^{m_3}Y_3$$
$$Z_4X_5 \longrightarrow$$
$$\bar{X} : (-1)^{m_1+m_3}Y_4Z_5 \longrightarrow$$
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Exercise



Hadamard-gate



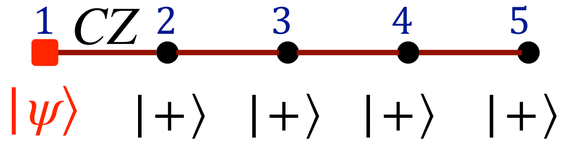


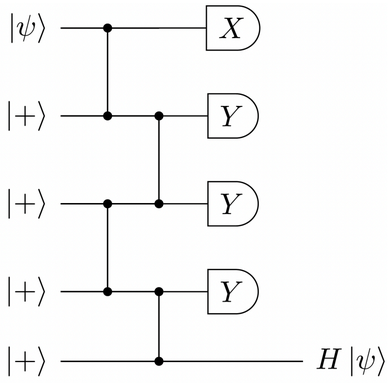
$$(-1)^{m_1}X_1 \xrightarrow{\Pi_{Y_4}} (-1)^{m_1}X_1$$
$$(-1)^{m_2}Y_2 \longrightarrow (-1)^{m_2}Y_2$$
$$(-1)^{m_3}Y_3 \longrightarrow (-1)^{m_3}Y_3$$
$$\boxed{Z_4X_5} \longrightarrow$$
$$\bar{X} : (-1)^{m_1+m_3}Y_4Z_5 \longrightarrow$$
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Exercise



Hadamard-gate



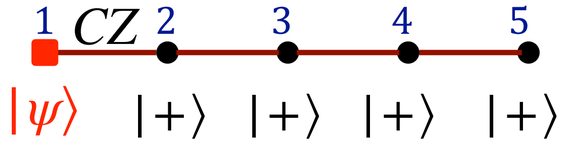


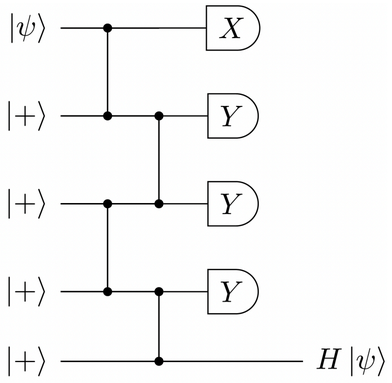
$$(-1)^{m_1}X_1 \xrightarrow{\Pi_{Y_4}} (-1)^{m_1}X_1$$
$$(-1)^{m_2}Y_2 \longrightarrow (-1)^{m_2}Y_2$$
$$(-1)^{m_3}Y_3 \longrightarrow (-1)^{m_3}Y_3$$
$$\boxed{Z_4X_5} \longrightarrow (-1)^{m_4}Y_4$$
$$\bar{X} : (-1)^{m_1+m_3}Y_4Z_5 \longrightarrow$$
$$\bar{Z} : (-1)^{m_2+m_3}Z_4 \longrightarrow$$

Exercise



Hadamard-gate



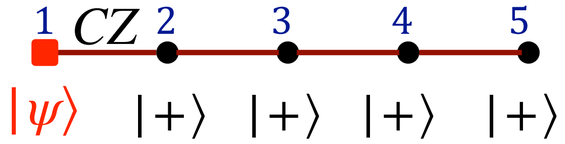


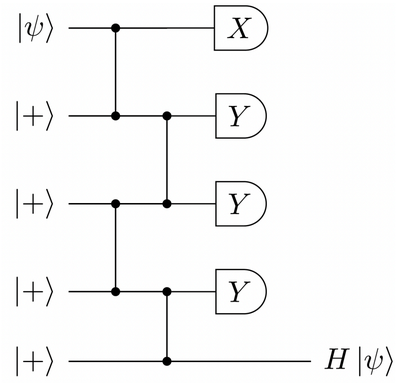
$$(-1)^{m_1}X_1 \xrightarrow{\Pi_{Y_4}} (-1)^{m_1}X_1$$
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Exercise



Hadamard-gate



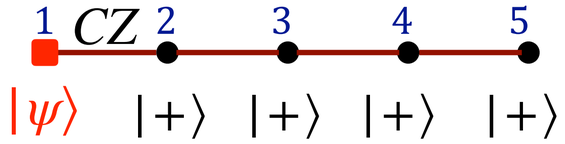


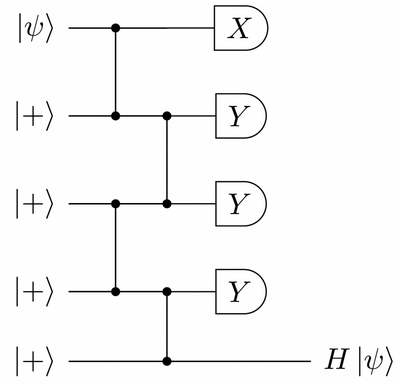
$$
\begin{aligned}
(-1)^{m_1}X_1 &\xrightarrow{\Pi_{Y_4}} (-1)^{m_1}X_1 \\
(-1)^{m_2}Y_2 &\longrightarrow (-1)^{m_2}Y_2 \\
(-1)^{m_3}Y_3 &\longrightarrow (-1)^{m_3}Y_3 \\
Z_4X_5 &\longrightarrow (-1)^{m_4}Y_4 \\
\bar{X} : (-1)^{m_1+m_3}Y_4Z_5 &\longrightarrow (-1)^{m_1+m_3+m_4}Z_5 \\
\bar{Z} : (-1)^{m_2+m_3}Z_4 &\longrightarrow
\end{aligned}
$$

Exercise



Hadamard-gate



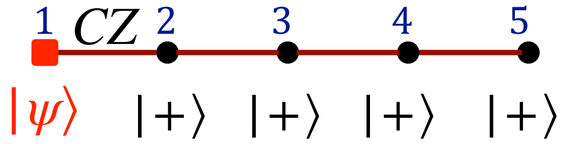


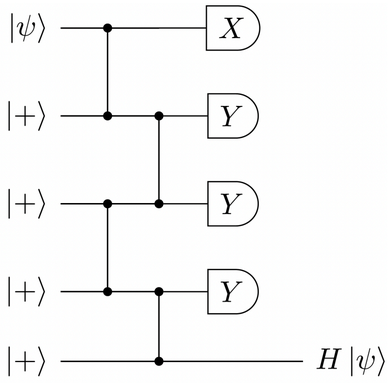
$$(-1)^{m_1}X_1 \xrightarrow{\Pi_{Y_4}} (-1)^{m_1}X_1$$
$$(-1)^{m_2}Y_2 \longrightarrow (-1)^{m_2}Y_2$$
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$$\bar{Z}: (-1)^{m_2+m_3}Z_4 \longrightarrow$$

Exercise



Hadamard-gate





$$\xrightarrow{\Pi_{Y_4}}$$

$$(-1)^{m_1}X_1 \longrightarrow (-1)^{m_1}X_1$$

$$(-1)^{m_2}Y_2 \longrightarrow (-1)^{m_2}Y_2$$

$$(-1)^{m_3}Y_3 \longrightarrow (-1)^{m_3}Y_3$$

$$Z_4X_5 \longrightarrow (-1)^{m_4}Y_4$$

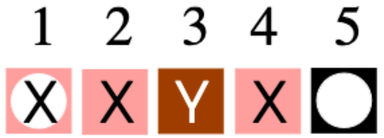
$$\bar{X}: (-1)^{m_1+m_3}Y_4Z_5 \longrightarrow (-1)^{m_1+m_3+m_4}Z_5$$

$$\bar{Z}: (-1)^{m_2+m_3}Z_4 \longrightarrow (-1)^{m_2+m_3}X_5$$

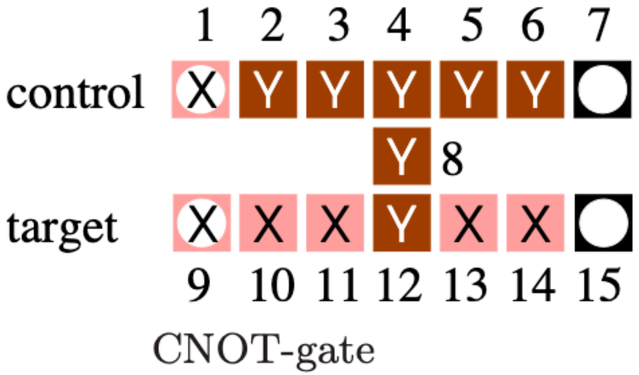
Clifford gates



Hadamard-gate

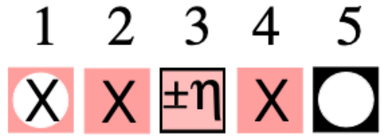


$\pi/2$-phase gate

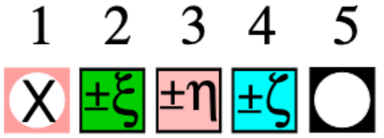


CNOT-gate

Other gates



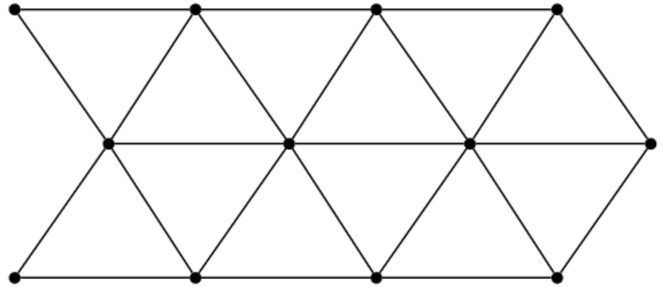
z-rotation

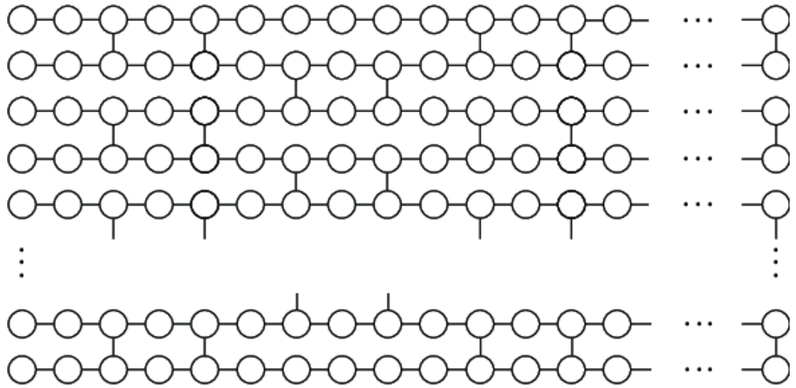


general rotation

What are the "good" resource states for 1WQC?

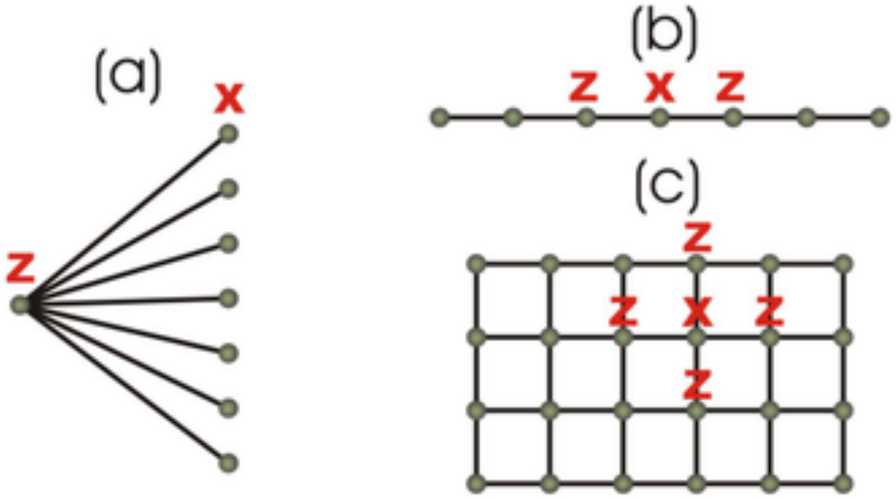
Resource state	Measurement basis	Class
Cluster state[37]	X, Y, $TXT^{\dagger}$[38]	graph state
Brickwork state[40]	X, Y, $TXT^{\dagger}$[40]	graph state
Triangular lattice state[41]	X, Z, H, XHX[41]	graph state
Raussendorf-Harrington-Goyal (RHG) lattice[42,43]	X, Y, Z, $TXT^{\dagger}$[43,44]	graph state
Decorated RHG lattice[45]	X, Y, $TXT^{\dagger}$[45]	graph state
Affleck-Kennedy-Lieb-Tasaki (AKLT) state[46,47]	qutrit bases[47]	matrix-product state
Union Jack state[33]	X, Y, Z[33]	hypergraph state
Three-uniform hypergraph state[36]	X, Y, Z[36]	hypergraph state
Mølmer-Sørensen graph state[15]	X, Z[15]	weighted graph state
Our state	X, Z	hypergraph state





[Takeuchi, Morimae, and Hayashi, Scientific Reports **9**, 13585 (2019).]

Graph states



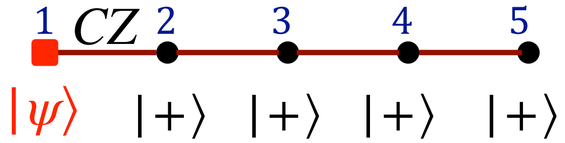
They can be described in terms of their stabilizer $\mathcal{S} := \langle G_1, \ldots, G_n \rangle$ where the generators are given by

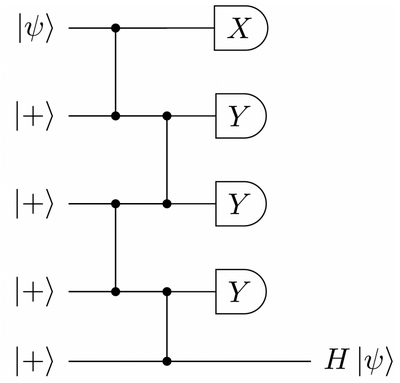
$$G_i = X_i \prod_{j \in \mathcal{N}(i)} Z_j \,.$$

Recall our earlier exercise



Hadamard-gate





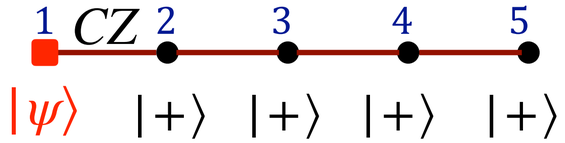
CZs

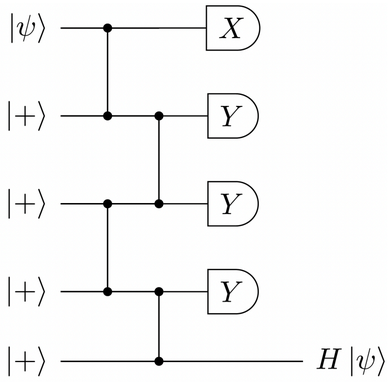
$$X_2 \longrightarrow Z_1 X_2 Z_3$$
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$$\bar{X} : X_1 \longrightarrow X_1 Z_2$$
$$\bar{Z} : Z_1 \longrightarrow Z_1$$

Recall our earlier exercise



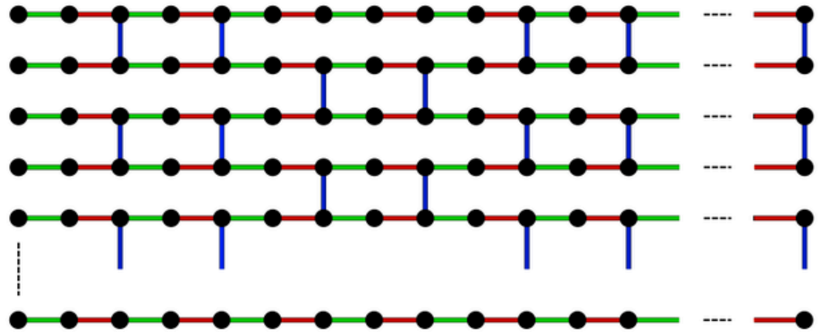
Hadamard-gate





CZs

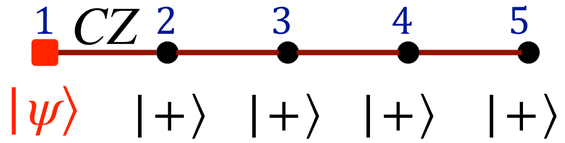
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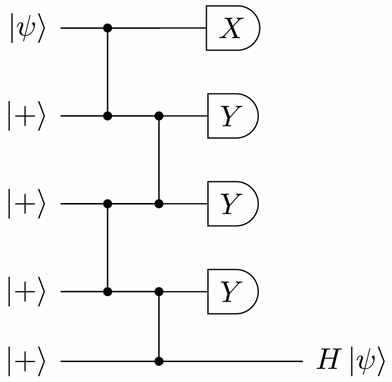


Recall our earlier exercise



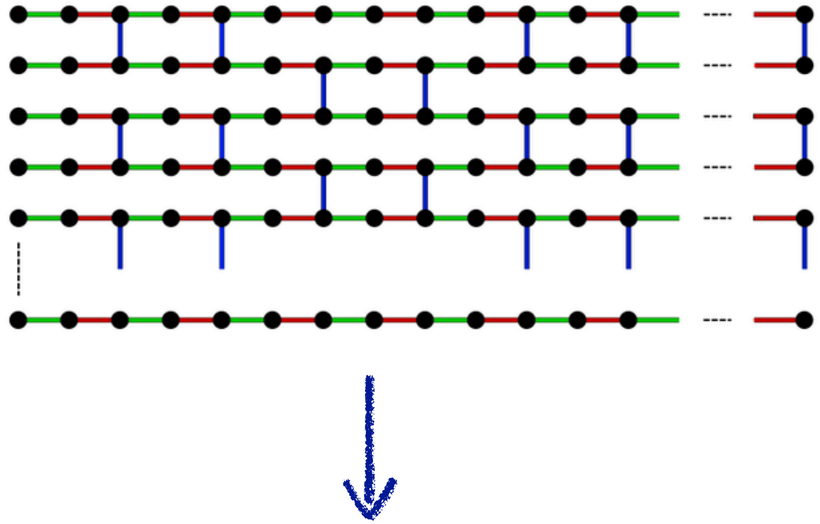
Hadamard-gate





CZs

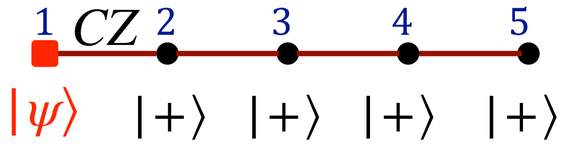
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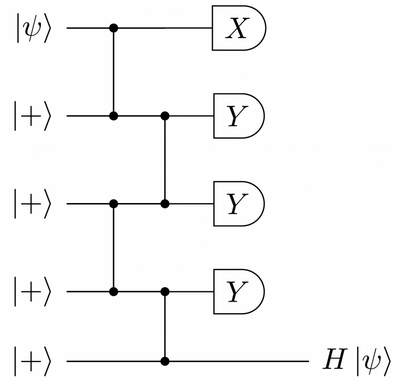


Recall our earlier exercise



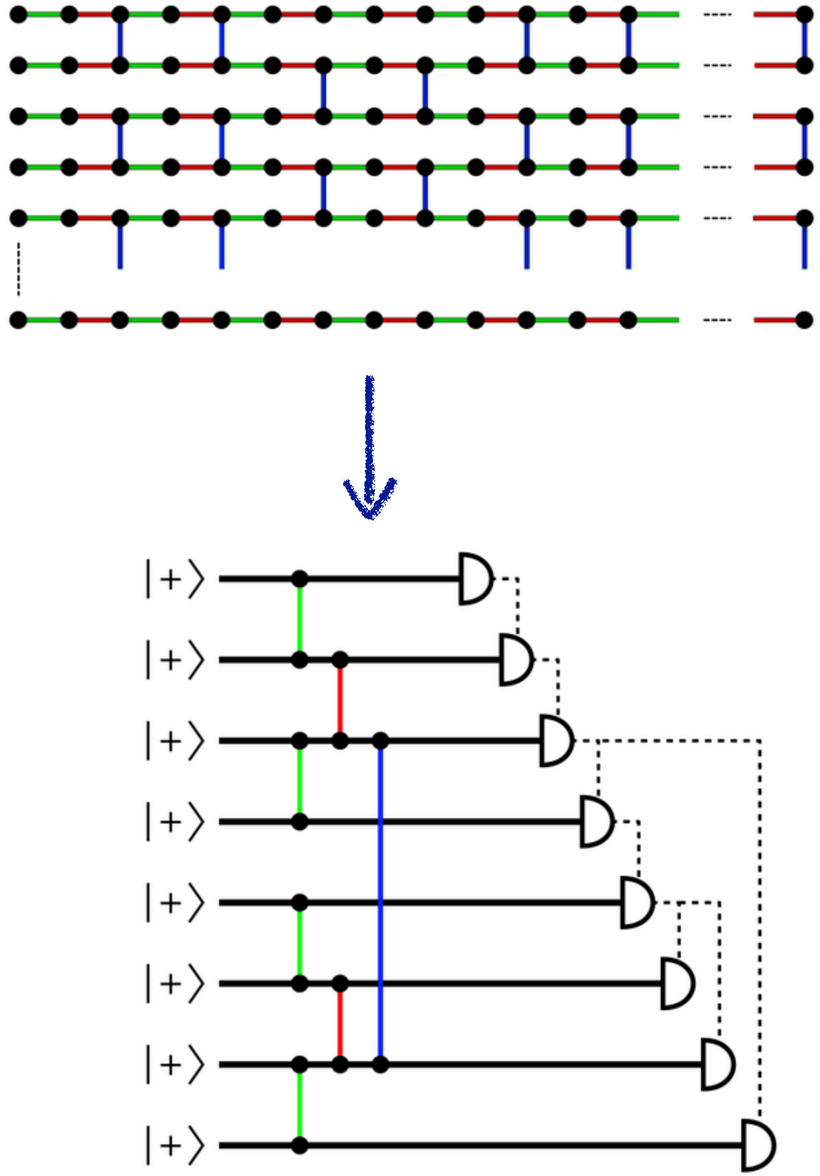
Hadamard-gate





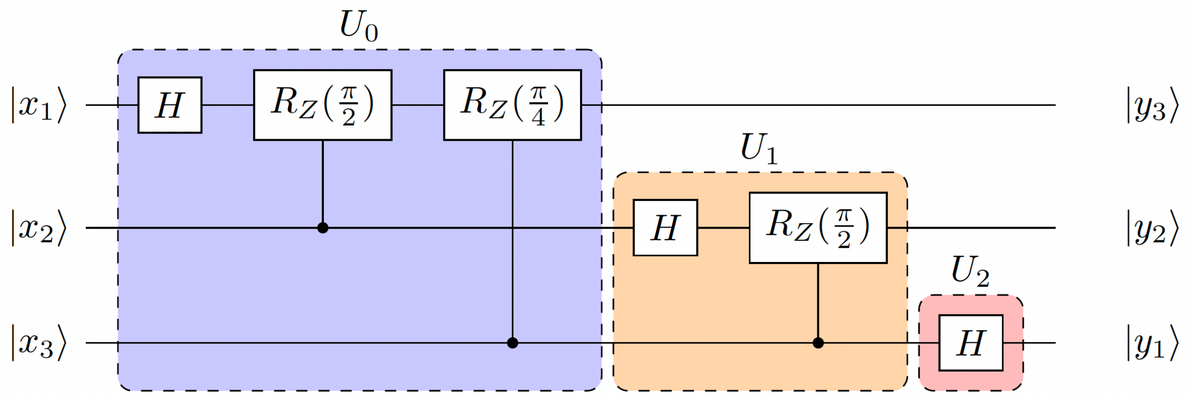
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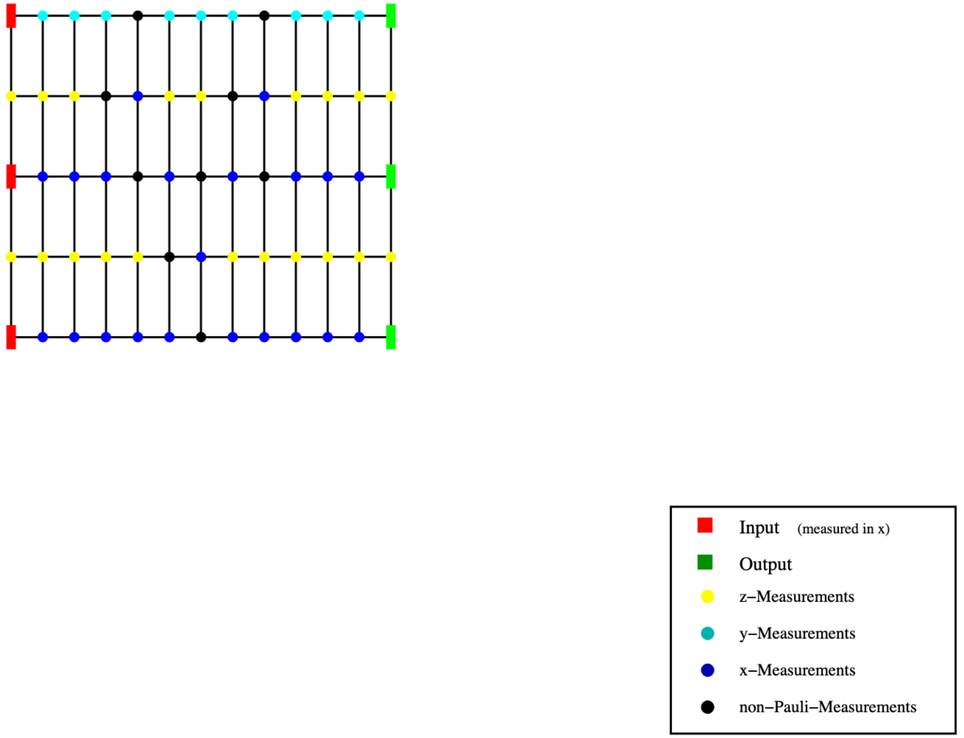


3-qubit QFT

Quantum circuit



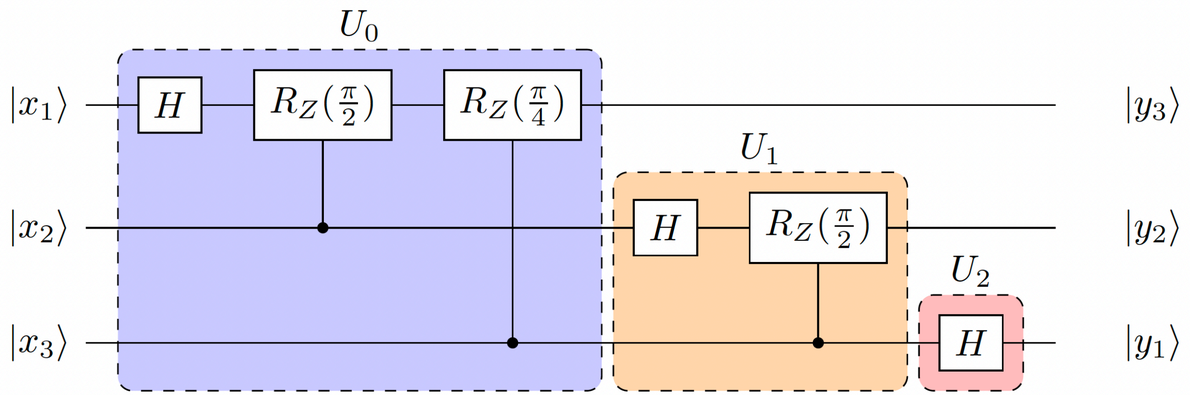
One-way quantum computation



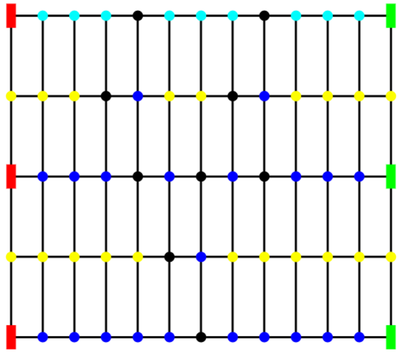
[Hein *et al.,* Proceedings of the International School of Physics "Enrico Fermi": Quantum Computers, Algorithms and Chaos 115-218 (2006).]

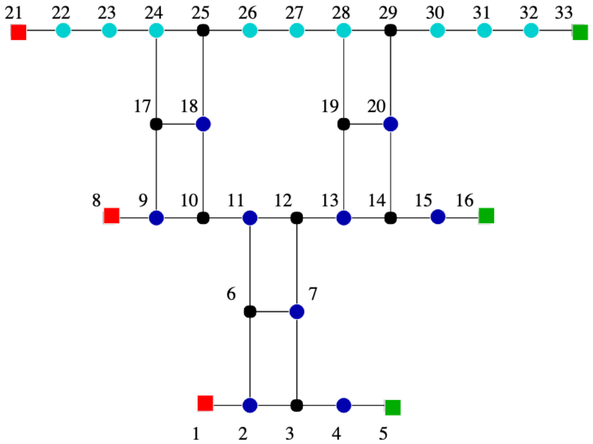
3-qubit QFT

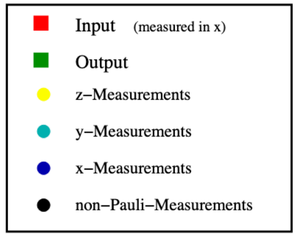
Quantum circuit



One-way quantum computation



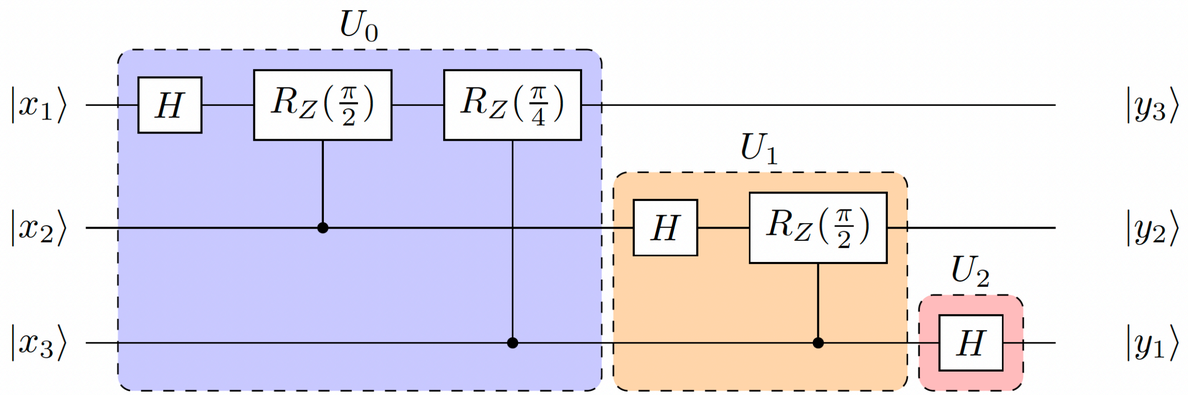




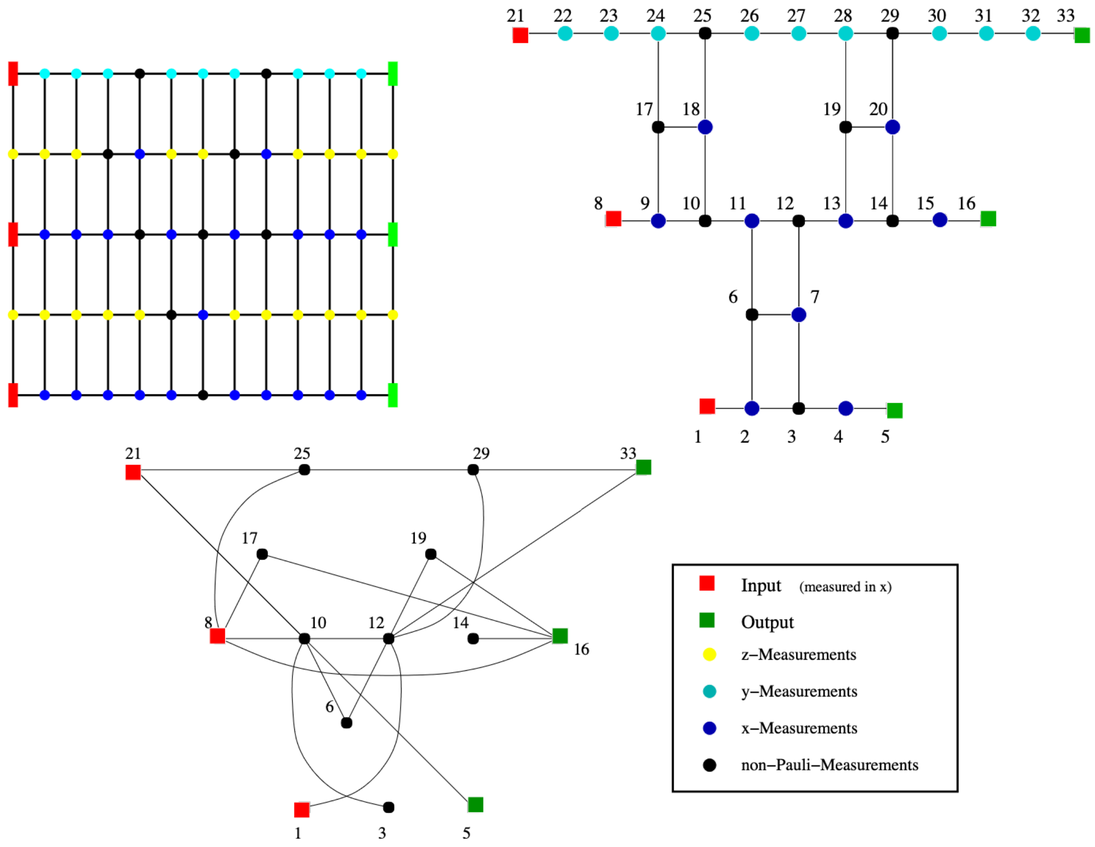
[Hein *et al.,* Proceedings of the International School of Physics "Enrico Fermi": Quantum Computers, Algorithms and Chaos 115-218 (2006).]

3-qubit QFT

Quantum circuit



One-way quantum computation



Computation by measurements: A unifying picture

PDF

Panos Aliferis* and Debbie W. Leung†

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Abstract

The ability to perform a universal set of quantum operations based solely on static resources and measurements presents us with a striking viewpoint for thin
We consider the two major models for doing quantum comp
appeared in the literature and show that they are conceptua
local mapping between them. This way we effectively unify t
interchangeable primitives. With the tools developed for this
effective methods for performing computation within both r
arbitrary graph states employing two-qubit measurements a

1 Jun 2005 | Conference Proceedings

Unifying quantum computation with projective measurements only and one-way quantum computation

Philippe Jorrand, Simon Perdrix | Author Affiliations

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Abstract

Quantum measurement is universal for quantum computation. Two models for performing measurement-based quantum computation exist: the one-way quantum computater was introduced by Briegel and Raussendorf and quantum computation via projective measurements only by Nielsen. The more recent development of this second model is based on state transfers instead of teleportation. From this development a finite but approximate quantum universal family of observables is exhibited which includes only one two-qubit observable while others are
versal family of observables
he paper is dedicated to
tion i.e. one-way quantum
only. From this comparison
nnections appear between
easurement-based

Unified derivations of measurement-based schemes for quantum computation

PDF

Andrew M. Childs[1,2,*], Debbie W. Leung[1,†], and Michael A. Nielsen[1,3,‡]

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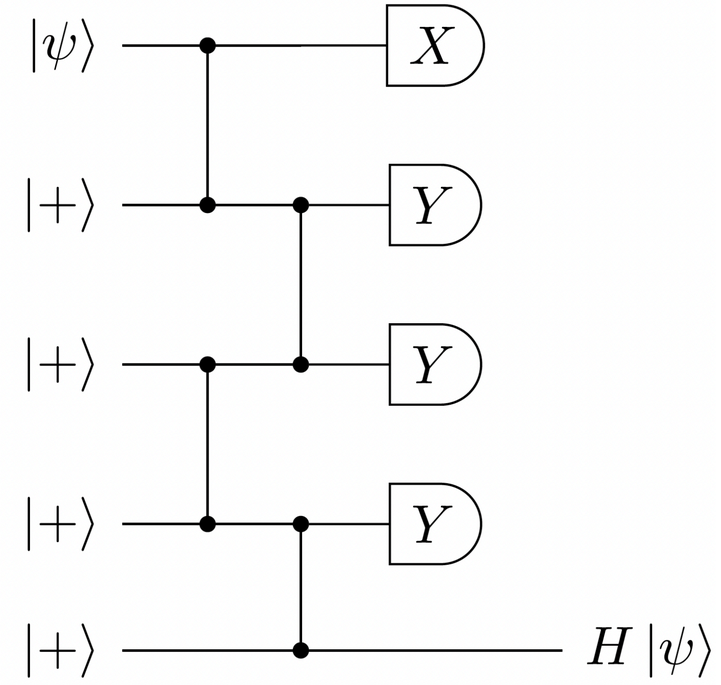
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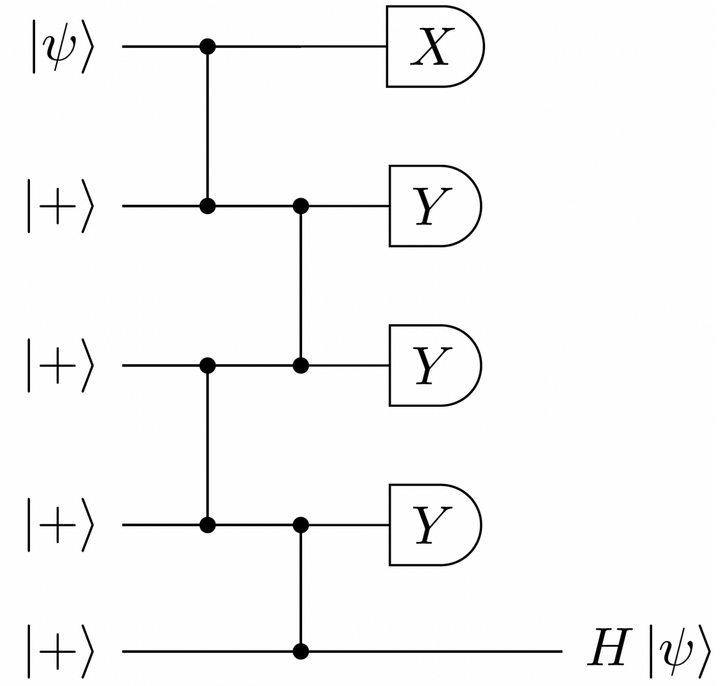
Abstract

We present unified, systematic derivations of schemes in the two known measurement-based models of quantum computation. The first model **(**introduced by Raussendorf and Briegel, [Phys. Rev. Lett. **86**, 5188 (2001)]**)** uses a fixed entangled state, adaptive measurements on single qubits, and feedforward of the measurement results. The second model **(**proposed by Nielsen, [Phys. Lett. A **308**, 96 (2003)] and further simplified by Leung, [Int. J. Quant. Inf. **2**, 33 (2004)]**)** uses adaptive two-qubit measurements that can be applied to arbitrary pairs of qubits, and feedforward of the measurement results. The underlying principle of our derivations is a variant of teleportation introduced by Zhou, Leung, and Chuang, [Phys. Rev. A **62**, 052316 (2000)]. Our derivations unify these two measurement-based models of quantum computation and provide significantly simpler schemes.

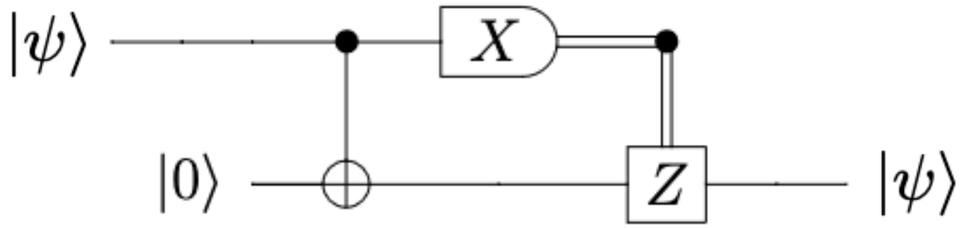
Recall our exercise



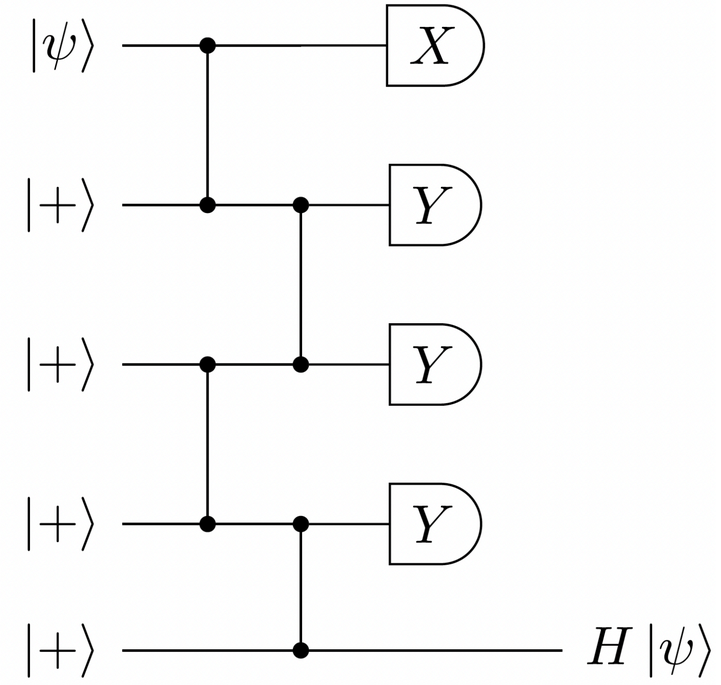
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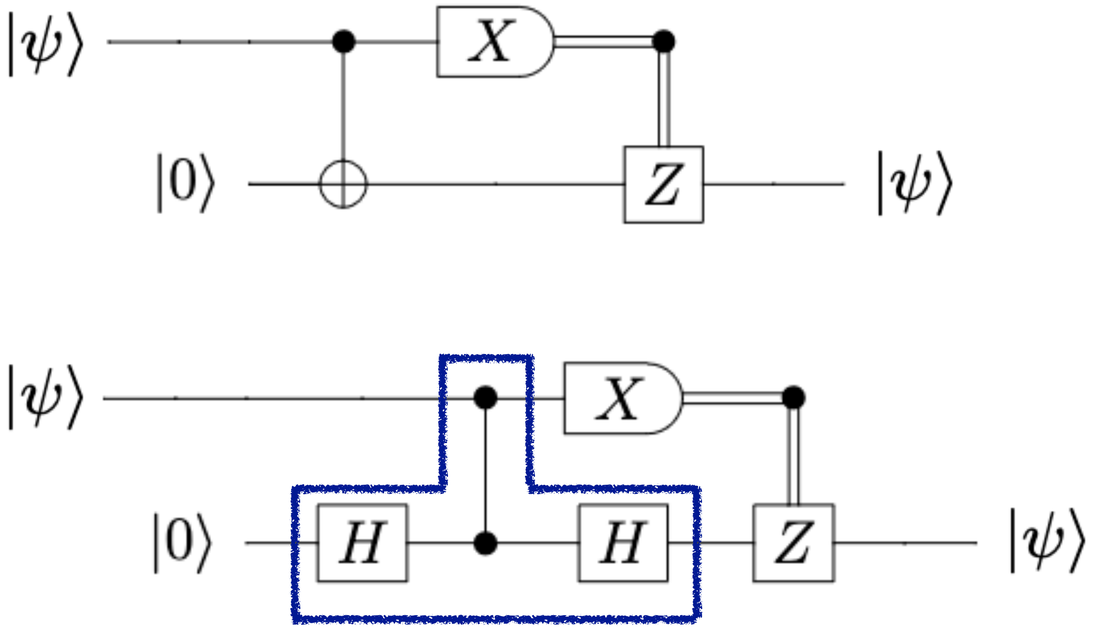
Recall 1-bit Z-teleportation



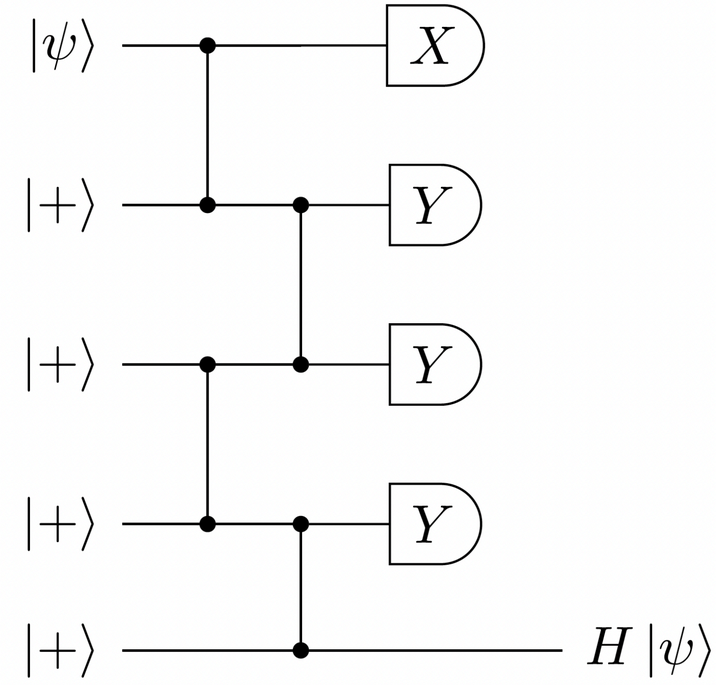
Recall our exercise



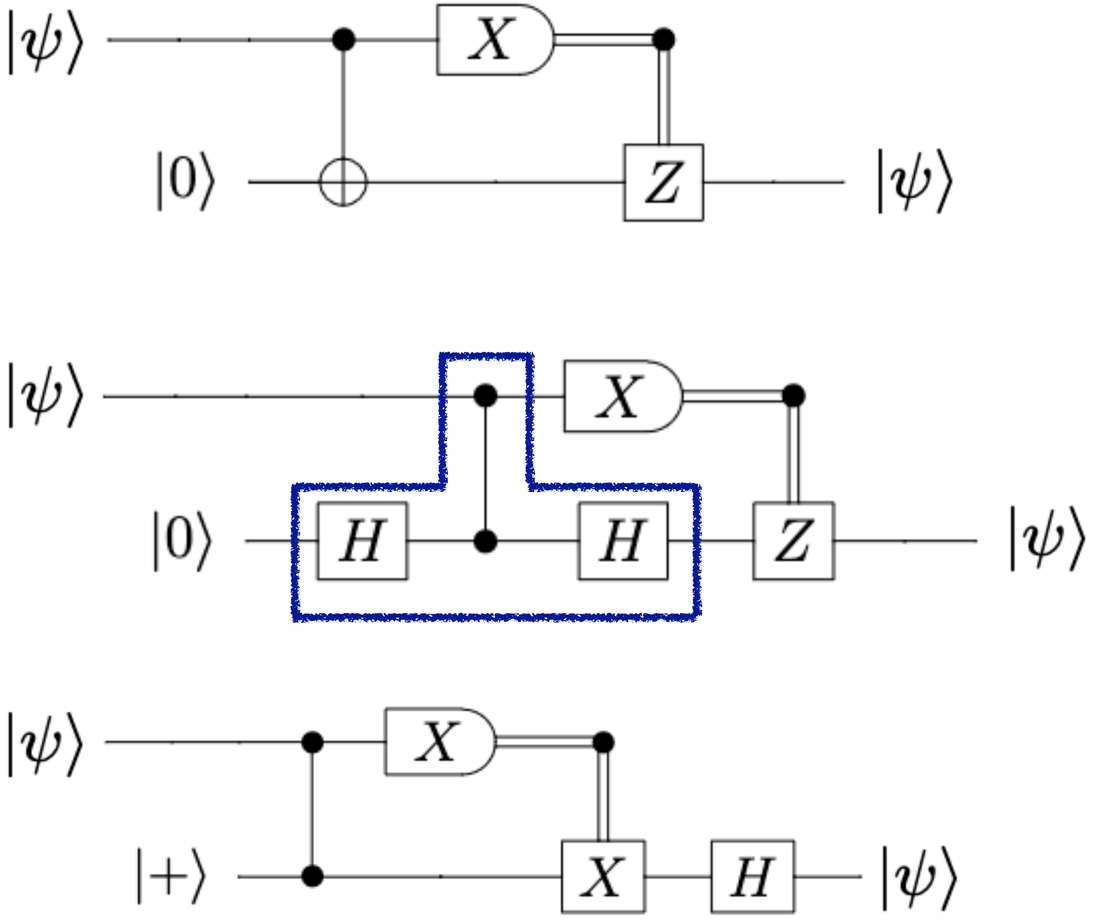
Recall 1-bit Z-teleportation



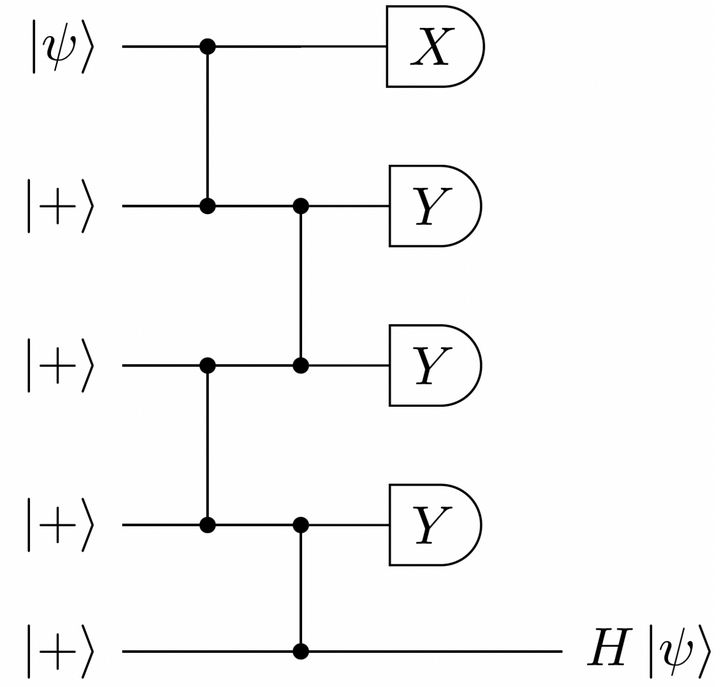
Recall our exercise



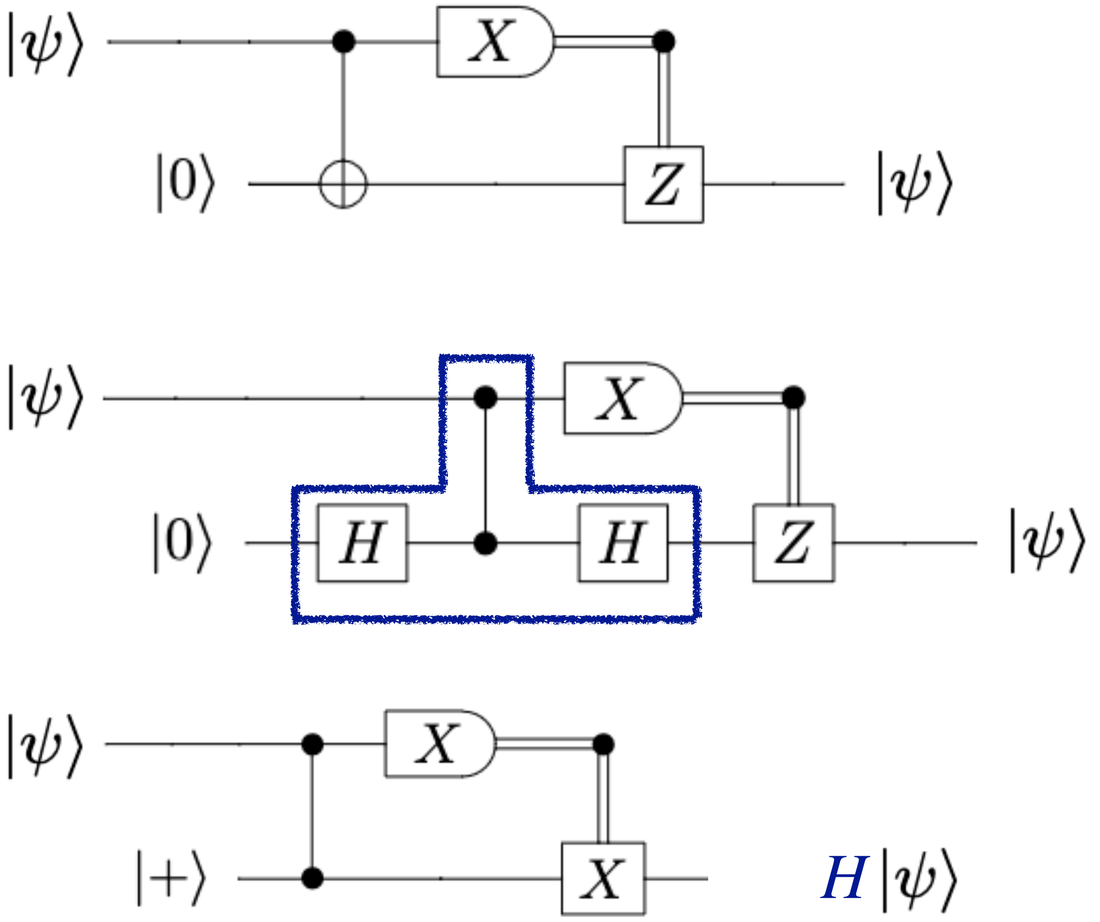
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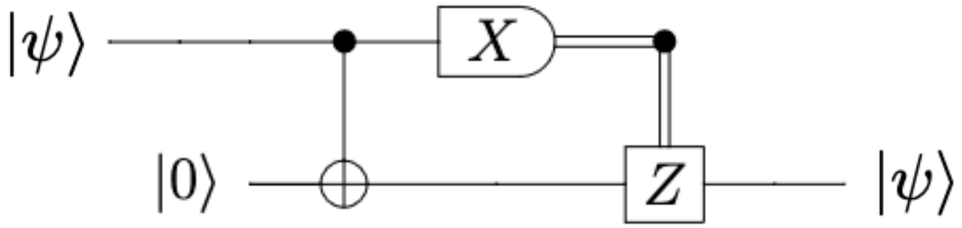
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Recall 1-bit Z-teleportation

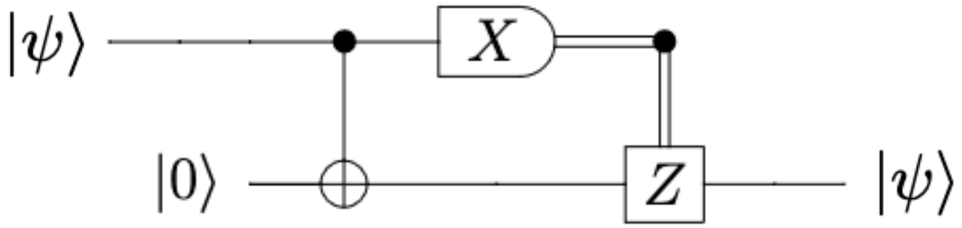


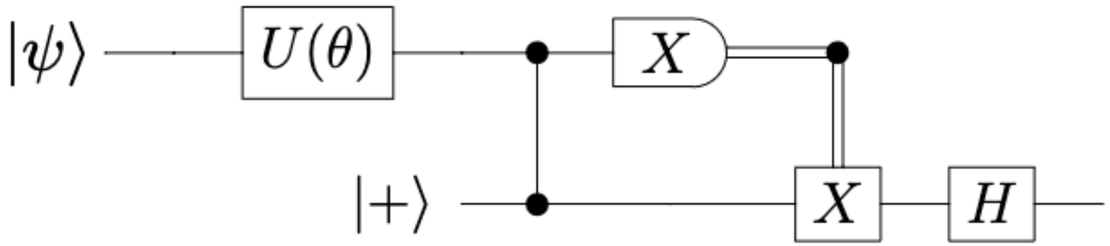
Let's use 1-qubit Z-teleportation to understand the essence of 1WQC:



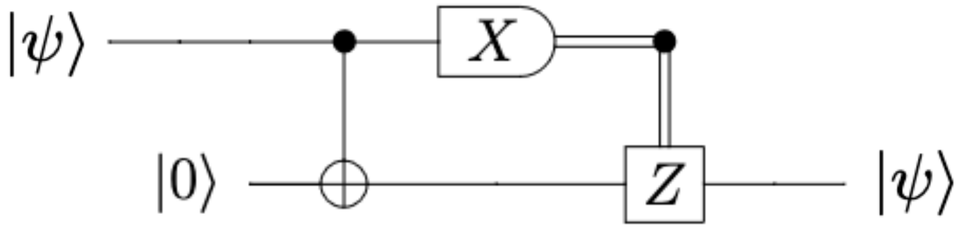
[McKague, Theory of Computing **12** 1-12 (2016).]

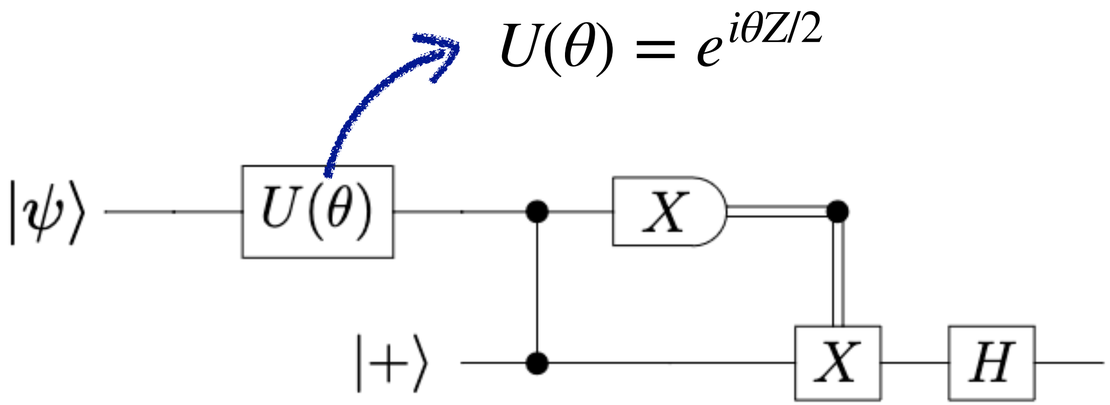
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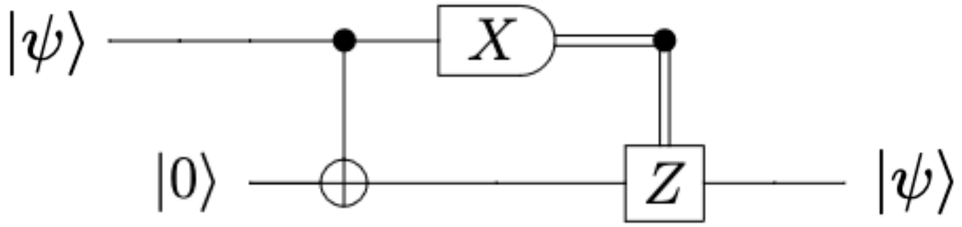
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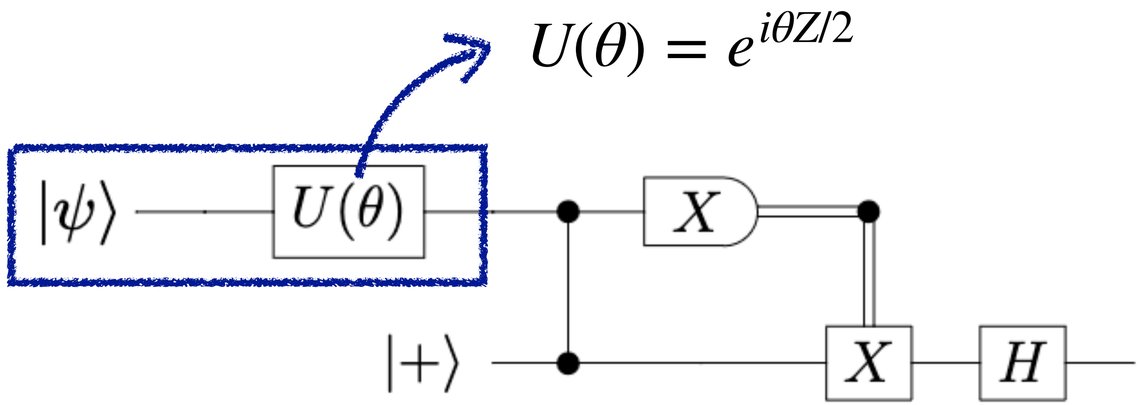




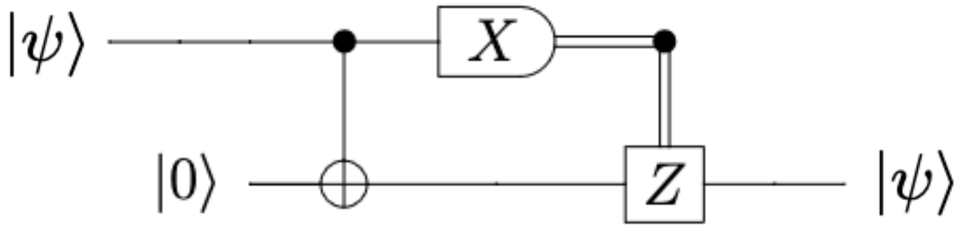
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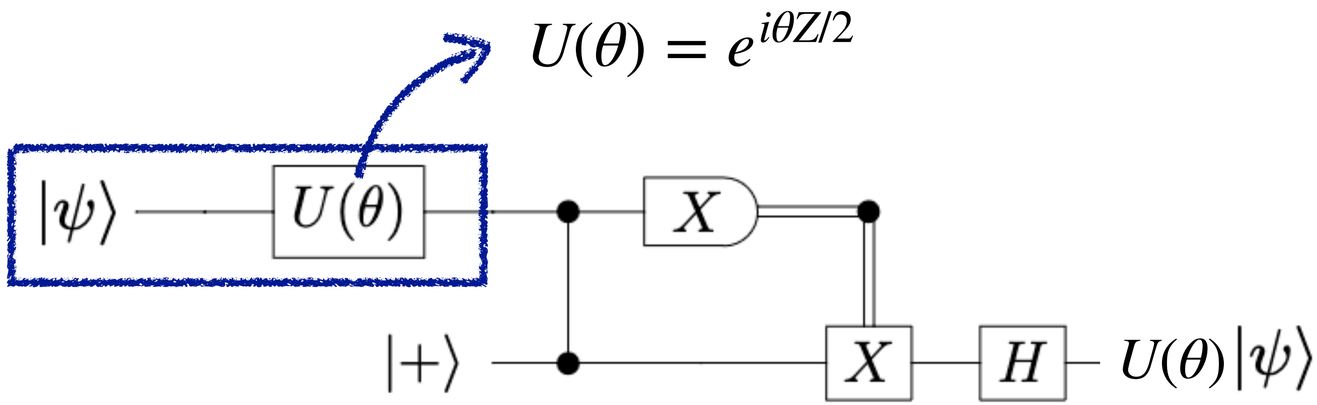
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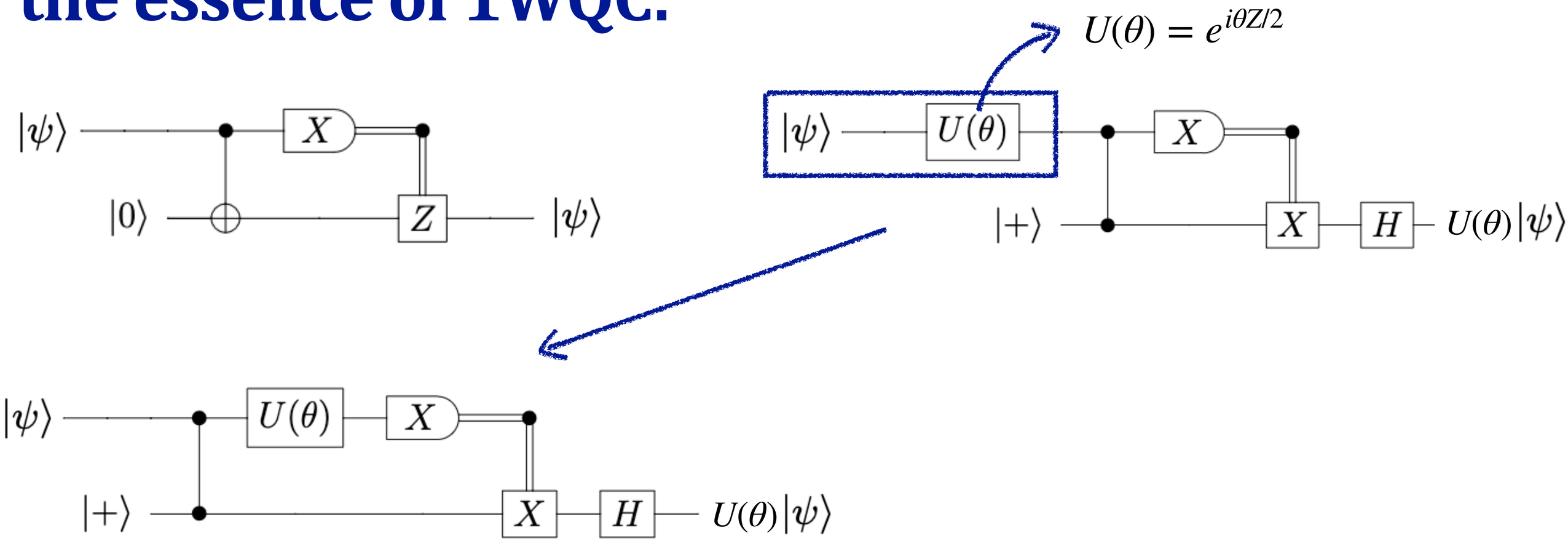


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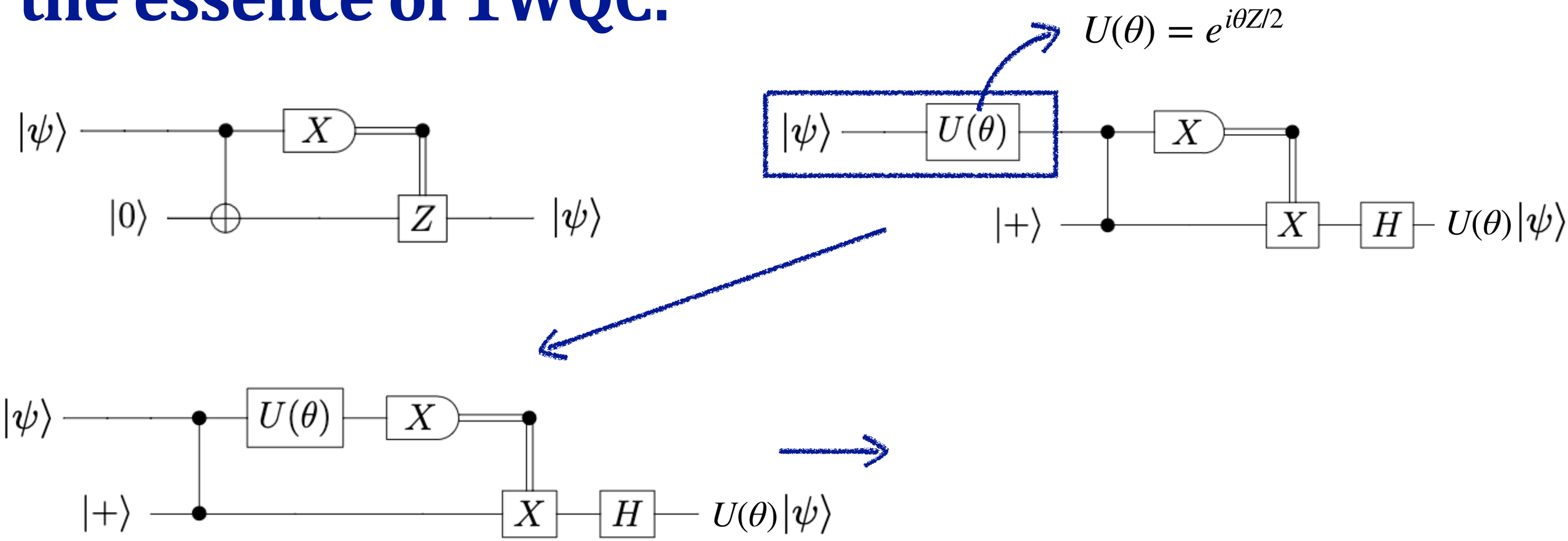




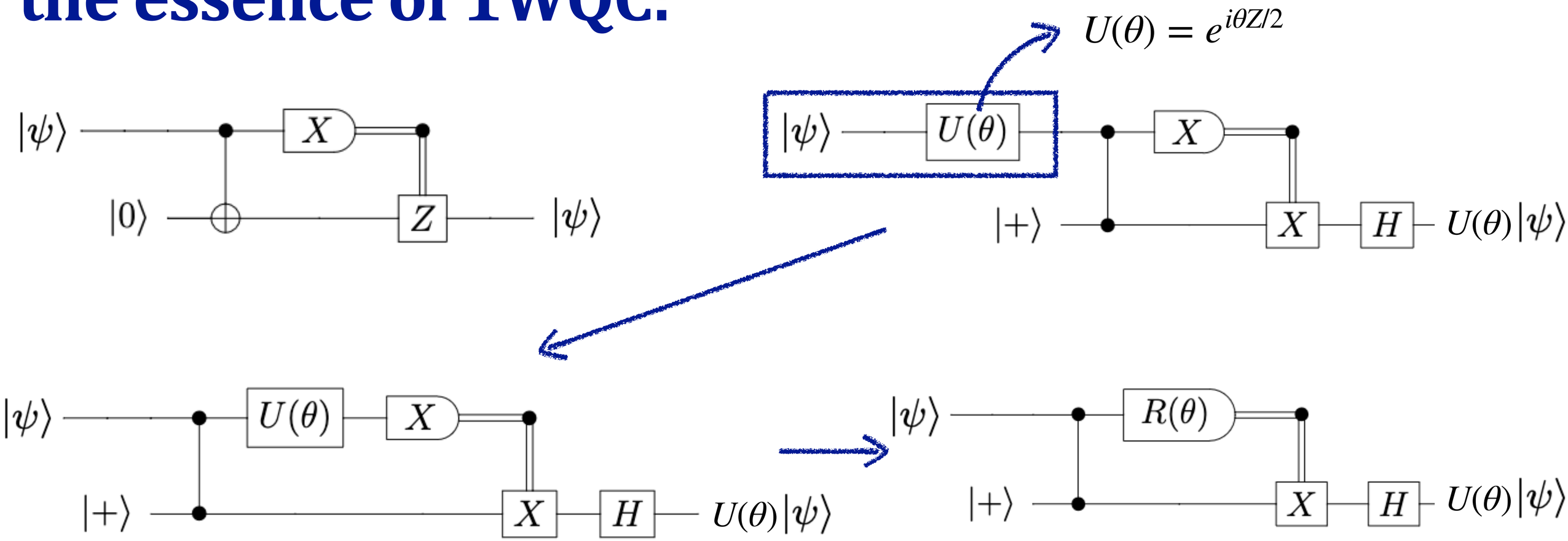
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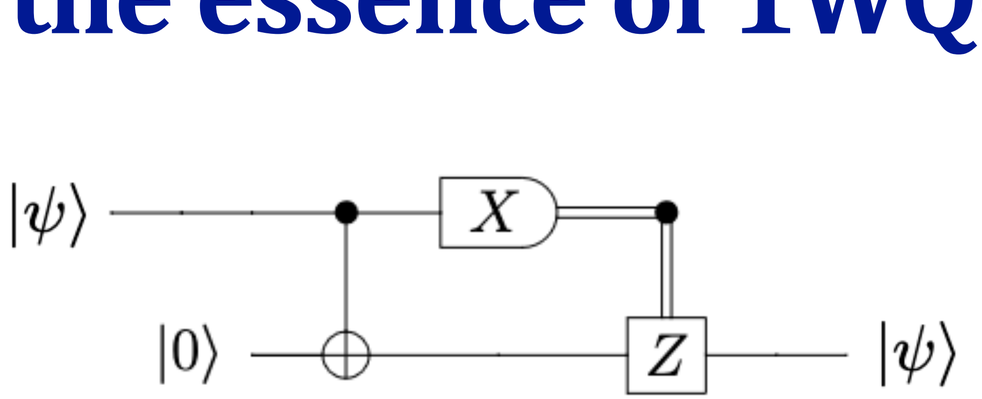
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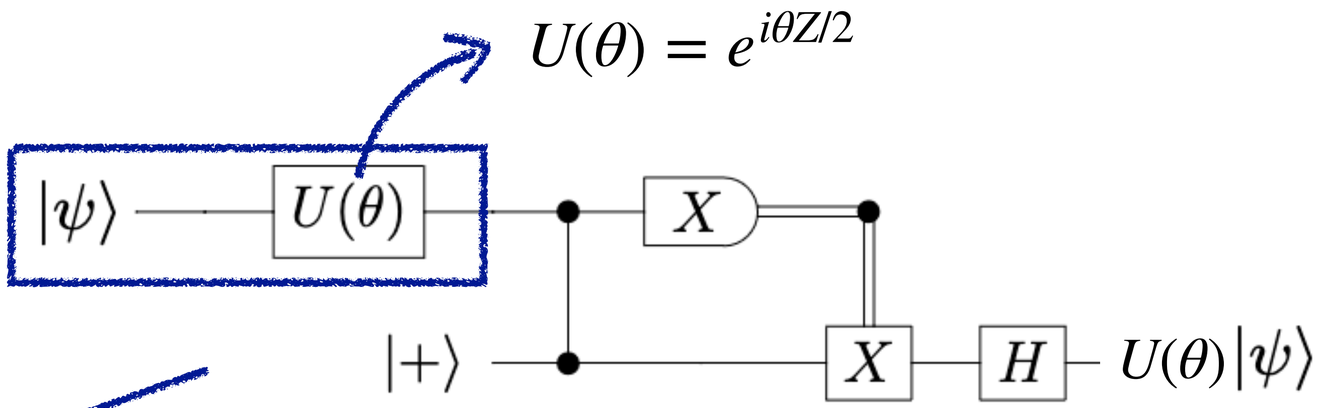
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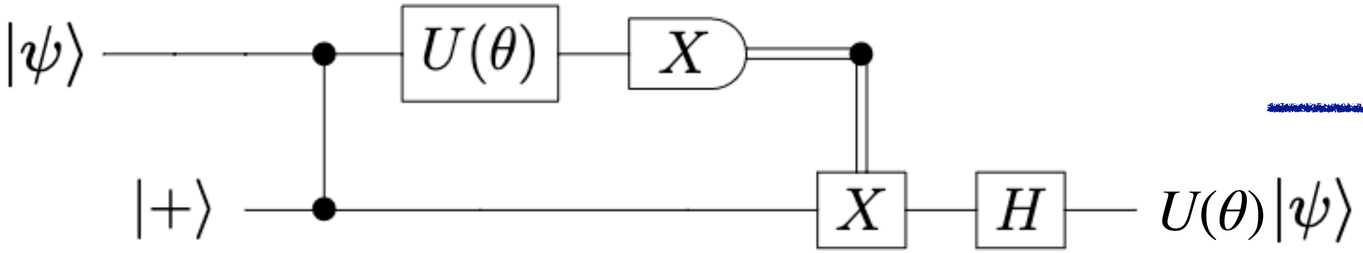


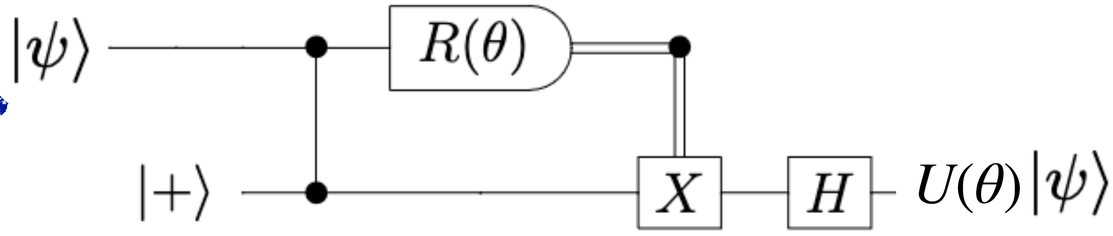
Let's use 1-qubit Z-teleportation to understand the essence of 1WQC:



$U(\theta) = e^{i\theta Z/2}$

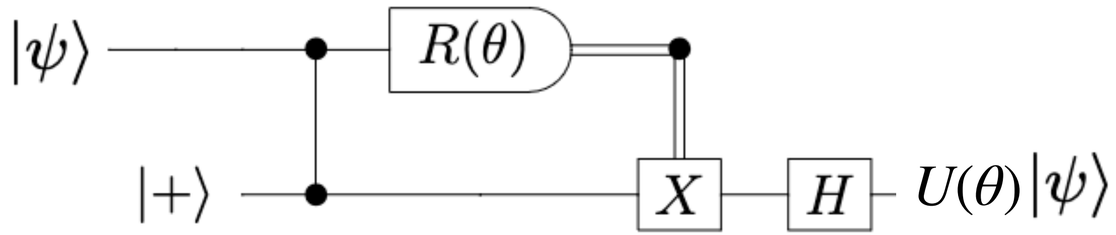


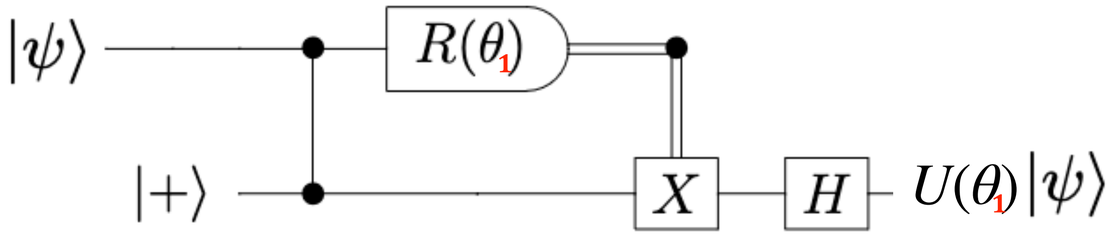


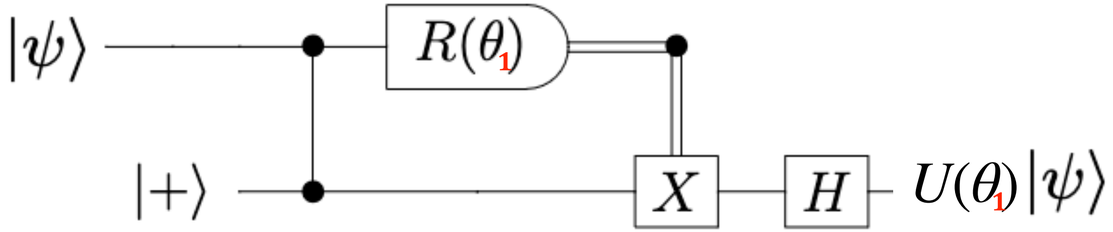


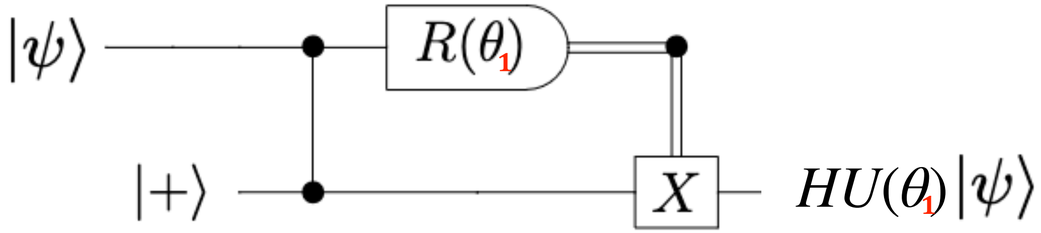
$$U(\theta)^\dagger X U = R(\theta) = \cos(\theta)X + \sin(\theta)Y$$

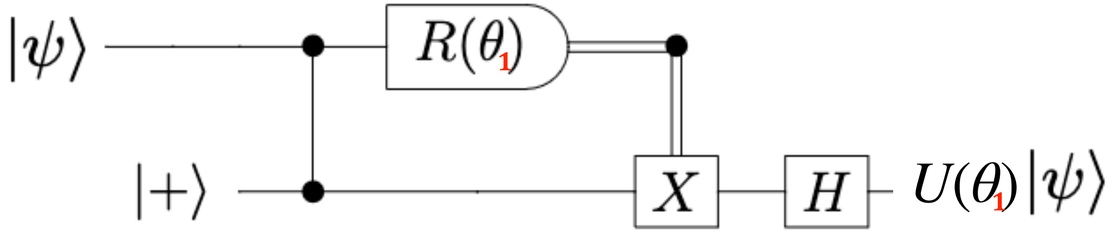
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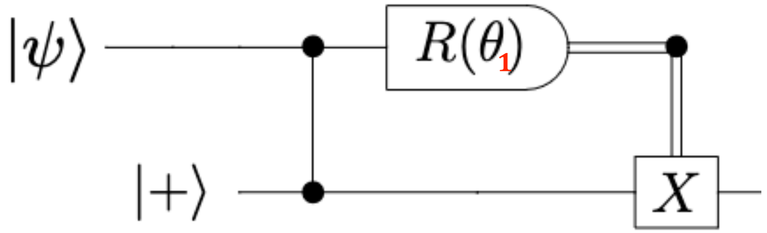

$|\psi\rangle$
$R(\theta)$
$|+\rangle$
X
H
$U(\theta)|\psi\rangle$


$|\psi\rangle$
$R(\theta_1)$
$|+\rangle$
X
H
$U(\theta_1)|\psi\rangle$

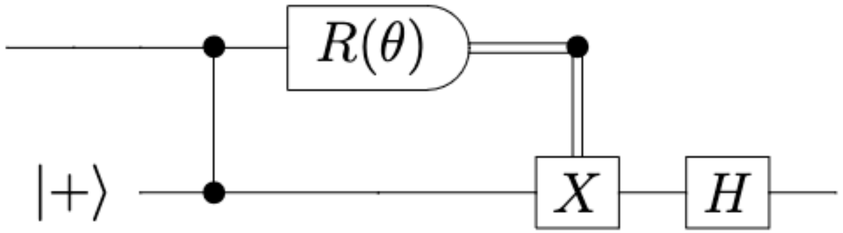

$|\psi\rangle$
$R(\theta_1)$
$|+\rangle$
X
H
$U(\theta_1)|\psi\rangle$

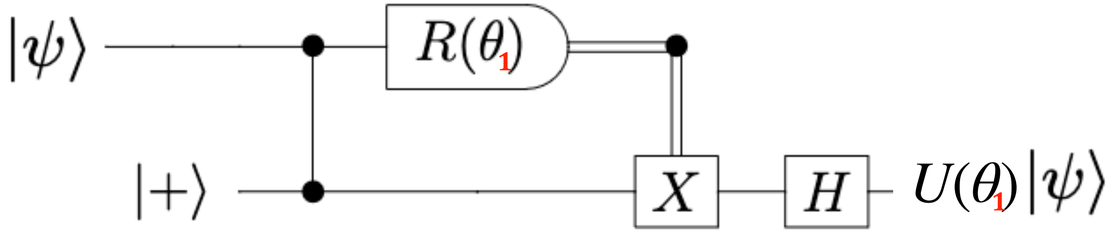

$|\psi\rangle$
$R(\theta_1)$
$|+\rangle$
X
$HU(\theta_1)|\psi\rangle$

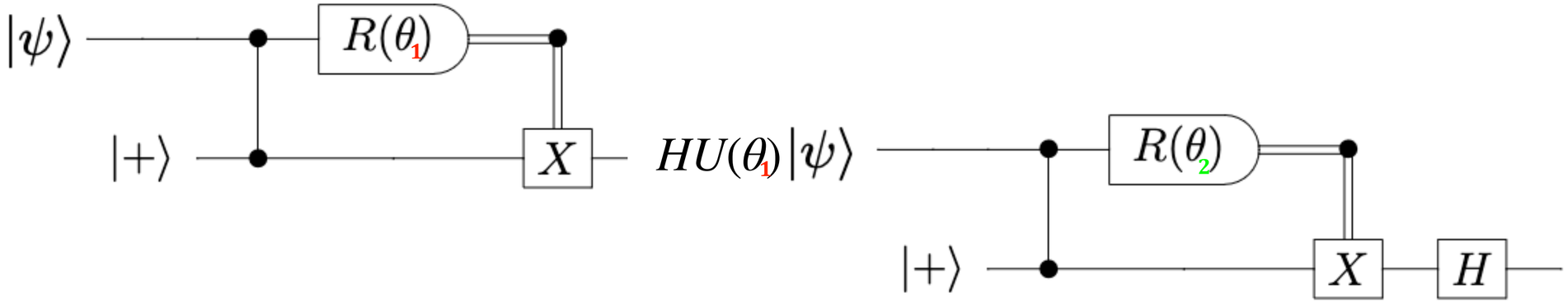


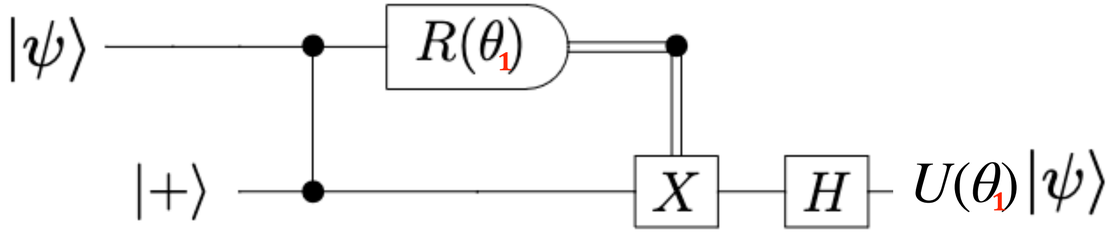


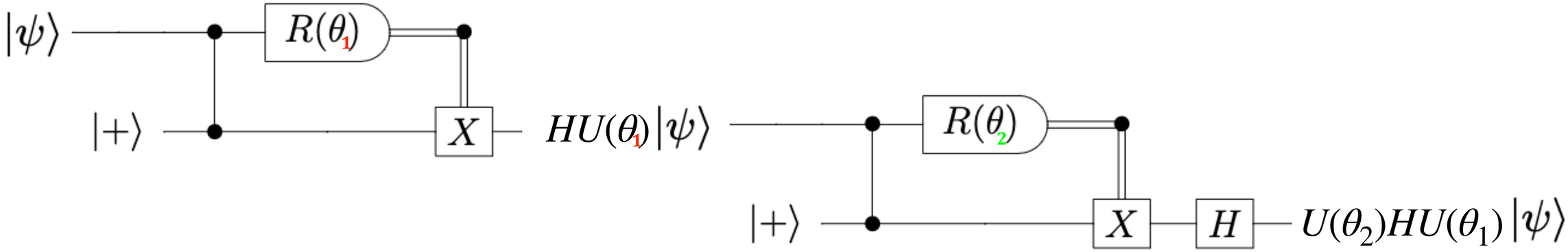
$HU(\theta_1)|\psi\rangle$

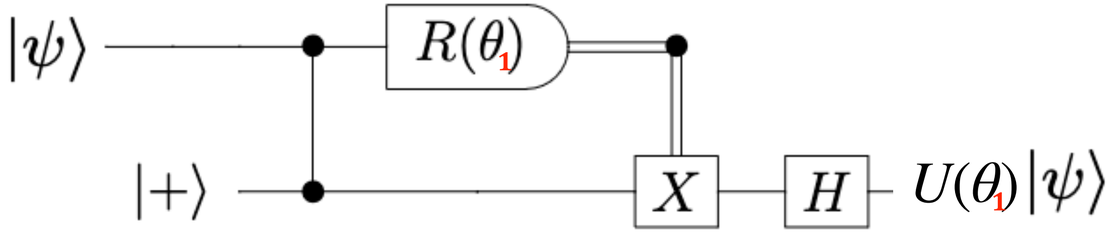


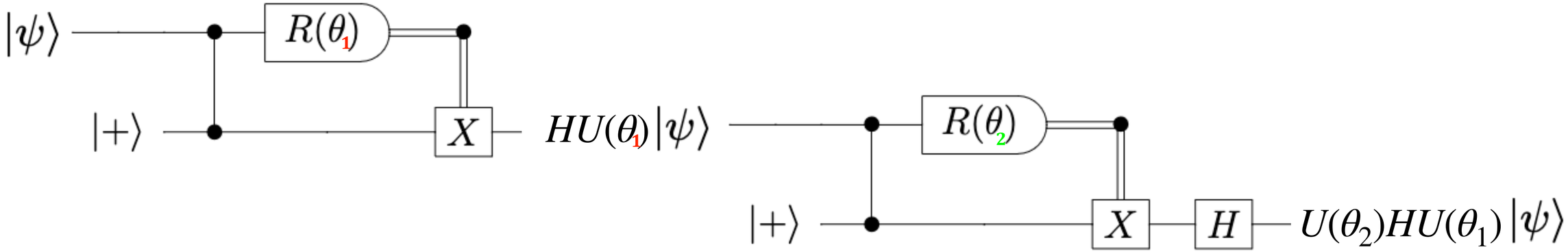

$|\psi\rangle$
$R(\theta_1)$
$|+\rangle$
X
H
$U(\theta_1)|\psi\rangle$

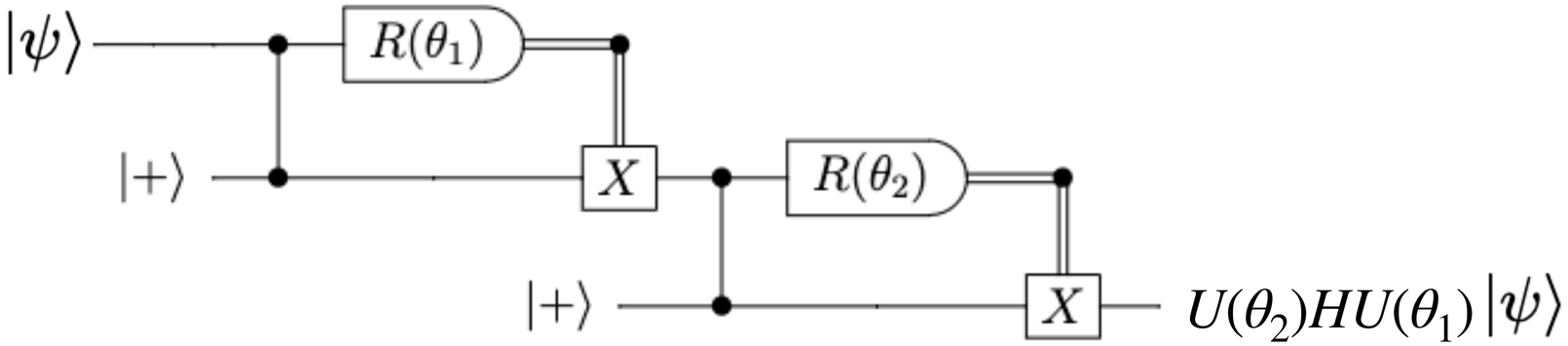

$|\psi\rangle$
$R(\theta_1)$
$|+\rangle$
X
$HU(\theta_1)|\psi\rangle$
$R(\theta_2)$
$|+\rangle$
X
H


$|\psi\rangle$
$R(\theta_1)$
$|+\rangle$
X
H
$U(\theta_1)|\psi\rangle$

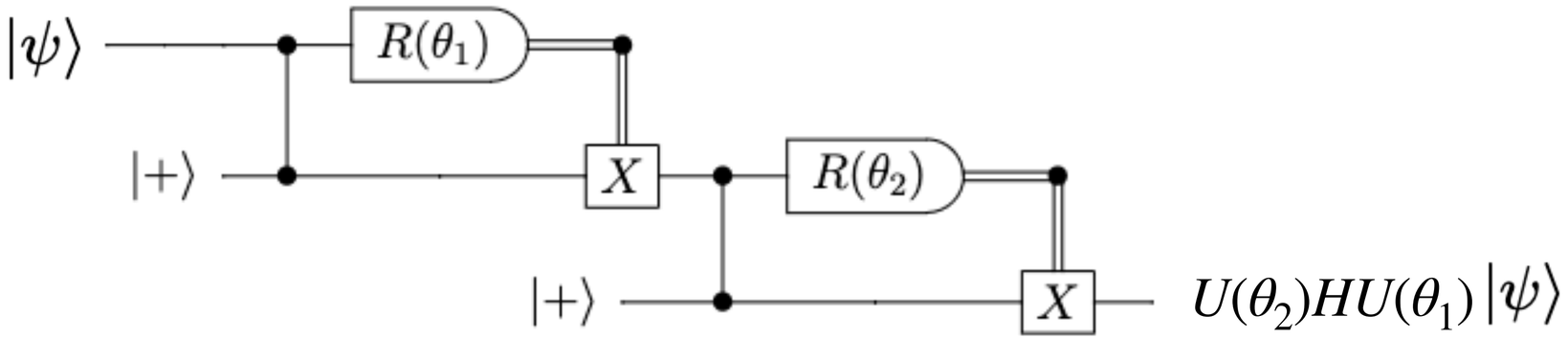

$|\psi\rangle$
$R(\theta_1)$
$|+\rangle$
X
$HU(\theta_1)|\psi\rangle$
$R(\theta_2)$
$|+\rangle$
X
H
$U(\theta_2)HU(\theta_1)|\psi\rangle$



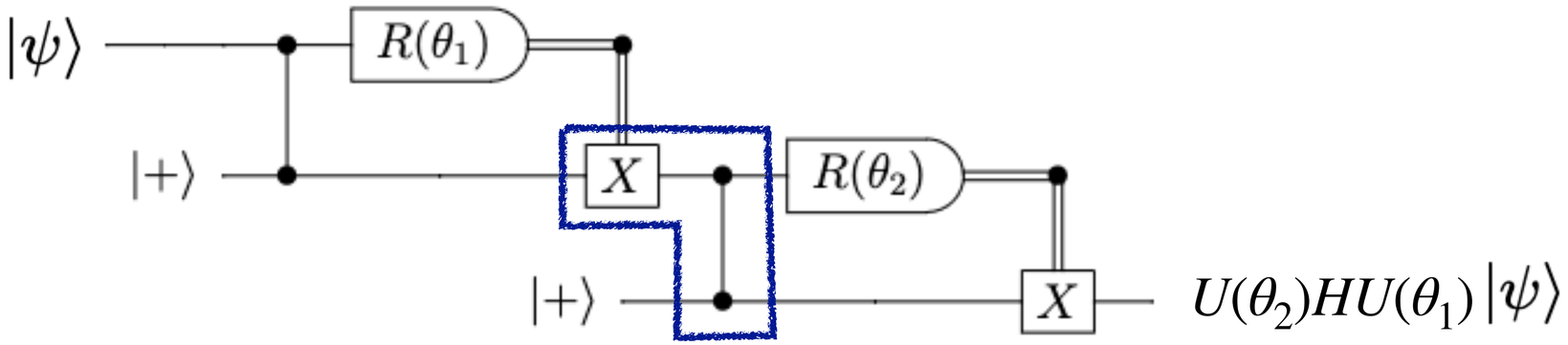


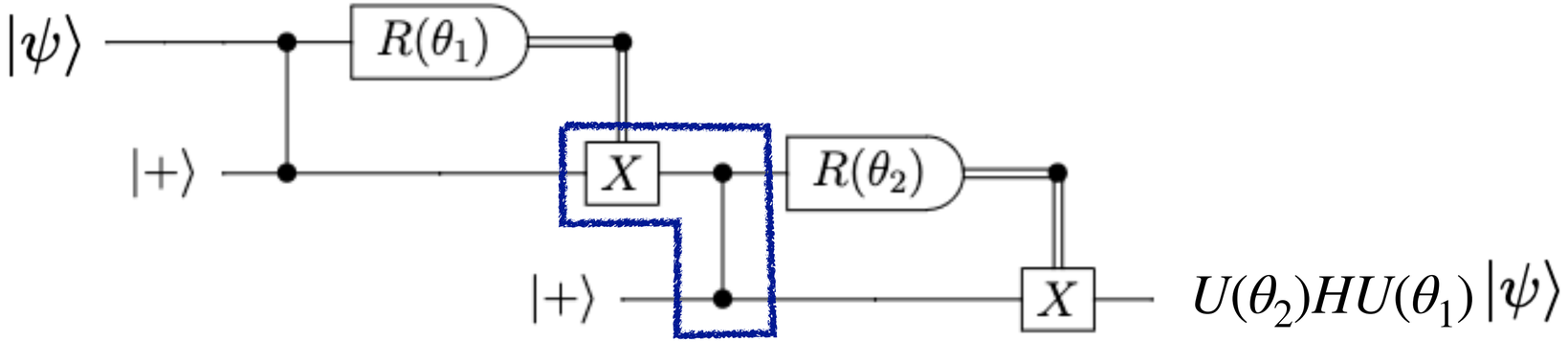


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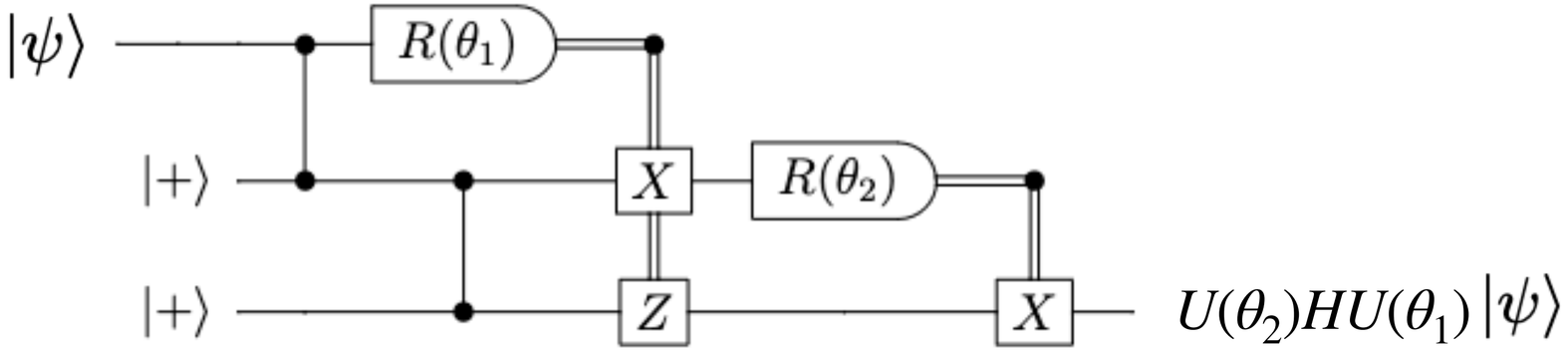


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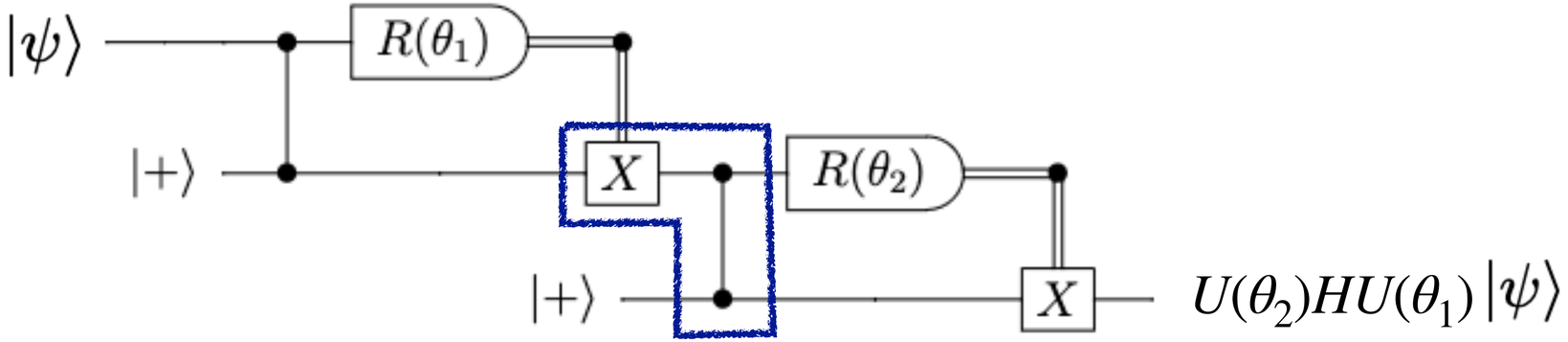

$|\psi\rangle$
$R(\theta_1)$
$|+\rangle$
X
$R(\theta_2)$
$|+\rangle$
X
$U(\theta_2)HU(\theta_1)|\psi\rangle$



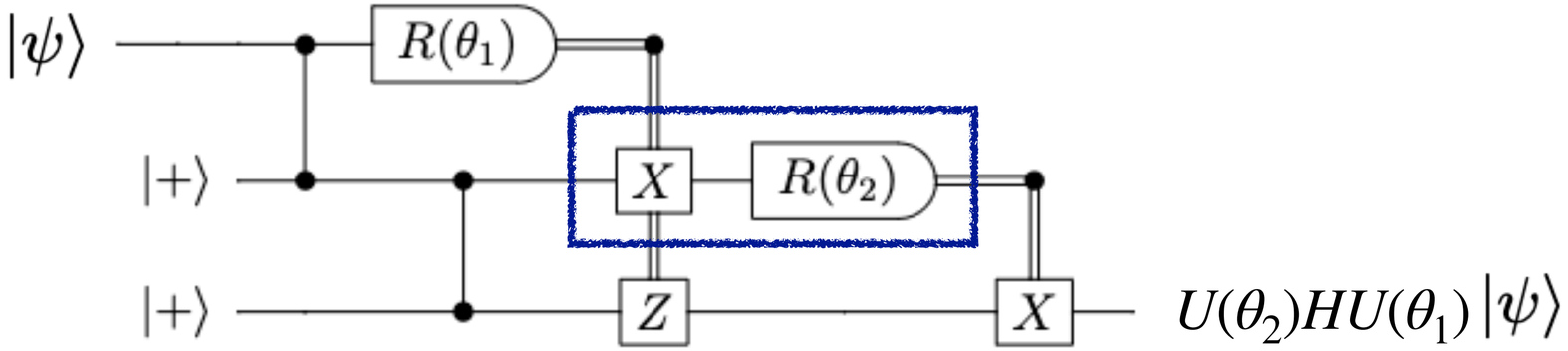
$U(\theta_2)HU(\theta_1)|\psi\rangle$



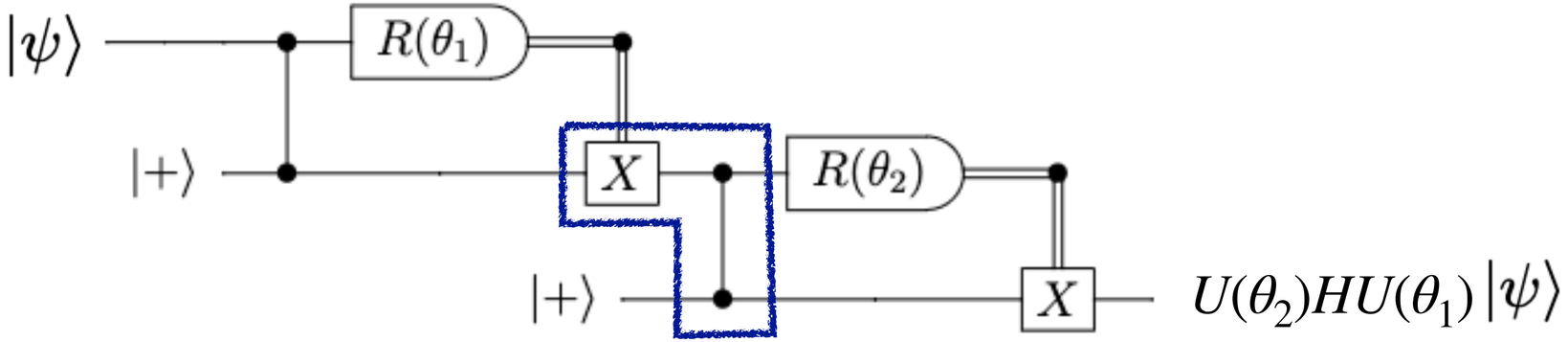
$U(\theta_2)HU(\theta_1)|\psi\rangle$



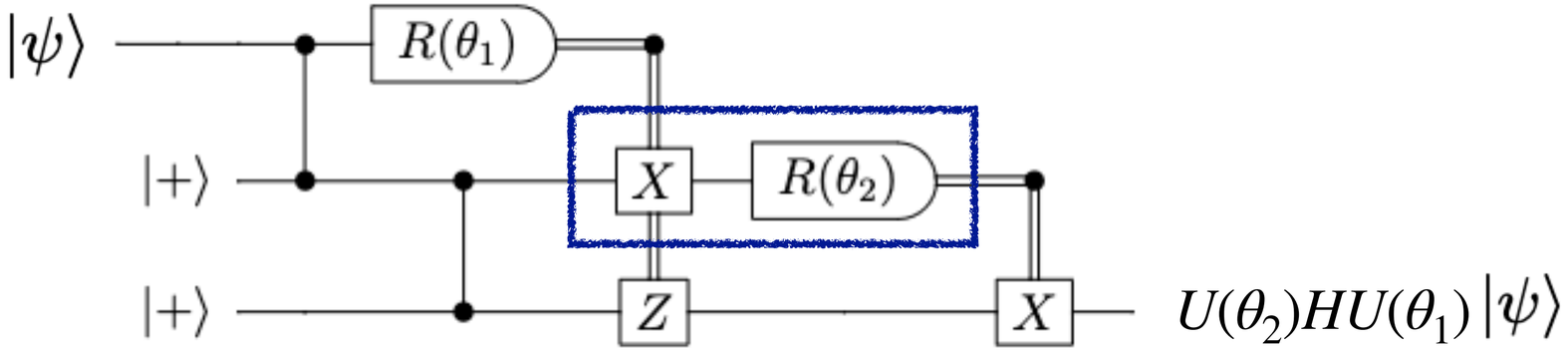
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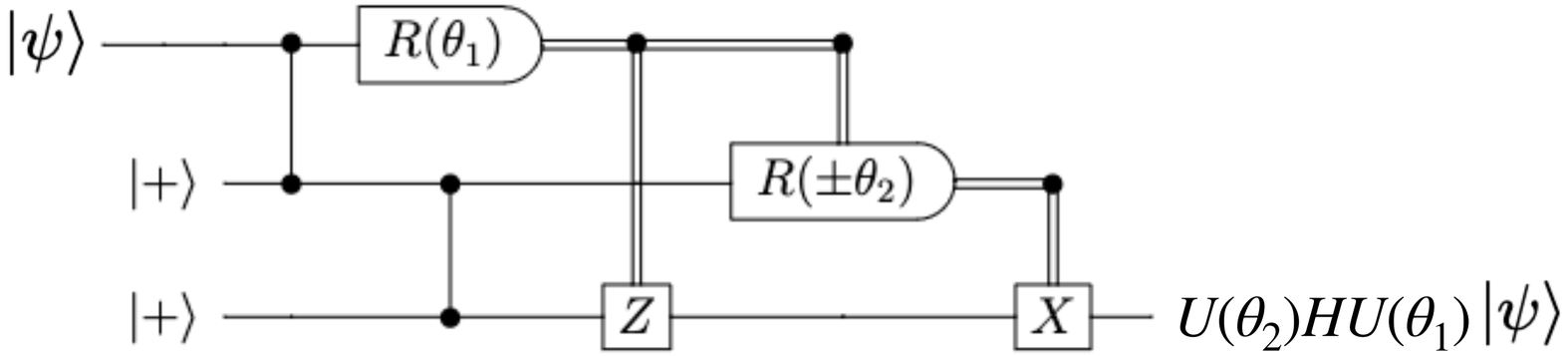
$U(\theta_2)HU(\theta_1)|\psi\rangle$



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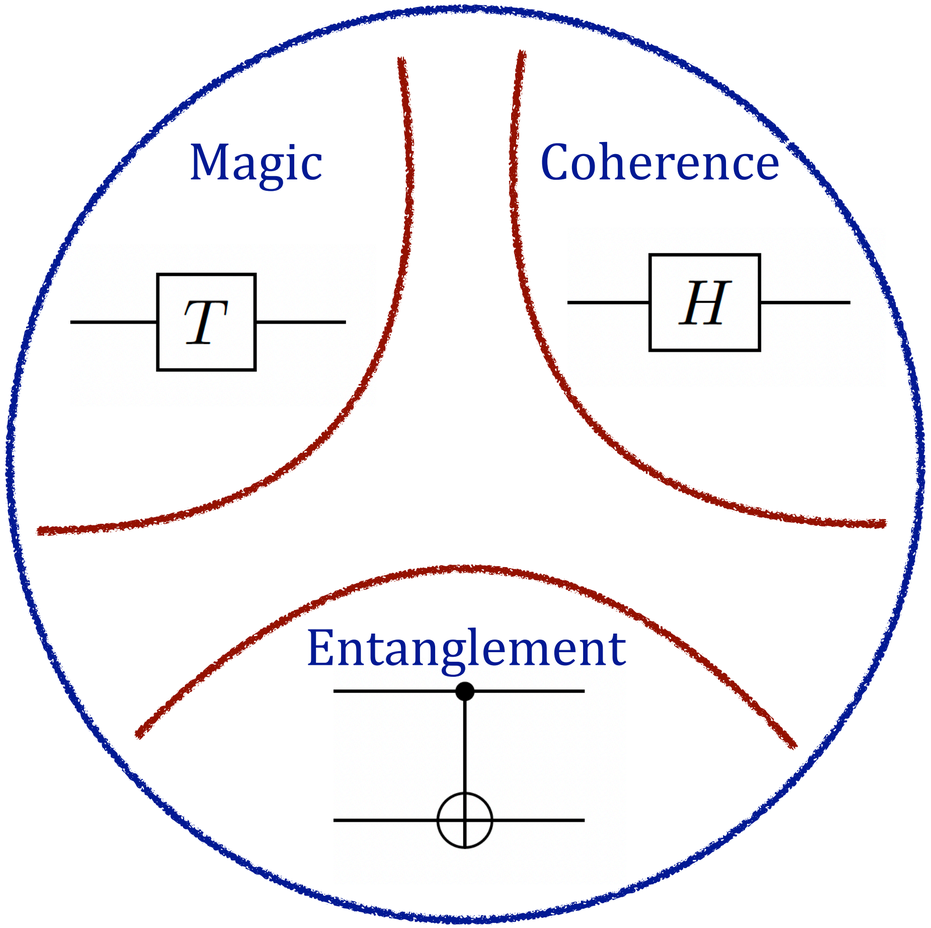


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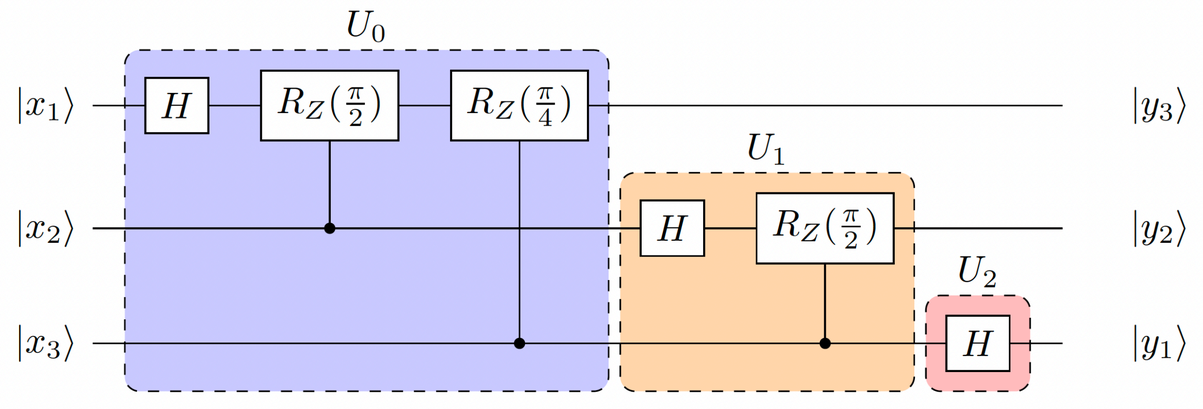


$U(\theta_2)HU(\theta_1)|\psi\rangle$

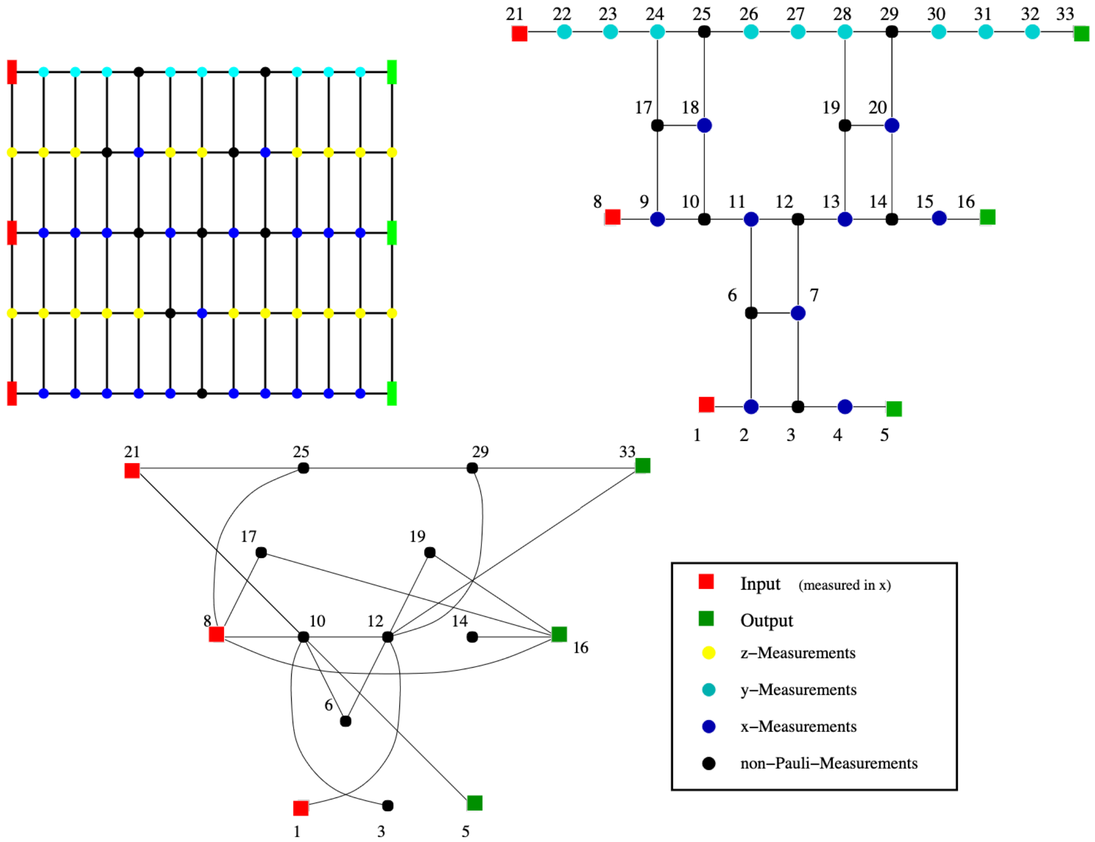
Why do we care about MBQC?


Magic
T
Coherence
H
Entanglement

Quantum circuit



One-way quantum computation



[Hein *et al.,* Proceedings of the International School of Physics "Enrico Fermi": Quantum Computers, Algorithms and Chaos 115-218 (2006).]

Universal Blind Quantum Computation

Authors: Anne Broadbent, Joseph Fitzsimons, Elham Kashefi | Authors Info & Claims

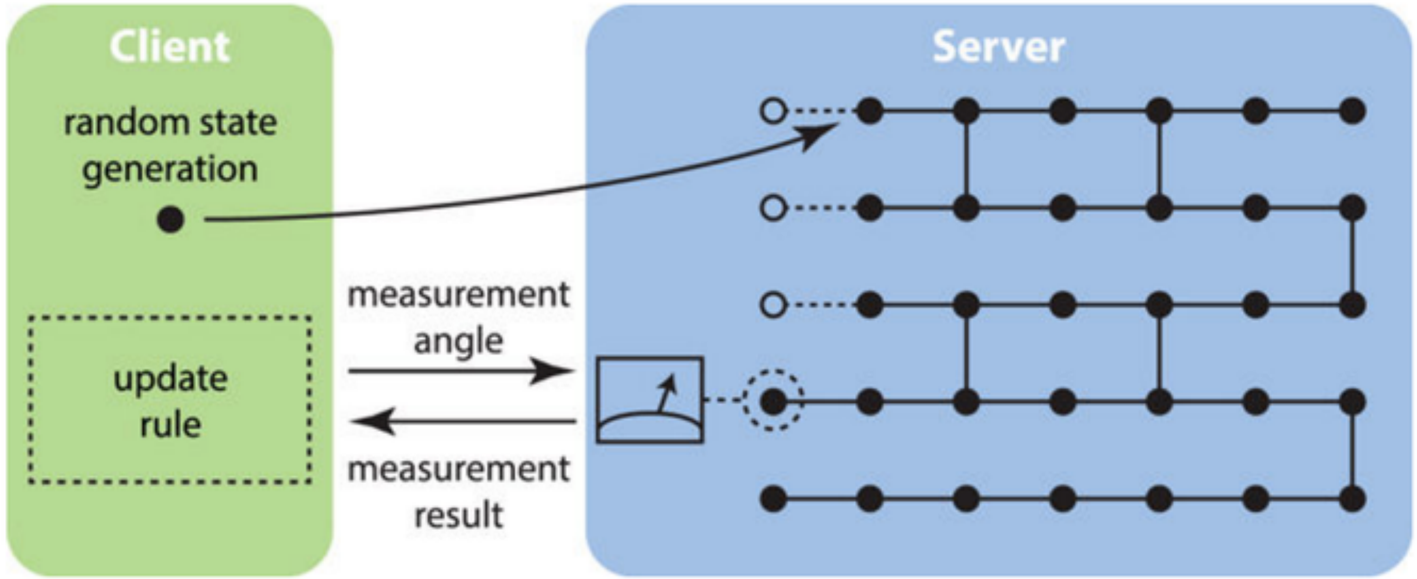
FOCS '09: Proceedings of the 2009 50th Annual IEEE Symposium on Foundations of Computer Science • Pages 517 - 526
https://doi.org/10.1109/FOCS.2009.36

Published: 25 October 2009 Publication History

59 0

Abstract

We present a protocol which allows a client to have a server carry out a quantum computation for her such that the client's inputs, outputs and computation remain perfectly private, and where she does not require any quantum computational power or memory. The client only needs to be able to prepare single qubits randomly chosen from a finite set and send them to the server, who has the balance of the required quantum computational resources. Our protocol is interactive: after the initial preparation of quantum states, the client and server use two-way classical communication which enables the client to drive the computation, giving single-qubit measurement instructions to the server, depending on previous measurement outcomes. Our protocol works for inputs and outputs that are either classical or quantum. We give an authentication protocol that allows the client to detect an interfering server; our scheme can also be made fault-tolerant. We also generalize our result to the setting of a purely classical client who communicates classically with two non-communicating entangled servers, in order to perform a blind quantum computation. By incorporating the authentication protocol, we show that any problem in BQP has an entangled two-prover interactive proof with a purely classical verifier. Our protocol is the first universal scheme which detects a cheating server, as well as the first protocol which does not require any quantum computation whatsoever on the client's side. The novelty of our approach is in using the unique features of measurement-based quantum computing which allows us to clearly distinguish between the quantum and classical aspects of a quantum computation.



[Broadbent, Fitzsimons, Kashefi, Proceedings of the 50th Annual IEEE Symposium on Foundations of Computer Science (FOCS '09) (2009).]

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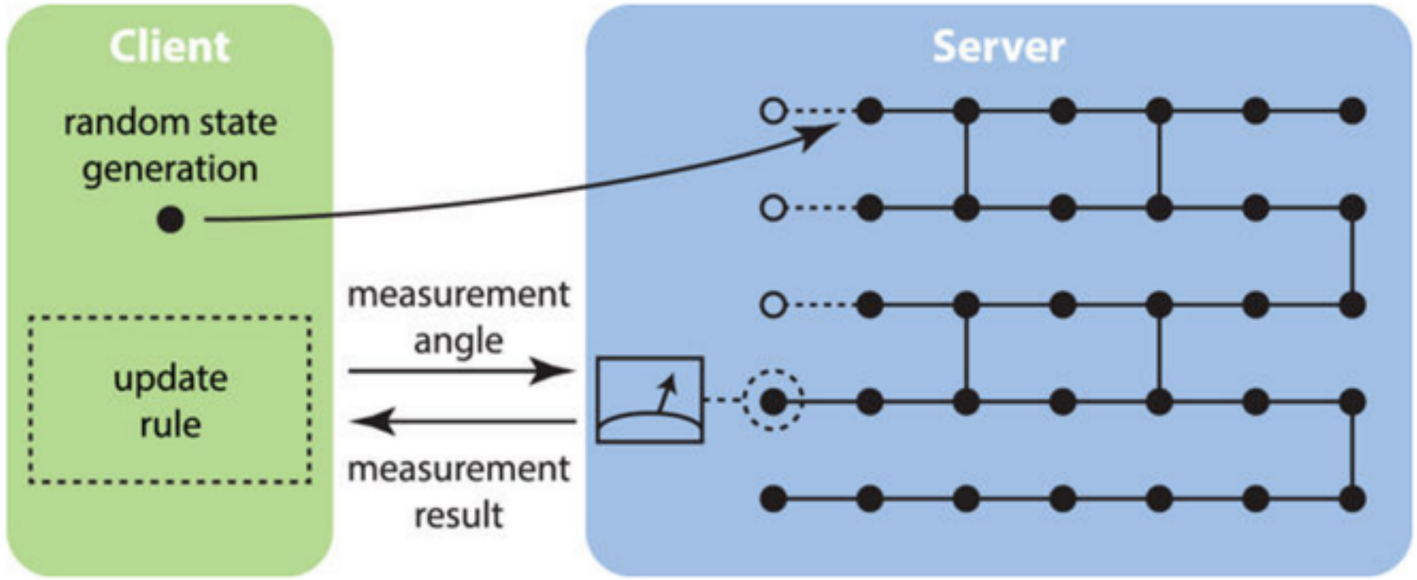
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Optical Quantum Computing

JEREMY L. O'BRIEN Authors Info & Affiliations

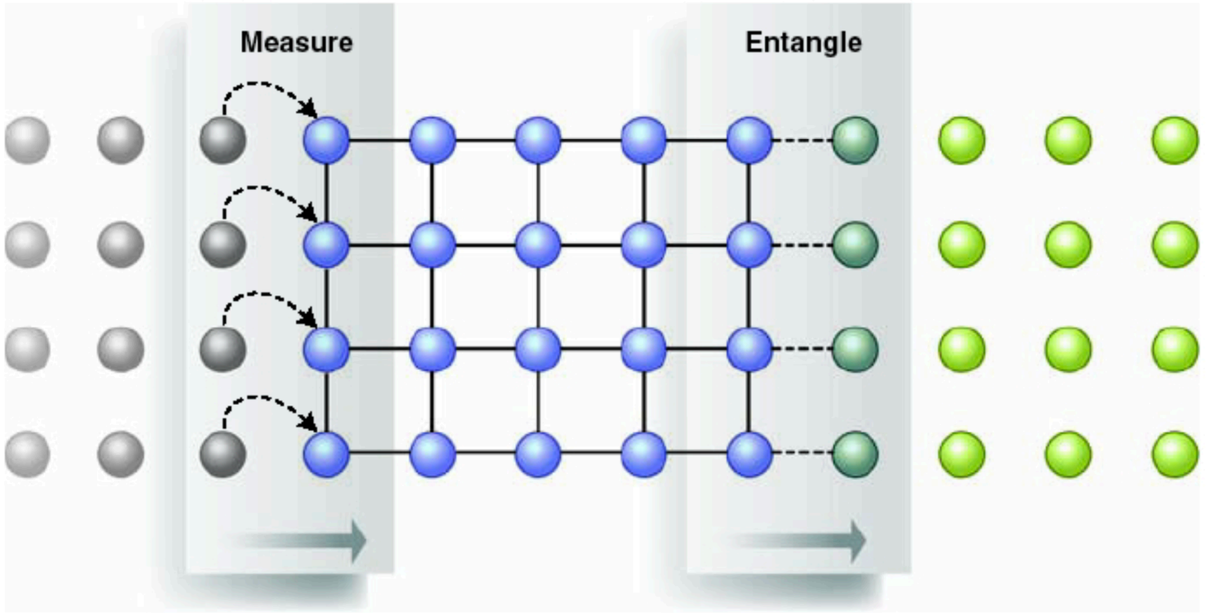
SCIENCE • 7 Dec 2007 • Vol 318, Issue 5856 • pp. 1567-1570 • DOI: 10.1126/science.1142892

15,190 1,140

Abstract

In 2001, all-optical quantum computing became feasible with the discovery that scalable quantum computing is possible using only single-photon sources, linear optical elements, and single-photon detectors. Although it was in principle scalable, the massive resource overhead made the scheme practically daunting. However, several simplifications were followed by proof-of-principle demonstrations, and recent approaches based on cluster states or error encoding have dramatically reduced this worrying resource overhead, making an all-optical architecture a serious contender for the ultimate goal of a large-scale quantum computer. Key challenges will be the realization of high-efficiency sources of indistinguishable single photons, low-loss, scalable optical circuits, high-efficiency single-photon detectors, and low-loss interfacing of these components.

Entangling as we go…



[O'Brien, Science **318**, 1467 (2007).]

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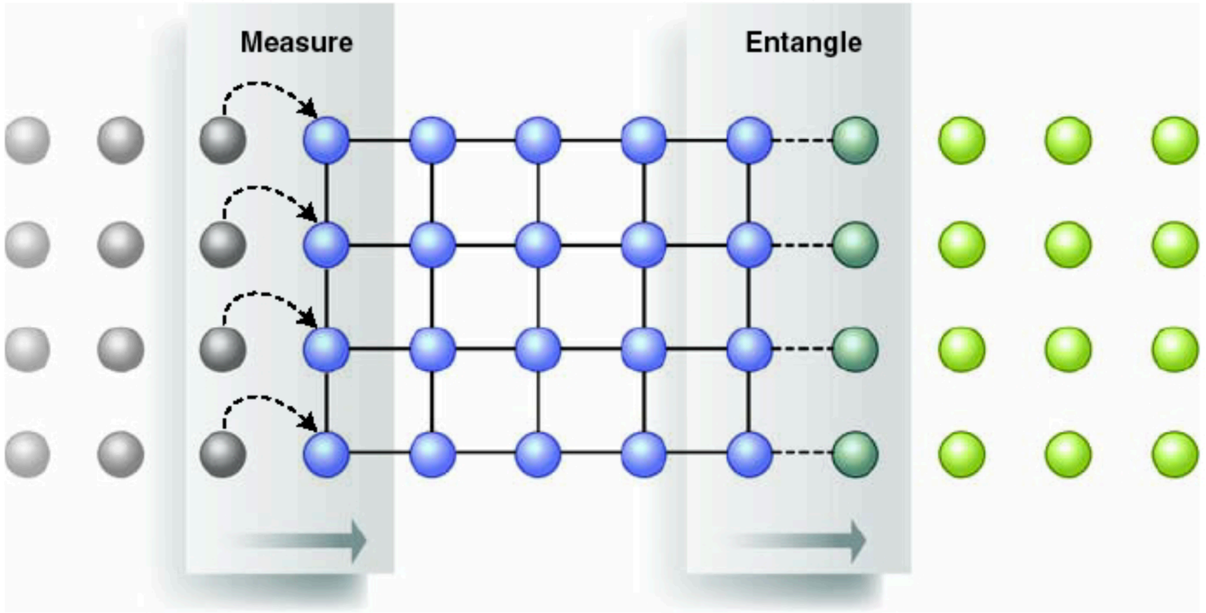
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Article | Open access | Published: 17 February 2023

Fusion-based quantum computation

Sara Bartolucci, Patrick Birchall, Hector Bombín, Hugo Cable, Chris Dawson, Mercedes Gimeno-Segovia, Eric Johnston, Konrad Kieling, Naomi Nickerson ✉, Mihir Pant ✉, Fernando Pastawski, Terry Rudolph & Chris Sparrow

Nature Communications **14**, Article number: 912 (2023) | Cite this article

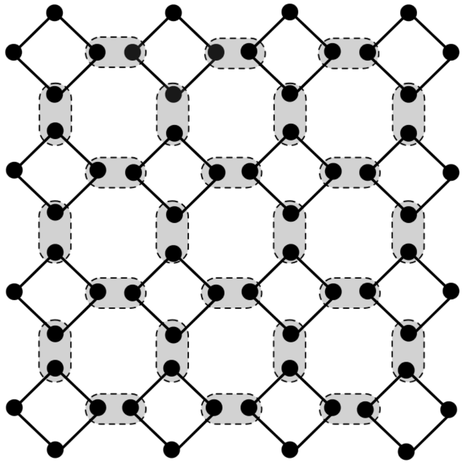
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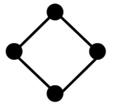
62k Accesses | **417** Citations | **114** Altmetric | Metrics

Abstract

The standard primitives of quantum computing include deterministic unitary entangling gates, which are not natural operations in many systems including photonics. Here, we present fusion-based quantum computation, a model for fault tolerant quantum computing constructed from physical primitives readily accessible in photonic systems. These are entangling measurements, called fusions, which are performed on the qubits of small constant sized entangled resource states. Probabilistic photonic gates as well as errors are directly dealt with by the quantum error correction protocol. We show that this computational model can achieve a higher threshold than schemes reported in literature. We present a ballistic scheme which can tolerate a 10.4% probability of suffering photon loss in each fusion, which corresponds to a 2.7% probability of loss of each individual photon. The architecture is also highly modular and has reduced classical processing requirements compared to previous photonic quantum computing architectures.

Fusion Network



i) Resource states 

ii) Fusion measurements 

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

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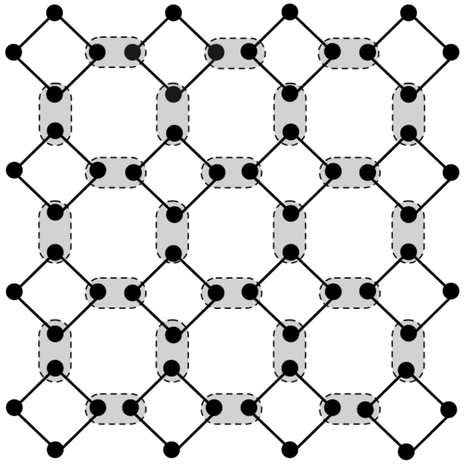
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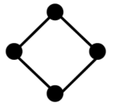
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Fusion Network

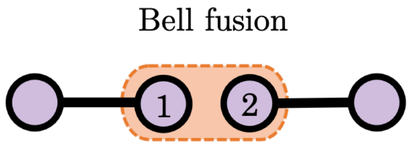


i) Resource states 

ii) Fusion measurements 

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

Exercise



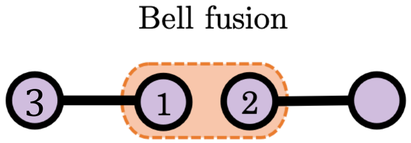
$M_1 = X_1X_2$
$M_2 = Z_1Z_2$

	Success	Failure	Erasure
Stabilizers measured	X_1X_2 Z_1Z_2	Z_1Z_2	None

In linear optics, Bell fusions are simple to perform but do not deterministically generate entanglement.

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

Exercise



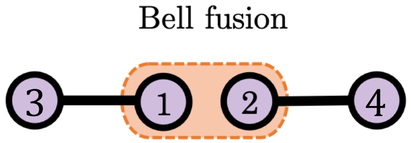
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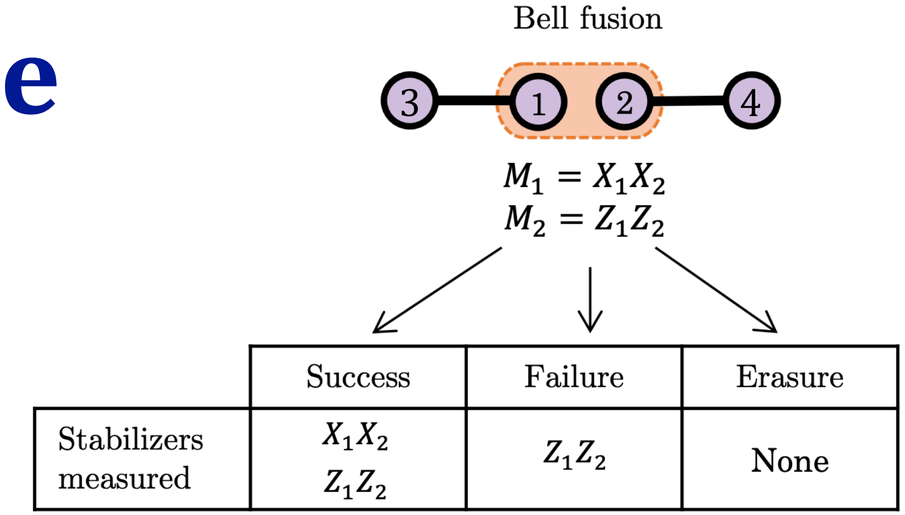
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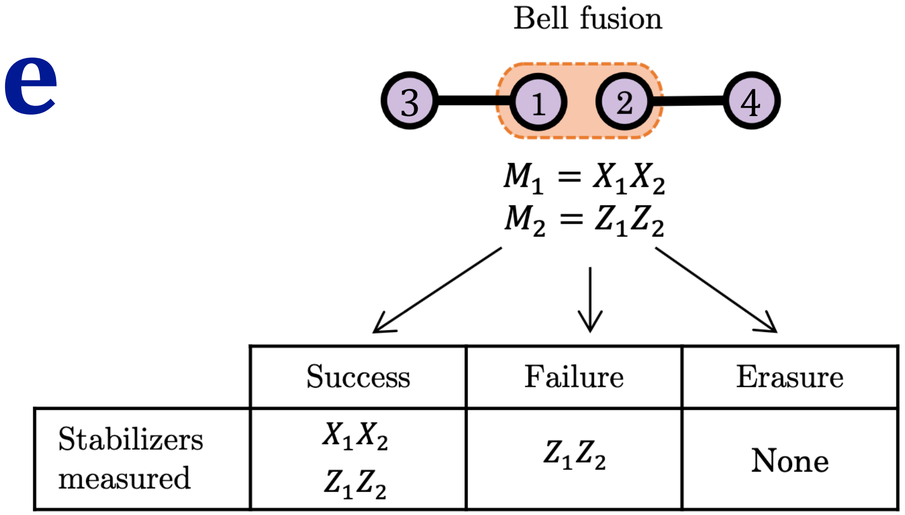
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In linear optics, Bell fusions are simple to perform but do not deterministically generate entanglement.

$$Z_1X_3 \longrightarrow$$
$$X_1Z_3 \longrightarrow$$
$$X_2Z_4 \longrightarrow$$
$$Z_2X_4 \longrightarrow$$

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

Exercise

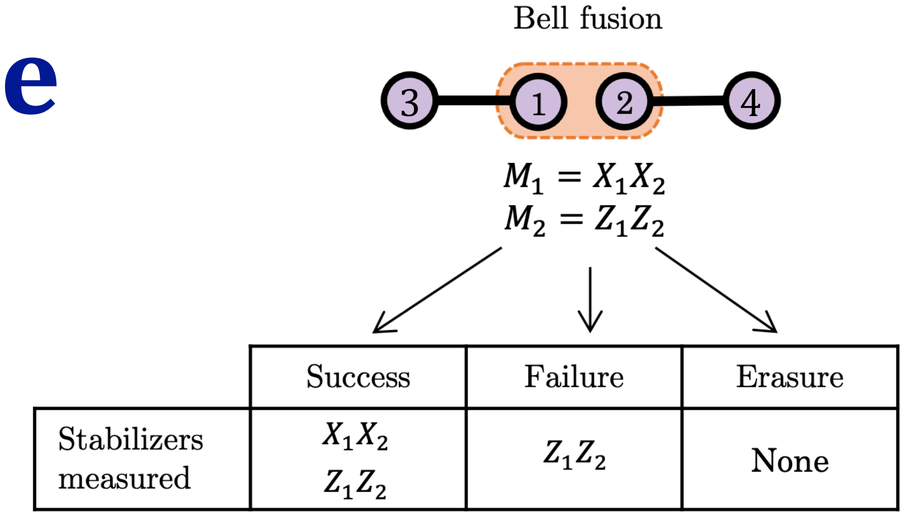


	Success	Failure	Erasure
Stabilizers measured	X_1X_2 Z_1Z_2	Z_1Z_2	None

In linear optics, Bell fusions are simple to perform but do not deterministically generate entanglement.

$$Z_1X_3 \xrightarrow{M_1}$$
$$X_1Z_3 \longrightarrow$$
$$X_2Z_4 \longrightarrow$$
$$Z_2X_4 \longrightarrow$$

Exercise



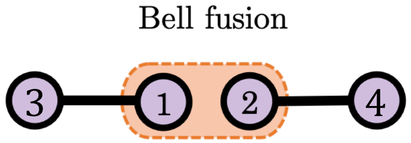
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$$Z_1X_3 \xrightarrow{M_1} (-1)^{m_1}X_1X_2 \longrightarrow$$
$$X_1Z_3 \longrightarrow$$
$$X_2Z_4 \longrightarrow$$
$$Z_2X_4 \longrightarrow$$

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

Exercise



$M_1 = X_1X_2$
$M_2 = Z_1Z_2$

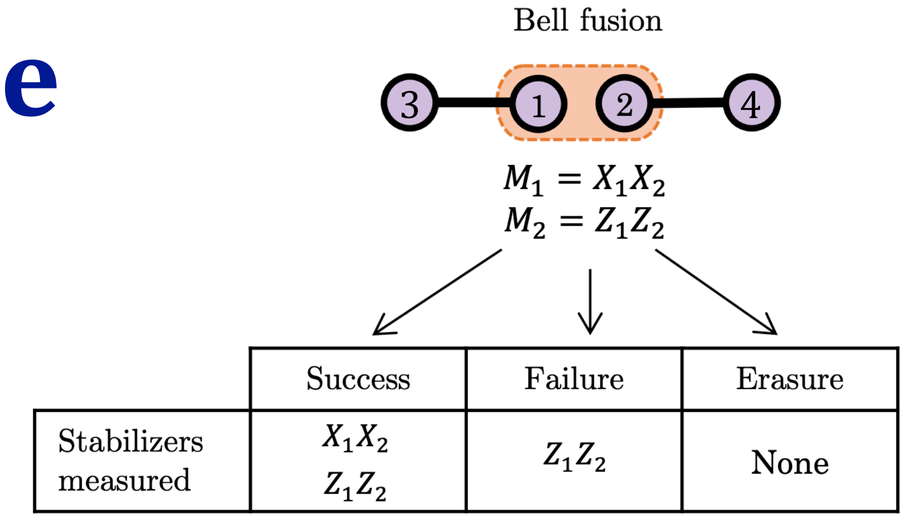
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Stabilizers measured	X_1X_2 Z_1Z_2	Z_1Z_2	None

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$$Z_1X_3 \xrightarrow{M_1} (-1)^{m_1}X_1X_2 \longrightarrow$$
$$X_1Z_3 \longrightarrow X_1Z_3 \longrightarrow$$
$$X_2Z_4 \longrightarrow$$
$$Z_2X_4 \longrightarrow$$

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

Exercise



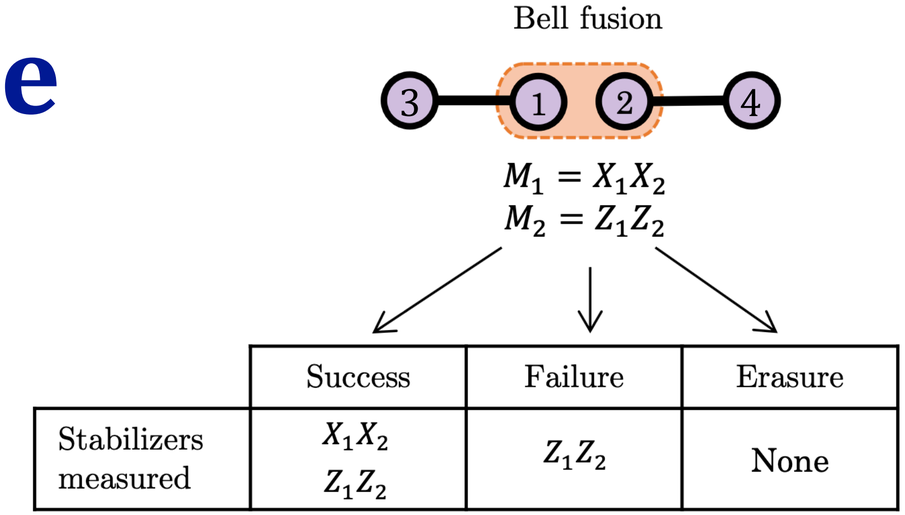
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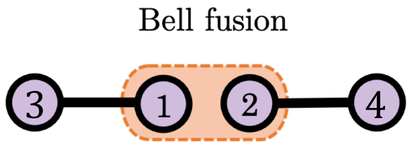
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$$X_1Z_3 \longrightarrow X_1Z_3 \longrightarrow$$
$$X_2Z_4 \longrightarrow X_2Z_4 \longrightarrow$$
$$Z_2X_4 \longrightarrow Z_1Z_2X_3X_4 \longrightarrow$$

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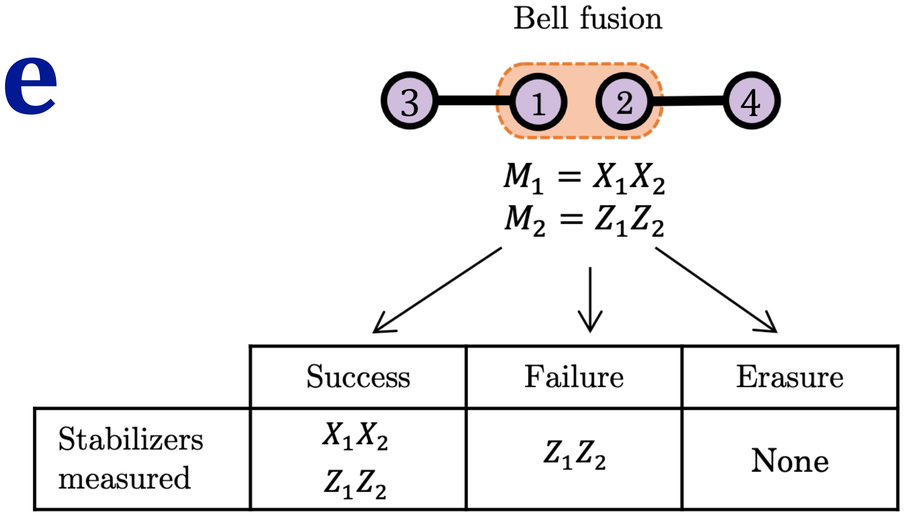
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$$
\begin{aligned}
Z_1X_3 &\xrightarrow{M_1} (-1)^{m_1}X_1X_2 \xrightarrow{M_2} \\
X_1Z_3 &\longrightarrow X_1Z_3 \longrightarrow \\
X_2Z_4 &\longrightarrow X_2Z_4 \longrightarrow \\
Z_2X_4 &\longrightarrow Z_1Z_2X_3X_4 \longrightarrow
\end{aligned}
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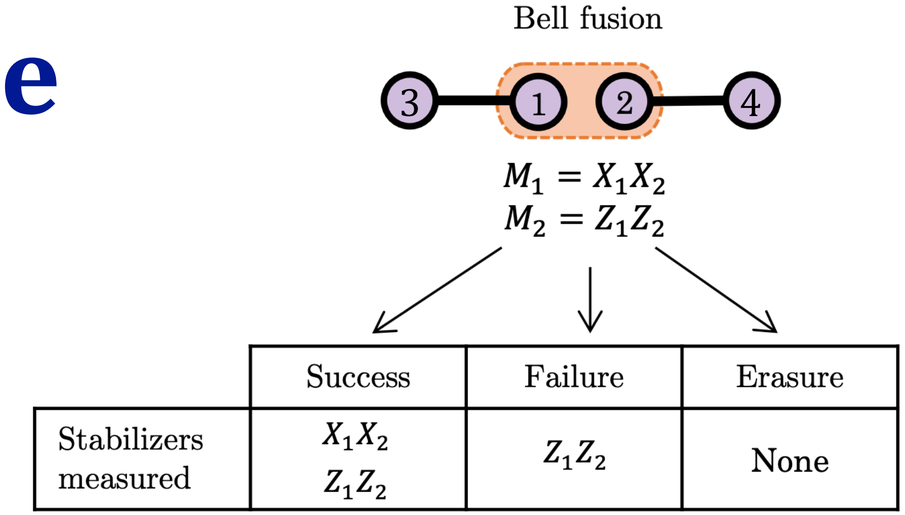
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Z_1X_3 &\xrightarrow{M_1} (-1)^{m_1}X_1X_2 &&\xrightarrow{M_2} (-1)^{m_1}X_1X_2\\
X_1Z_3 &\longrightarrow X_1Z_3 &&\longrightarrow\\
X_2Z_4 &\longrightarrow X_2Z_4 &&\longrightarrow\\
Z_2X_4 &\longrightarrow Z_1Z_2X_3X_4 &&\longrightarrow
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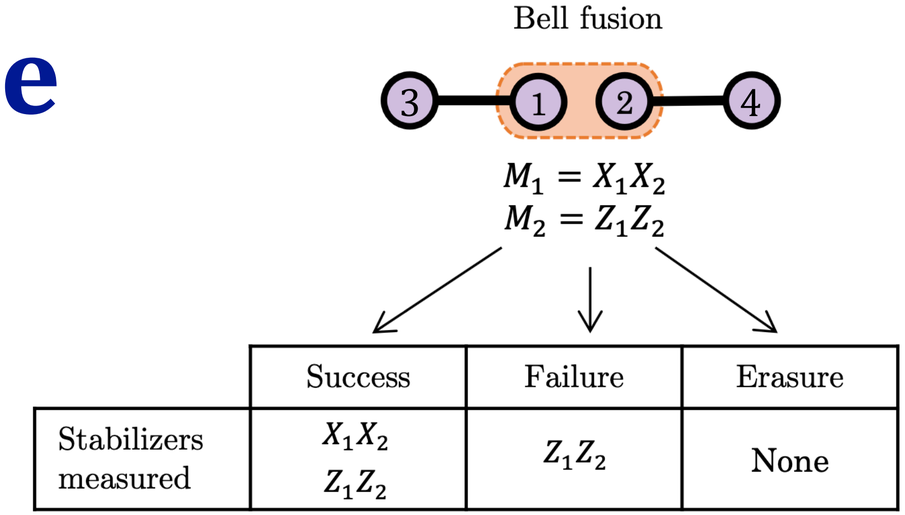
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X_1Z_3 &\longrightarrow X_1Z_3 &&\longrightarrow (-1)^{m_2}Z_1Z_2\\
X_2Z_4 &\longrightarrow X_2Z_4 &&\longrightarrow\\
Z_2X_4 &\longrightarrow Z_1Z_2X_3X_4 &&\longrightarrow
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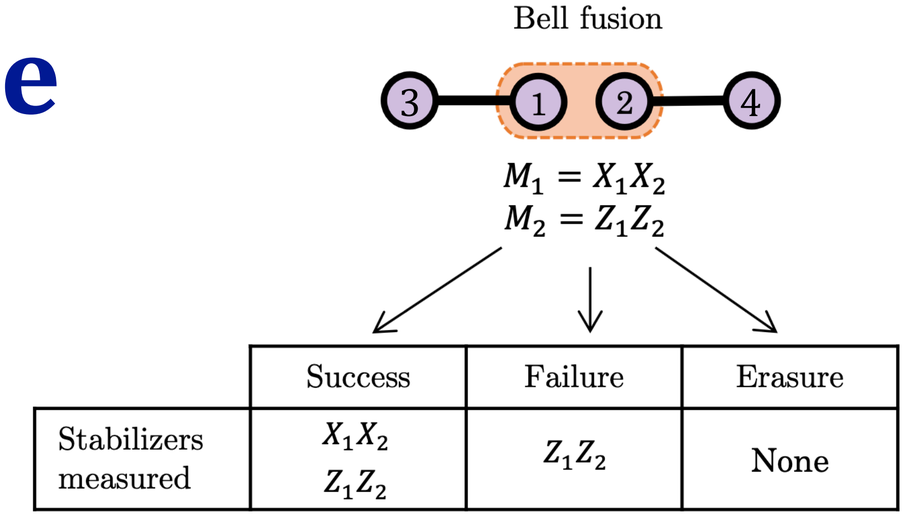
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X_1Z_3 &\longrightarrow X_1Z_3 &&\longrightarrow (-1)^{m_2}Z_1Z_2\\
X_2Z_4 &\longrightarrow X_2Z_4 &&\longrightarrow X_1X_2Z_3Z_4\\
Z_2X_4 &\longrightarrow Z_1Z_2X_3X_4 &&\longrightarrow
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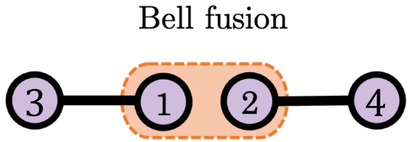
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X_1Z_3 &\longrightarrow X_1Z_3 &&\longrightarrow (-1)^{m_2}Z_1Z_2\\
X_2Z_4 &\longrightarrow X_2Z_4 &&\longrightarrow X_1X_2Z_3Z_4\\
Z_2X_4 &\longrightarrow Z_1Z_2X_3X_4 &&\longrightarrow Z_1Z_2X_3X_4
\end{aligned}
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[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

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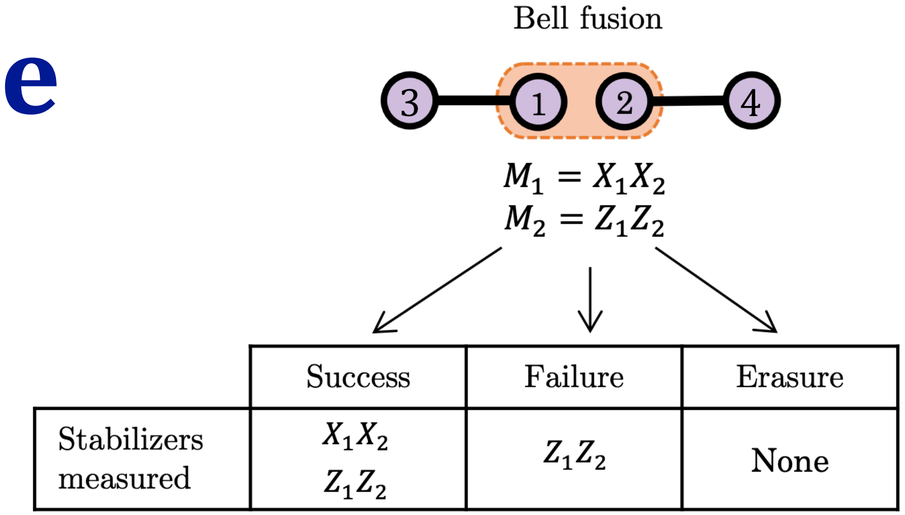
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X_1Z_3 &\longrightarrow X_1Z_3 &&\longrightarrow (-1)^{m_2}Z_1Z_2\\
X_2Z_4 &\longrightarrow X_2Z_4 &&\longrightarrow X_1X_2Z_3Z_4\\
Z_2X_4 &\longrightarrow Z_1Z_2X_3X_4 &&\longrightarrow Z_1Z_2X_3X_4
\end{aligned}
$$

Exercise



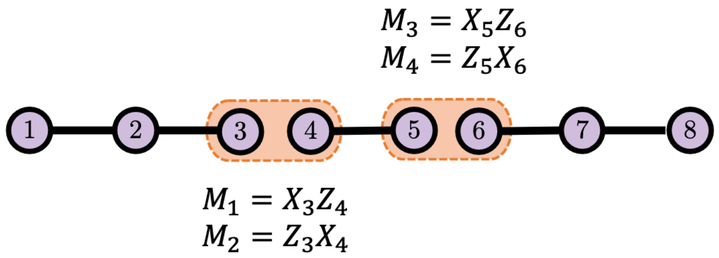
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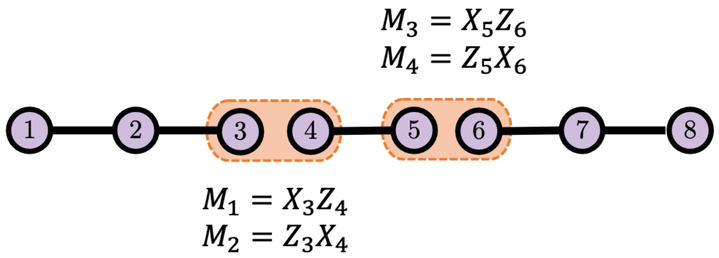
$$
\begin{aligned}
Z_1X_3 &\xrightarrow{M_1} (-1)^{m_1}X_1X_2 &&\xrightarrow{M_2} (-1)^{m_1}X_1X_2 \\
X_1Z_3 &\longrightarrow X_1Z_3 &&\longrightarrow (-1)^{m_2}Z_1Z_2 \\
X_2Z_4 &\longrightarrow X_2Z_4 &&\longrightarrow X_1X_2Z_3Z_4 \\
Z_2X_4 &\longrightarrow Z_1Z_2X_3X_4 &&\longrightarrow Z_1Z_2X_3X_4
\end{aligned}
\quad \rightarrow \quad
\begin{aligned}
&(-1)^{m_1}X_1X_2 \\
&(-1)^{m_2}Z_1Z_2 \\
&(-1)^{m_1}Z_3Z_4 \\
&(-1)^{m_2}X_3X_4
\end{aligned}
$$

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

Exercise



Exercise



X_1Z_2

$Z_1X_2Z_3$

Z_2X_3

X_4Z_5

Z_4X_5

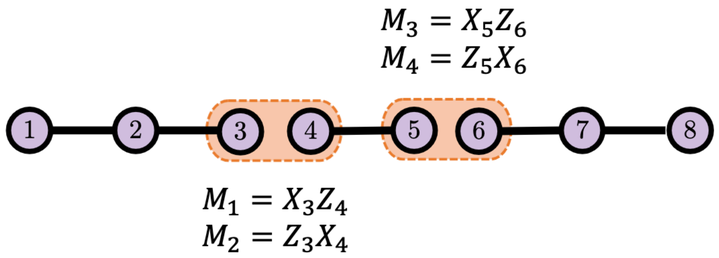
X_6Z_7

$Z_6X_7Z_8$

Z_7X_8

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

Exercise



X_1Z_2

$Z_1X_2Z_3$

Z_2X_3

X_4Z_5

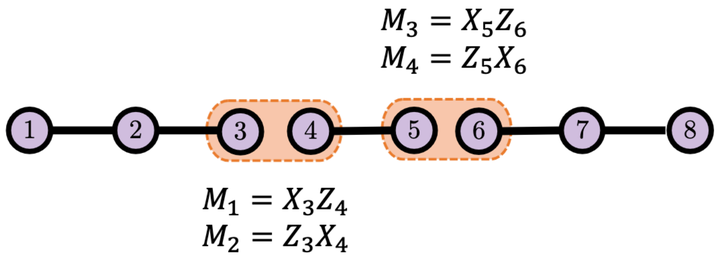
Z_4X_5

X_6Z_7

$Z_6X_7Z_8$

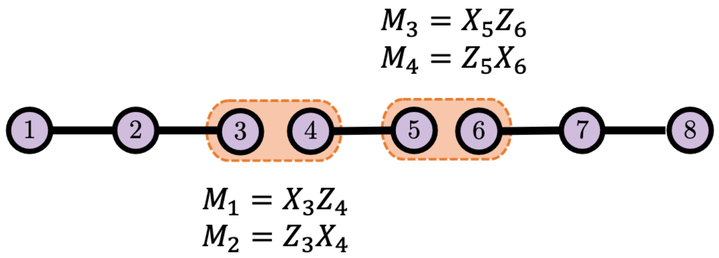
Z_7X_8

Exercise



$$
\begin{aligned}
&X_1Z_2 && X_1Z_2\\
&Z_1X_2Z_3 && (-1)^{m_1}X_3Z_4\\
&Z_2X_3 && (-1)^{m_2}Z_3X_4\\
&X_4Z_5 \quad\rightarrow && (-1)^{m_3}X_5Z_6\\
&Z_4X_5 && (-1)^{m_4}Z_5X_6\\
&X_6Z_7 && (-1)^{m_2+m_4}Z_1X_2Z_7\\
&Z_6X_7Z_8 && (-1)^{m_1+m_3}Z_2X_7Z_8\\
&Z_7X_8 && Z_7X_8
\end{aligned}
$$

Exercise



$$X_1Z_2$$
$$Z_1X_2Z_3$$
$$Z_2X_3$$
$$X_4Z_5$$
$$Z_4X_5$$
$$X_6Z_7$$
$$Z_6X_7Z_8$$
$$Z_7X_8$$



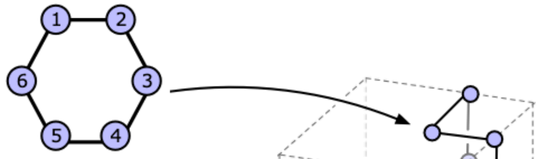
$$X_1Z_2$$
$$(-1)^{m_1}X_3Z_4$$
$$(-1)^{m_2}Z_3X_4$$
$$(-1)^{m_3}X_5Z_6$$
$$(-1)^{m_4}Z_5X_6$$
$$(-1)^{m_2+m_4}Z_1X_2Z_7$$
$$(-1)^{m_1+m_3}Z_2X_7Z_8$$
$$Z_7X_8$$

Solution:



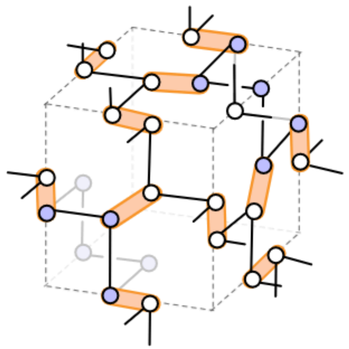
6-ring Fusion Network

a) 6-ring resource state unit cell



$S_1 = Z_1X_2Z_3I_4I_5I_6$
$S_2 = I_1Z_2X_3Z_4I_5I_6$
$S_3 = I_1I_2Z_3X_4Z_5I_6$
$S_4 = I_1I_2I_3Z_4X_5Z_6$
$S_5 = Z_1I_2I_3I_4Z_5X_6$
$S_6 = X_1Z_2I_3I_4I_5Z_6$

b) 6-ring fusion network

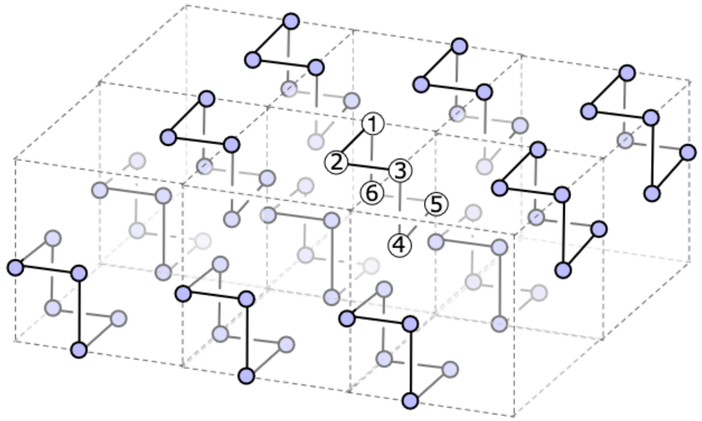


c) Fusion measurements



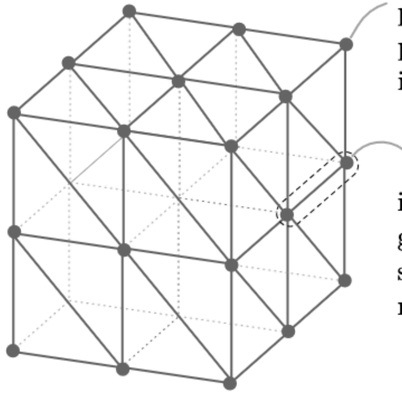
$M_1 = X_1X_2$
$M_2 = Z_1Z_2$

d) Multi unit cell layout



Resource states form planes in (1,1,1) direction. Each state fuses to 3 states in the layer above (e.g. 1,3,5) and 3 below (e.g. 2,4,6)

e) Syndrome graph



i) Vertices represent parity checks. Each parity check has 12 incident edges.

ii) Each edge on the graph represents a single fusion measurement bit.

Horizontal and vertical edges correspond to XX fusion outcomes. Diagonal edges correspond to ZZ outcomes.

[Bartolucci *et al.*, Nature Communications **14**, 912 (2023).]

PHYSICAL REVIEW X **6,** 021043 (2016)

Trading Classical and Quantum Computational Resources

Sergey Bravyi, Graeme Smith, and John A. Smolin
IBM T.J. Watson Research Center, 1101 Kitchawan Road, Yorktown Heights, New York 10598, USA
(Received 29 March 2016; published 29 June 2016)

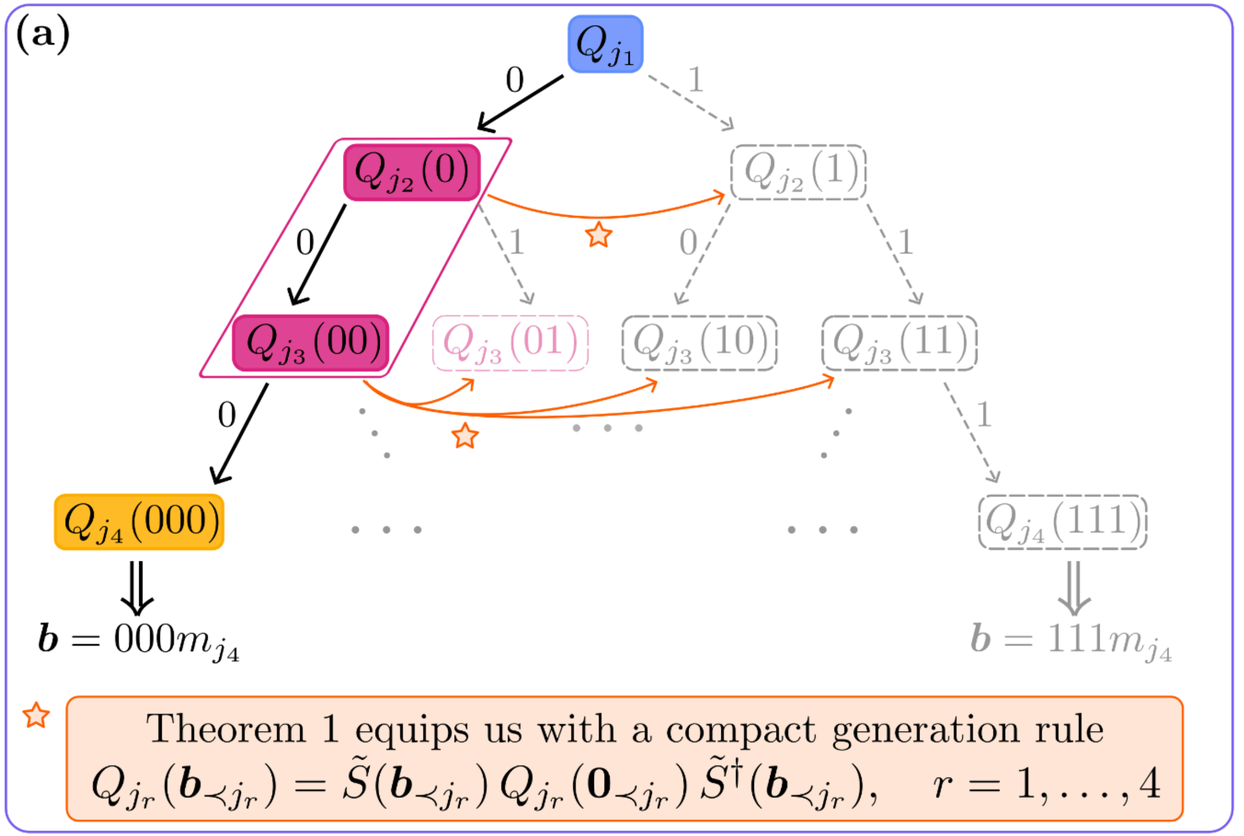
We propose examples of a hybrid quantum-classical simulation where a classical computer assisted by a small quantum processor can efficiently simulate a larger quantum system. First, we consider sparse quantum circuits such that each qubit participates in $O(1)$ two-qubit gates. It is shown that any sparse circuit on $n+k$ qubits can be simulated by sparse circuits on n qubits and a classical processing that takes time $2^{O(k)}\mathrm{poly}(n)$. Second, we study Pauli-based computation (PBC), where allowed operations are nondestructive eigenvalue measurements of n-qubit Pauli operators. The computation begins by initializing each qubit in the so-called magic state. This model is known to be equivalent to the universal quantum computer. We show that any PBC on $n+k$ qubits can be simulated by PBCs on n qubits and a classical processing that takes time $2^{O(k)}\mathrm{poly}(n)$. Finally, we propose a purely classical algorithm that can simulate a PBC on n qubits in a time $2^{\alpha n}\mathrm{poly}(n)$, where $\alpha \approx 0.94$. This improves upon the brute-force simulation method, which takes time $2^n\mathrm{poly}(n)$. Our algorithm exploits the fact that n-fold tensor products of magic states admit a low-rank decomposition into n-qubit stabilizer states.

DOI: 10.1103/PhysRevX.6.021043 Subject Areas: Quantum Information

Normal-Form Characterization of Adaptivity and Entanglement in Pauli-Based Computation

Filipa C. R. Peres,[1,2] Selman Ipek,[3,4] Daniel Lovsted,[5] Joshua Nevin,[5] and Anne Broadbent[5]
[1] *Quantum Thermodynamics and Computation Group, Electromagnetism and Matter Physics Department, University of Granada, Granada, Spain*
[2] *Department for Quantum Information and Computation at Kepler (QUICK), Johannes Kepler University, Linz, Austria*
[3] *Institut für Theoretische Physik, Leibniz Universität Hannover, Hannover, Germany*
[4] *Department of Mathematics, Bilkent University, Ankara, Turkey*
[5] *Department of Mathematics and Statistics, University of Ottawa, Ottawa, Canada*

Pauli-based computation (PBC) is a measurement-based model of quantum computation that samples from output distributions of universal Clifford+T circuits. Here, we provide a normal form (NF) that completely characterizes its adaptive and entanglement structures. These insights unify PBC with one-way quantum computing as dual or 'time-reversed' counterparts. We use the NF to advance strategies for distributed computation and investigate magic-state injection rates. We also demonstrate an exponential improvement in the detection of unnecessary magic-state qubits, allowing their efficient removal. Our NF further enables ahead-of-time circuit compilation using PBC, reducing CNOT gate count and quantum memory of random quantum circuits on average by 50% and 40%, respectively, beating other state-of-the-art compilers.



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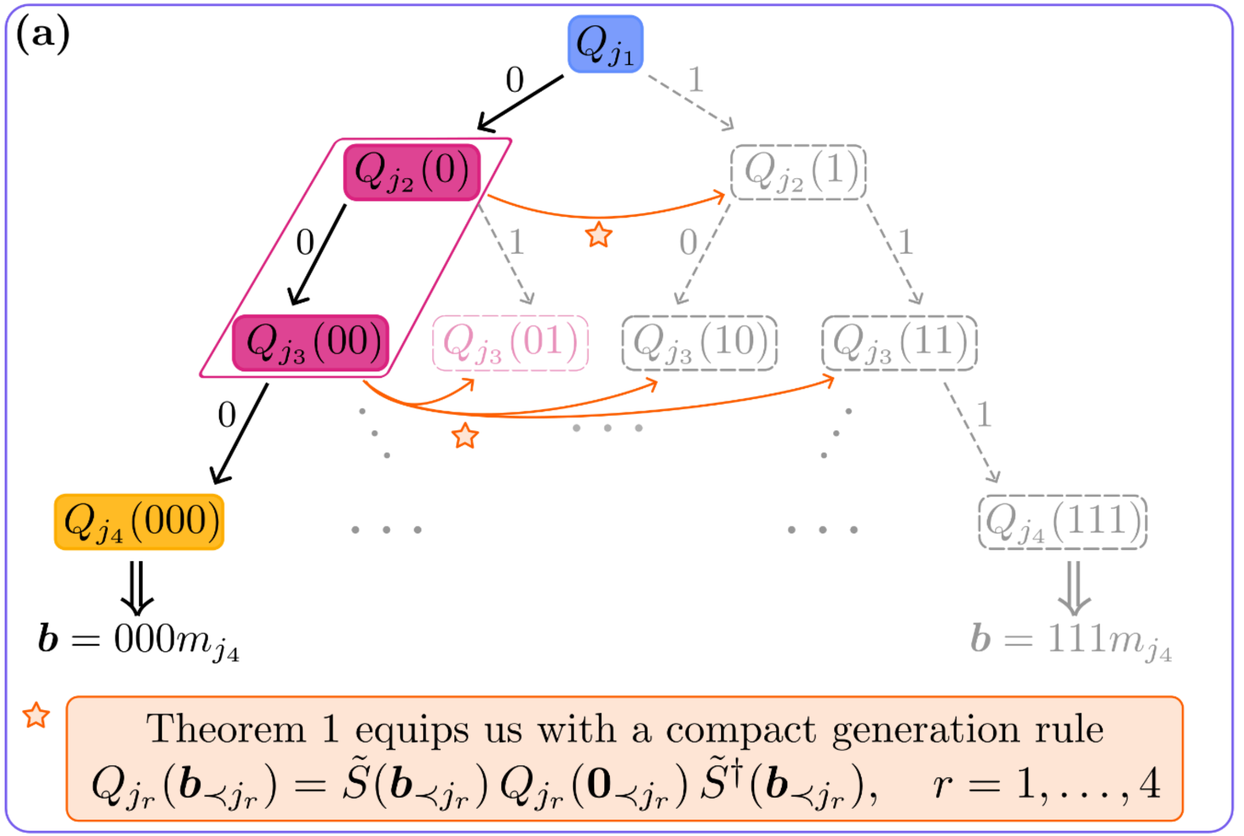
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Recap from previous classes…

Quantum circuits with:

- stabilizer state inputs,
- Clifford gates,
- and Pauli measurements

are efficiently classically simulable.

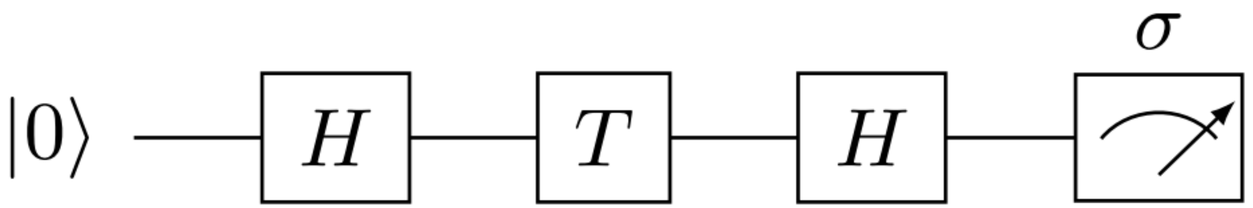
GOTTESMAN-KNILL THEOREM

But…

Clifford+T circuits are universal for quantum computation!

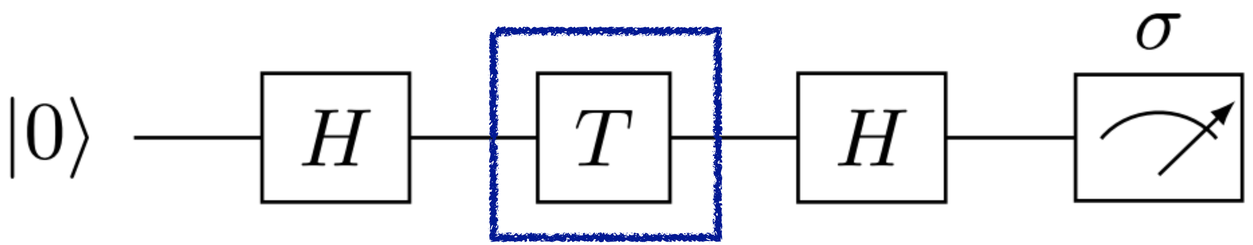
$$T = \mathrm{diag}(1, e^{i\pi/4}) \, .$$

Magic-state injection



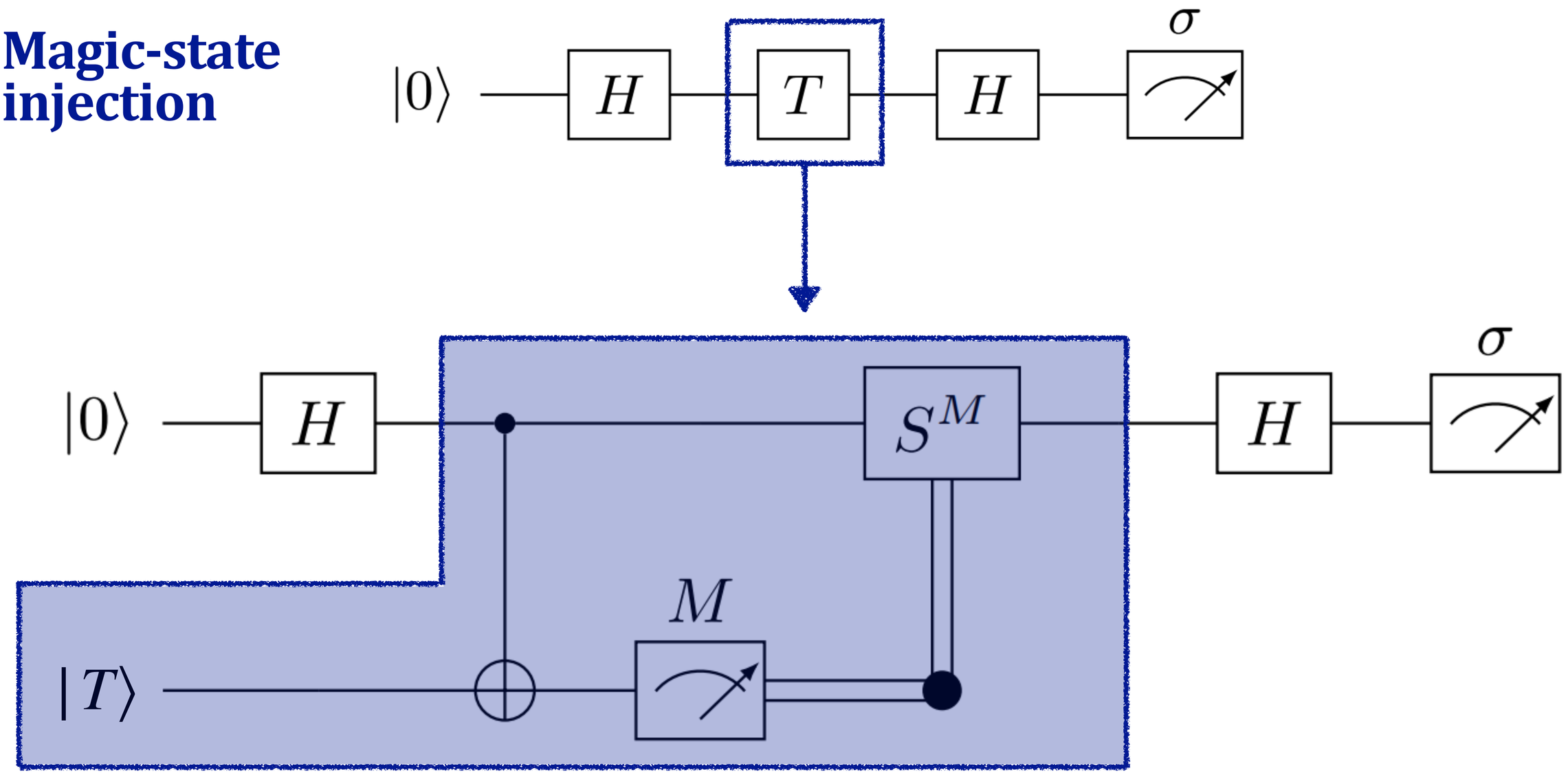
[Zhou, Leung, and Chuang, arXiv:quant-ph/0002039 (2000).]

Magic-state injection



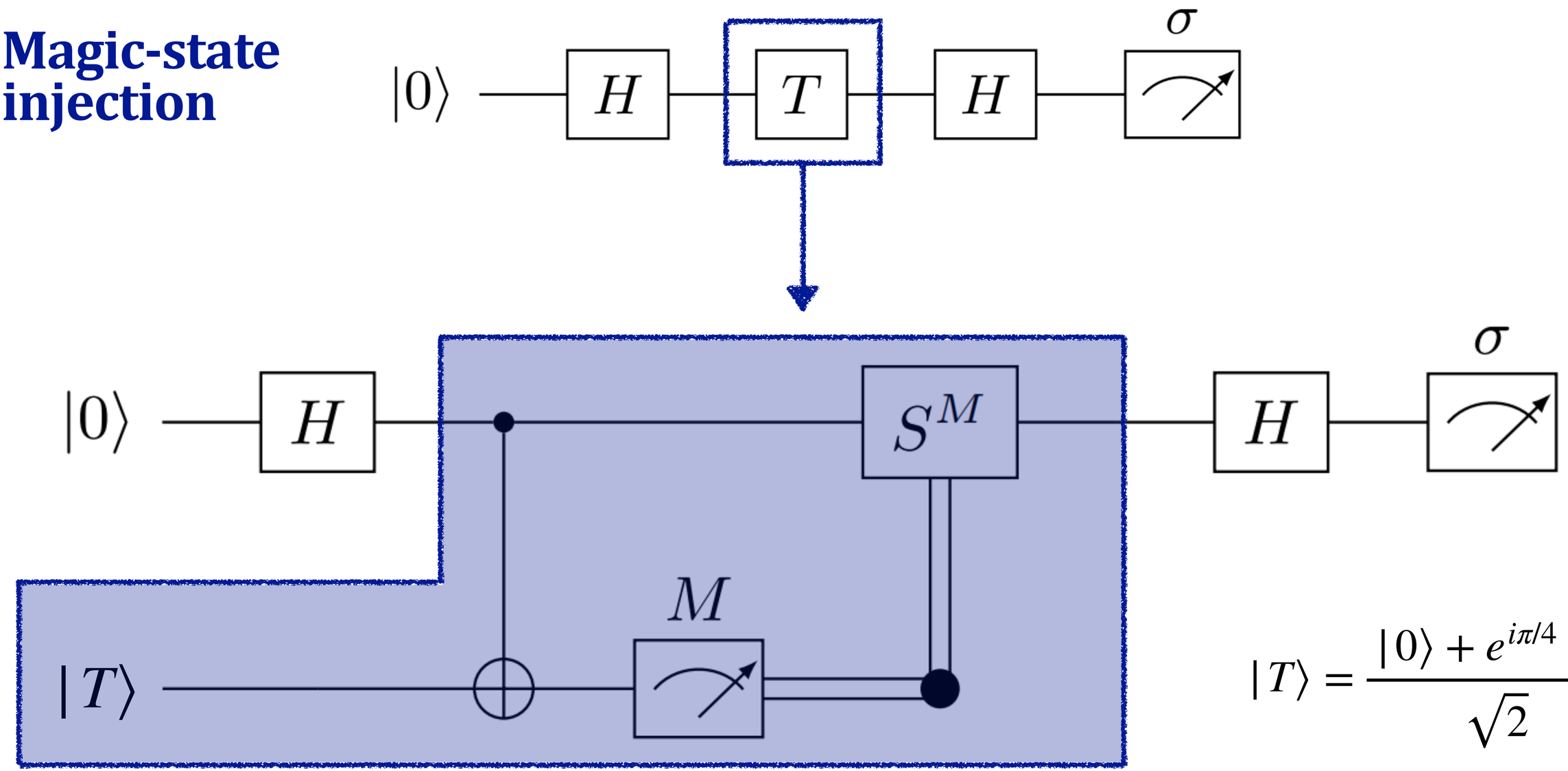
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Magic-state injection



[Zhou, Leung, and Chuang, arXiv:quant-ph/0002039 (2000).]

Magic-state injection



$$|T\rangle = \frac{|0\rangle + e^{i\pi/4}\,|1\rangle}{\sqrt{2}}$$

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$\mathscr{C}$

$\mathscr{Q}$

[Bravyi, Smith, and Smolin, Phys. Rev. X **6**, 021043 (2016).]

$\mathscr{C}$

- $|0\rangle^{\otimes n}$

$\mathscr{Q}$

[Bravyi, Smith, and Smolin, Phys. Rev. X **6**, 021043 (2016).]

$\mathscr{C}$

- $|0\rangle^{\otimes n}$
- t T gates

$\mathscr{Q}$

$\mathscr{C}$

- $|0\rangle^{\otimes n}$
- t T gates
- w Z-measurements

$\mathscr{Q}$

[Bravyi, Smith, and Smolin, Phys. Rev. X **6**, 021043 (2016).]

$\mathscr{C}$

- $|0\rangle^{\otimes n}$
- t T gates
- w Z-measurements



$\mathcal{Q}$

$\mathscr{C}$

- $|0\rangle^{\otimes n}$
- t T gates
- w Z-measurements



$\mathscr{Q}$

- $|T\rangle^{\otimes t}$

$\mathscr{C}$

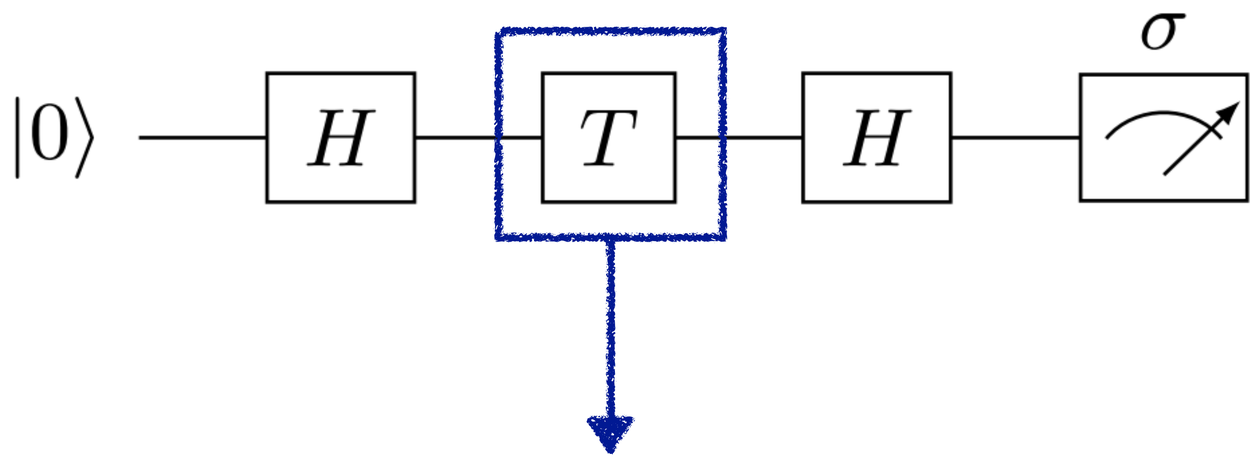
- $|0\rangle^{\otimes n}$
- t T gates
- w Z-measurements

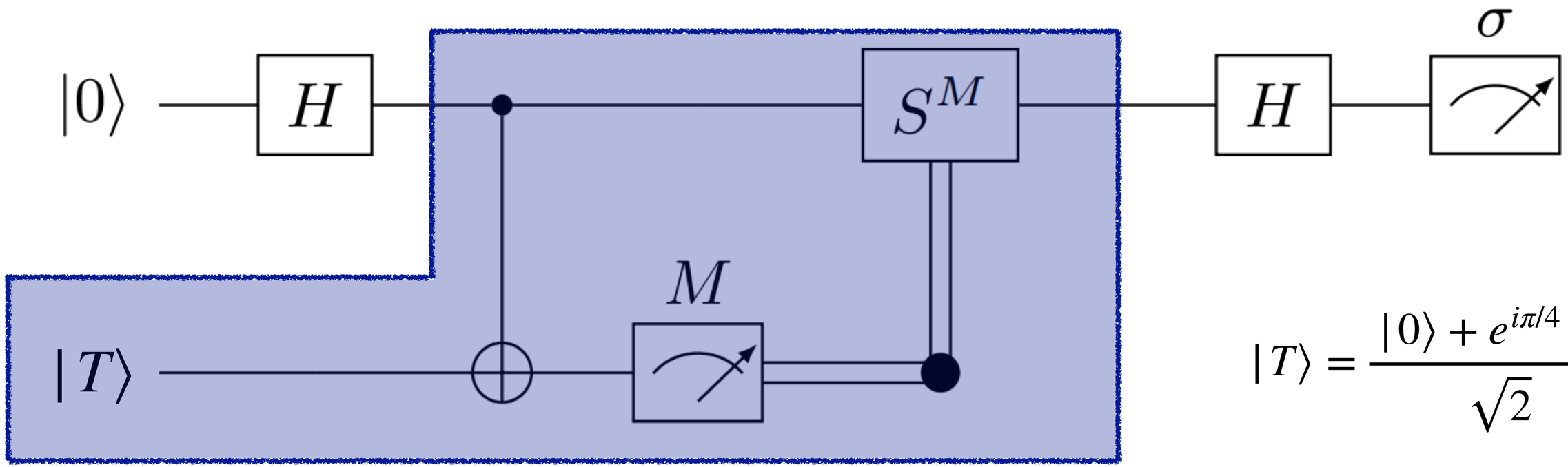
$\mathscr{Q}$

- $|T\rangle^{\otimes t}$
- at most t Pauli measurements

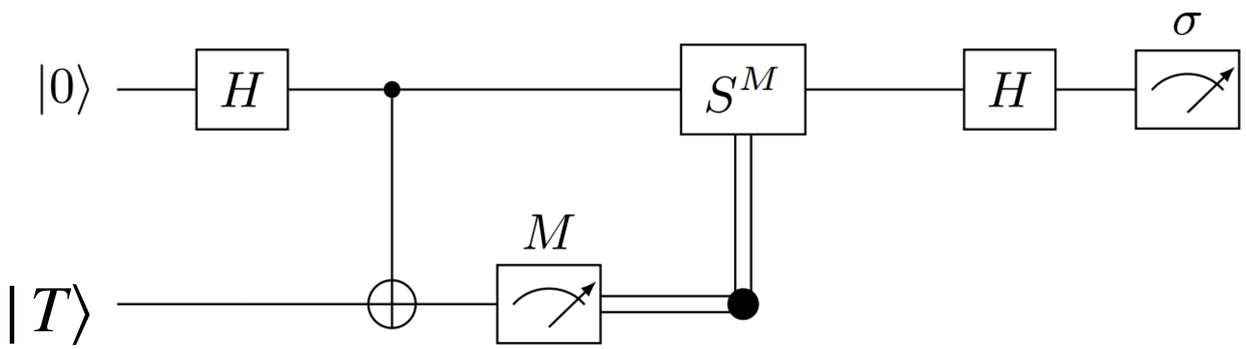
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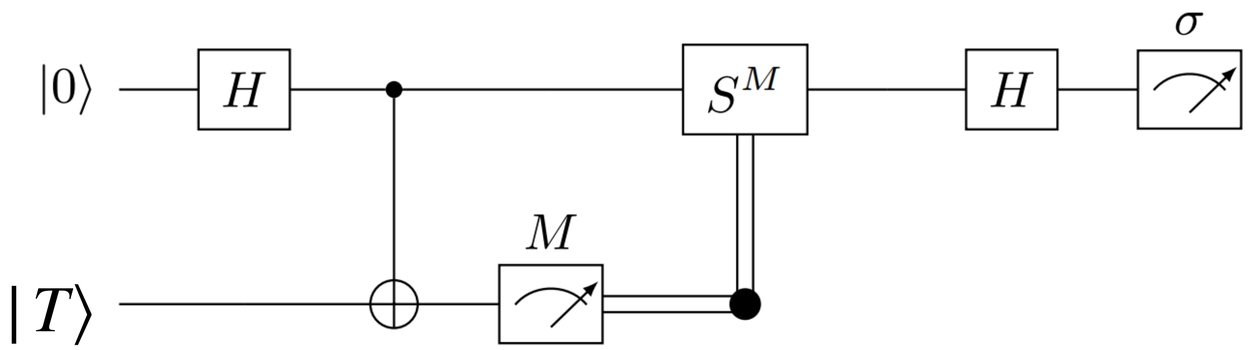
Example:

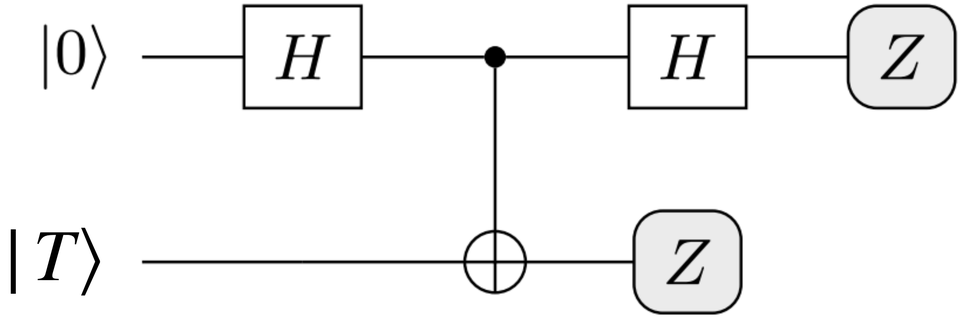


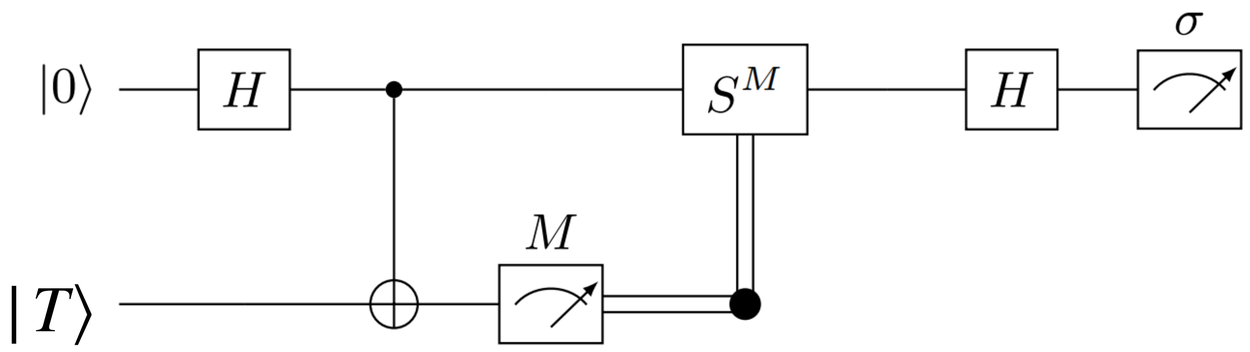


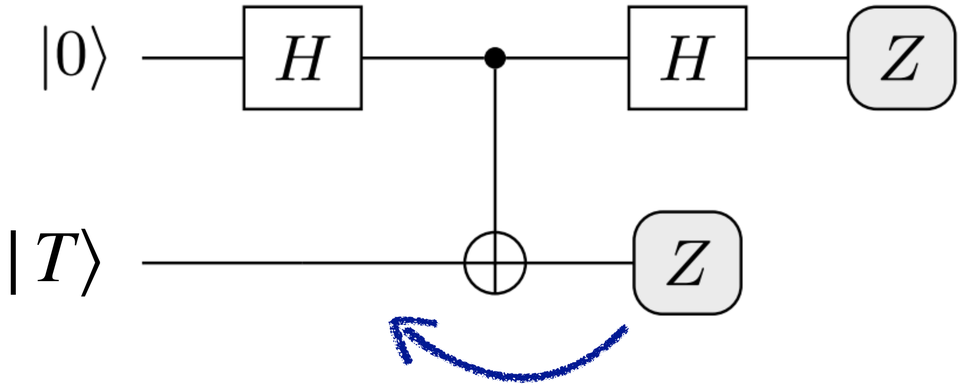
$$|T\rangle = \frac{|0\rangle + e^{i\pi/4}\,|1\rangle}{\sqrt{2}}$$

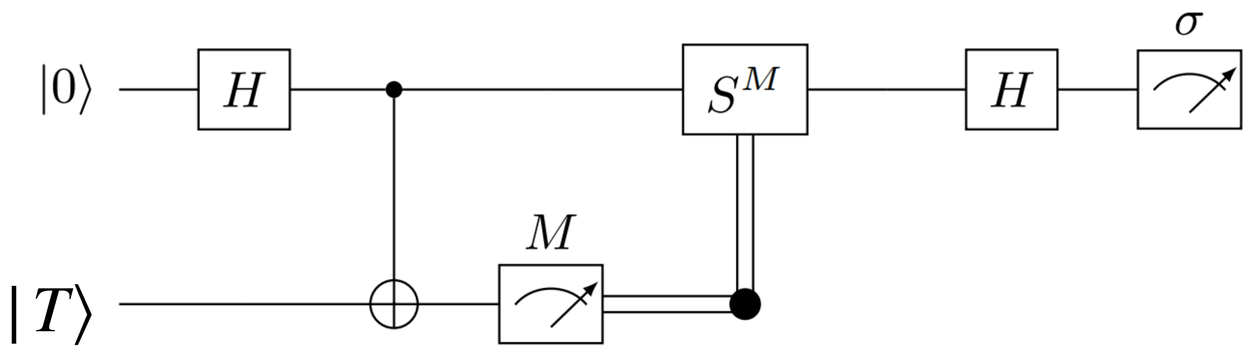

$|0\rangle$
$|T\rangle$
H
M
S^M
H
σ

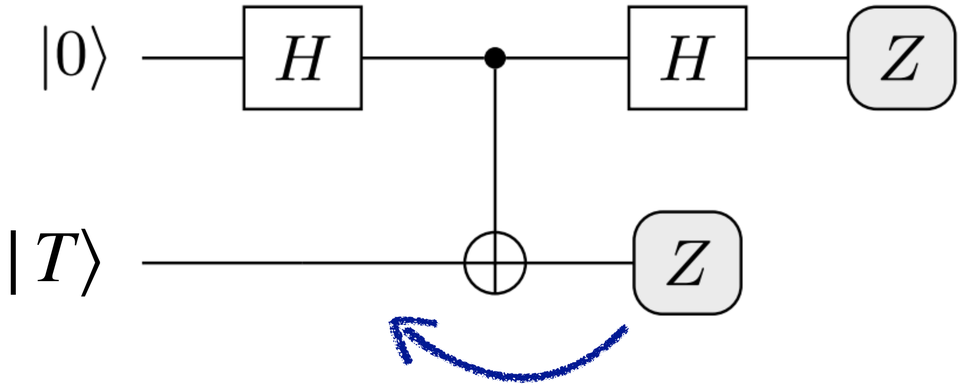

$|0\rangle$
$|T\rangle$
H
M
S^M
H
σ

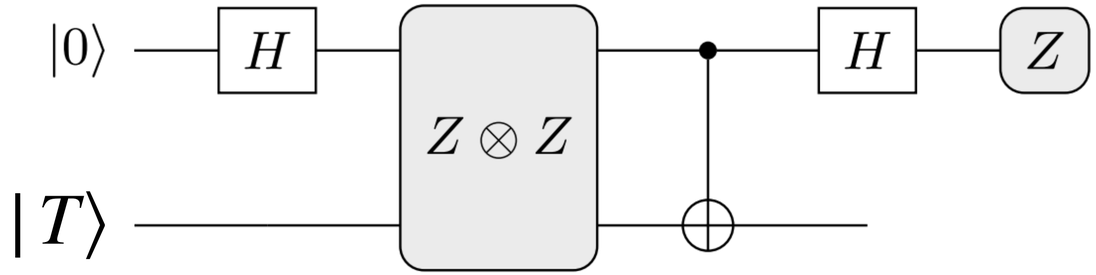

$|0\rangle$
$|T\rangle$
H
H
Z
Z

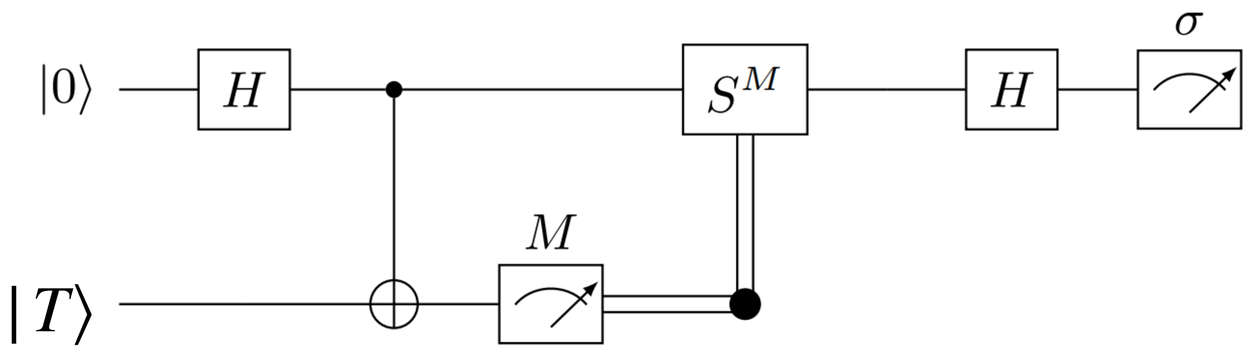

$|0\rangle$
$|T\rangle$
H
M
S^M
H
σ

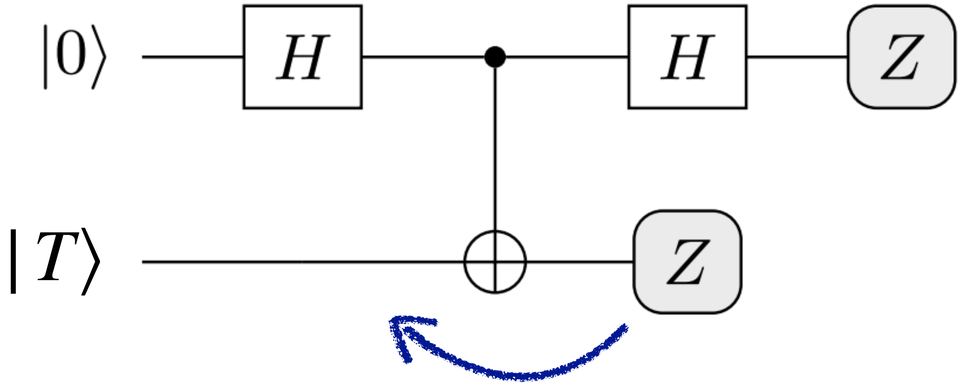

$|0\rangle$
$|T\rangle$
H
H
Z
Z

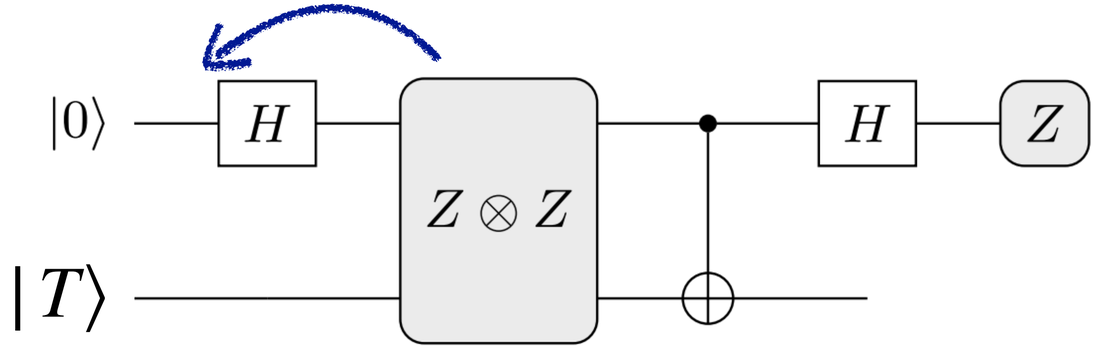

$|0\rangle$
H
S^M
H
σ
M
$|T\rangle$

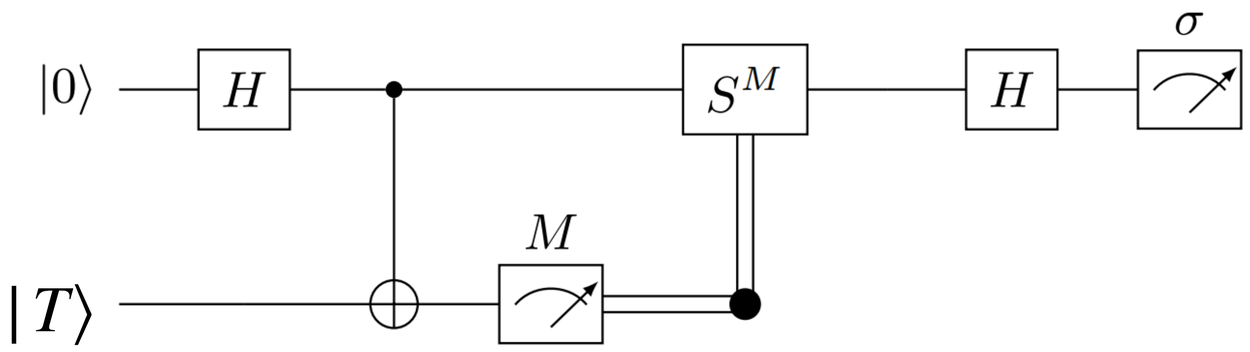

$|0\rangle$
H
H
Z
$|T\rangle$
Z

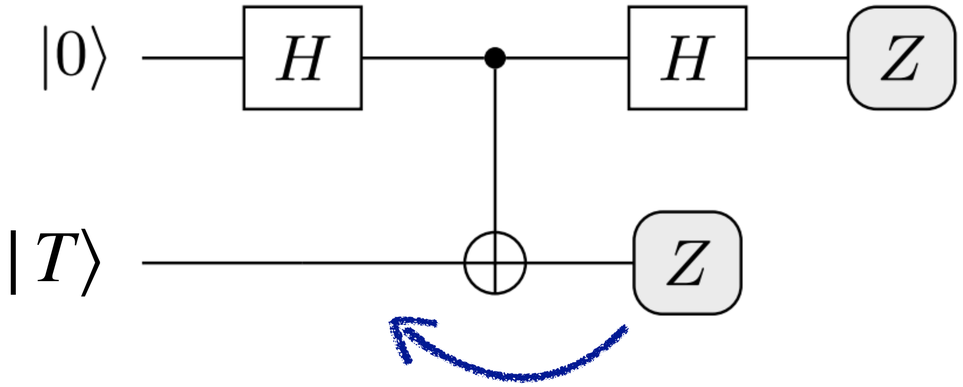

$|0\rangle$
H
$Z \otimes Z$
H
Z
$|T\rangle$

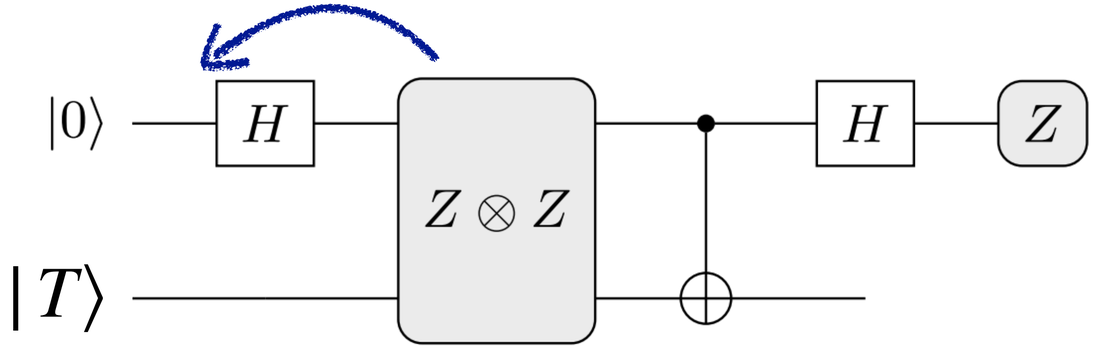

$|0\rangle$
H
S^M
H
σ
M
$|T\rangle$

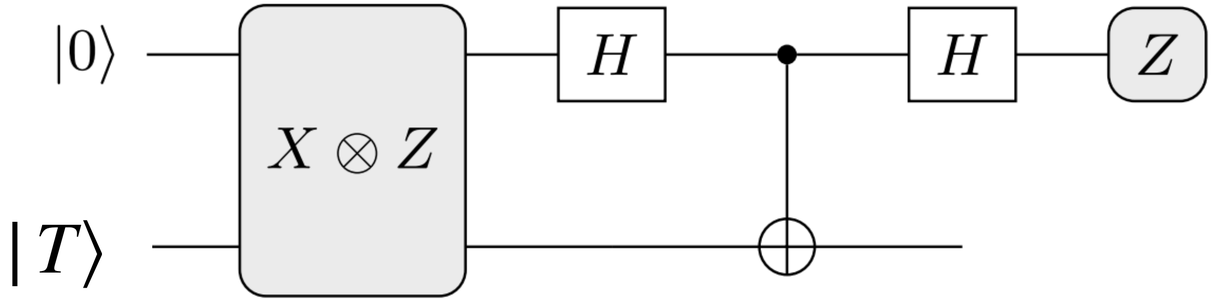

$|0\rangle$
H
H
Z
$|T\rangle$
Z

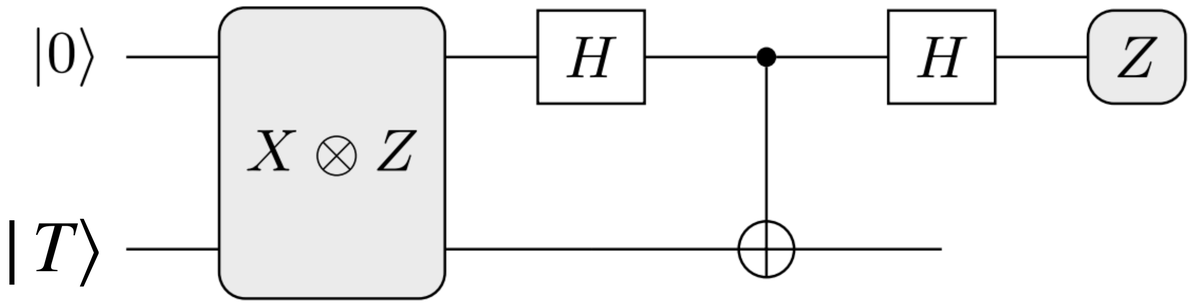

$|0\rangle$
H
$Z \otimes Z$
H
Z
$|T\rangle$

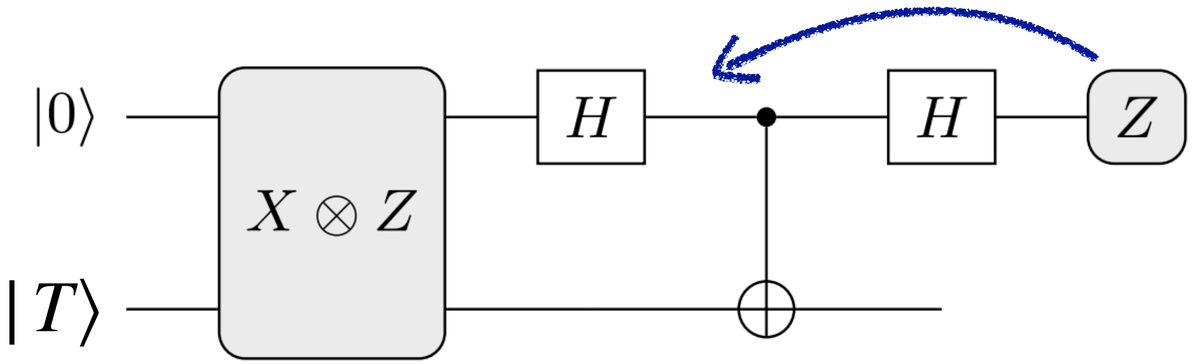

$|0\rangle$
$|T\rangle$
H
M
S^M
H
σ

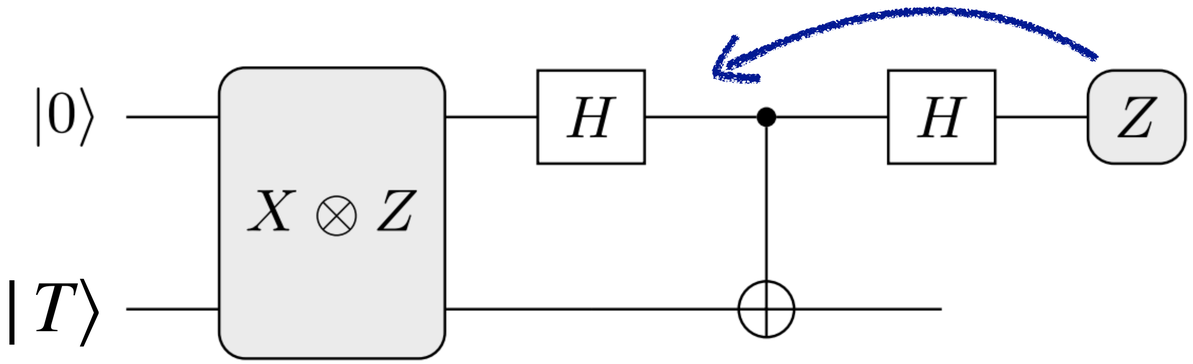

$|0\rangle$
$|T\rangle$
H
H
Z
Z

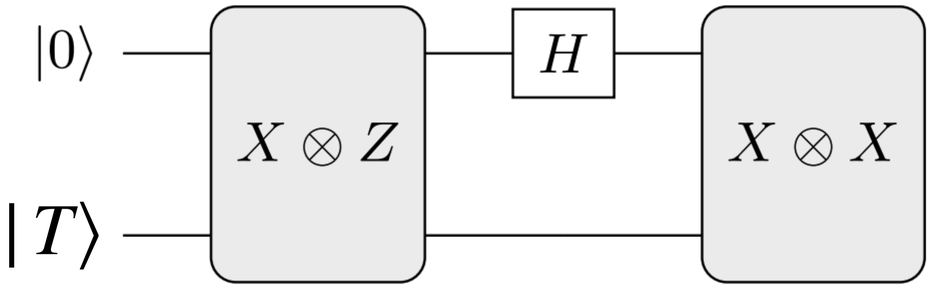

$|0\rangle$
$|T\rangle$
H
$Z \otimes Z$
H
Z

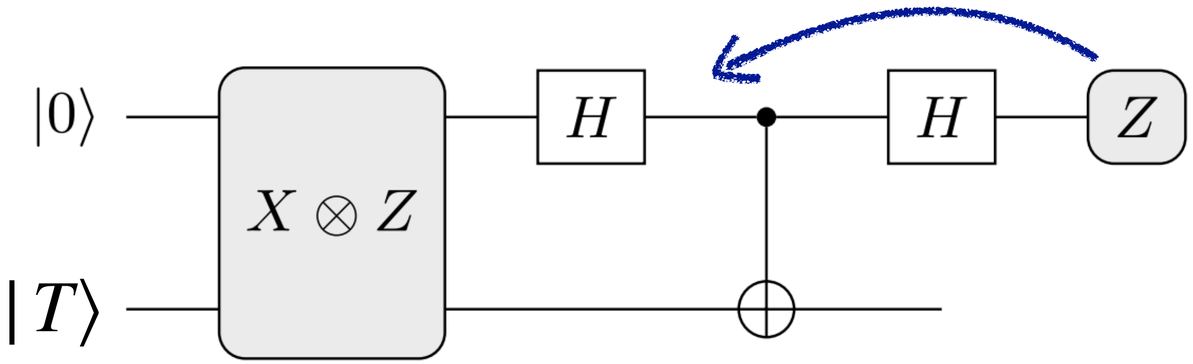

$|0\rangle$
$|T\rangle$
$X \otimes Z$
H
H
Z

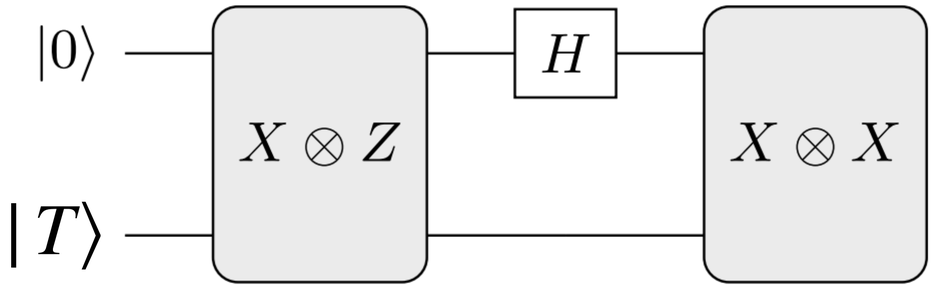

$|0\rangle$
$|T\rangle$
$X \otimes Z$
H
H
Z

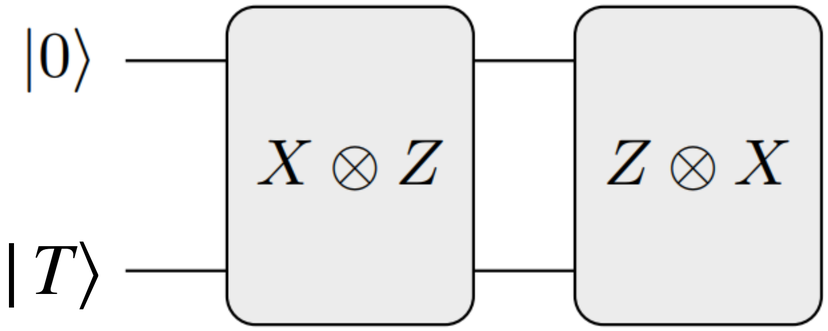

$|0\rangle$
$|T\rangle$
$X \otimes Z$
H
H
Z

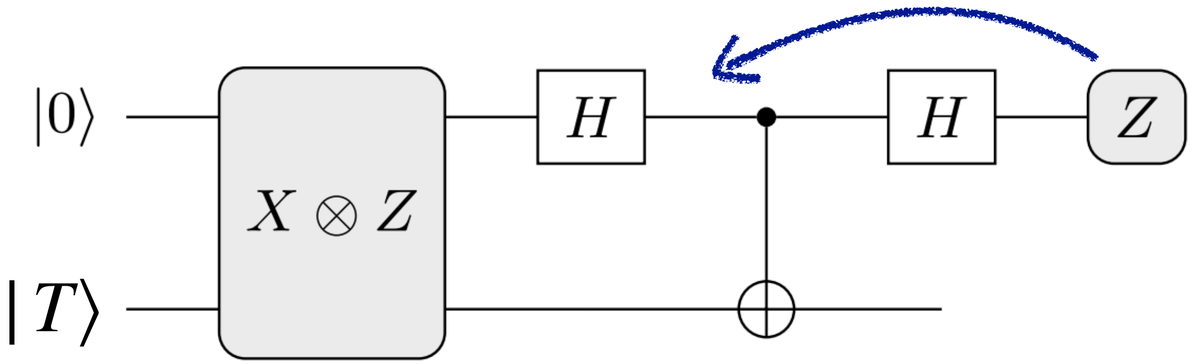

$|0\rangle$
$|T\rangle$
$X \otimes Z$
H
H
Z

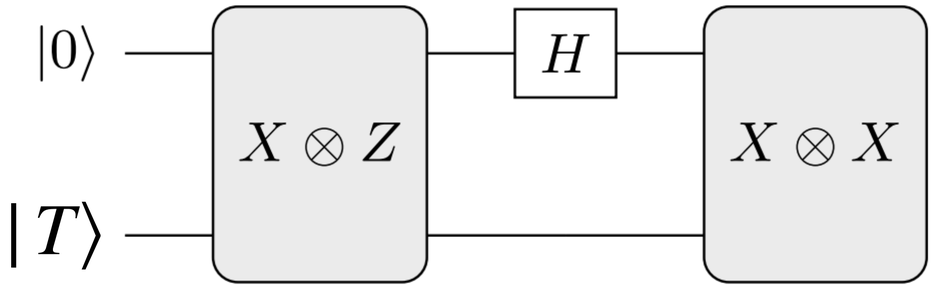

$|0\rangle$
$|T\rangle$
$X \otimes Z$
H
$X \otimes X$


$|0\rangle$
$|T\rangle$
$X \otimes Z$
H
H
Z


$|0\rangle$
$|T\rangle$
$X \otimes Z$
H
$X \otimes X$

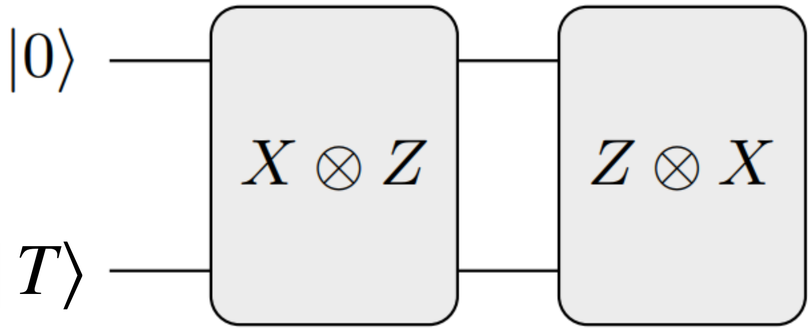

$|0\rangle$
$|T\rangle$
$X \otimes Z$
$Z \otimes X$

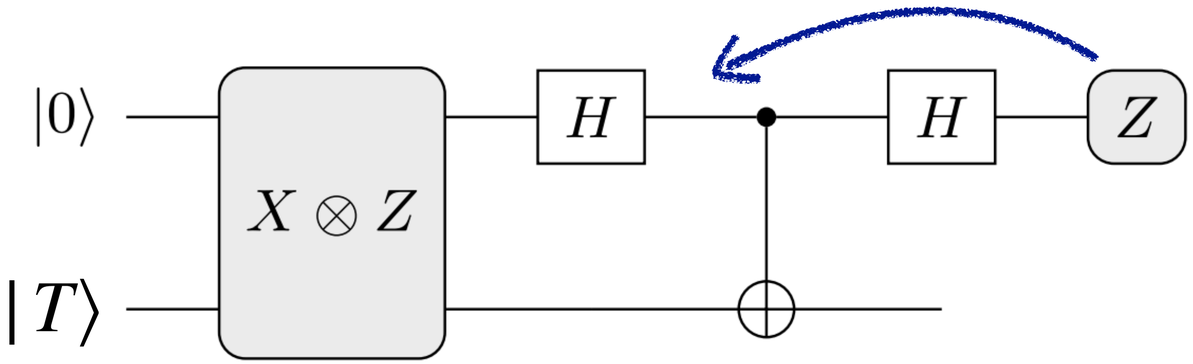


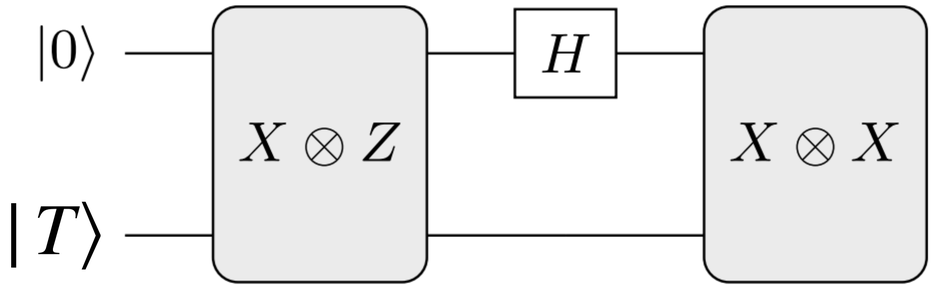


Now…

This does **not** fit the definition of PBC!

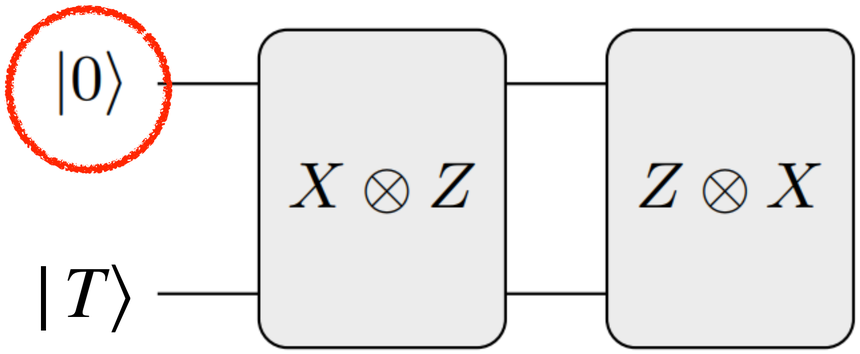


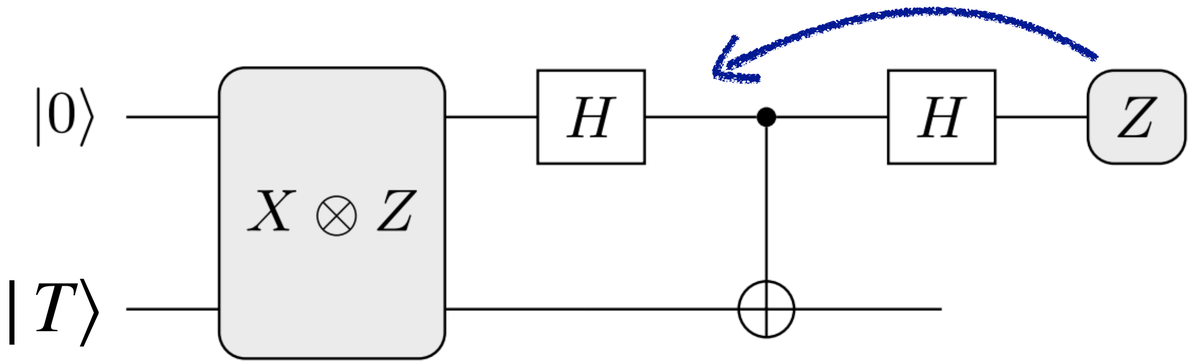


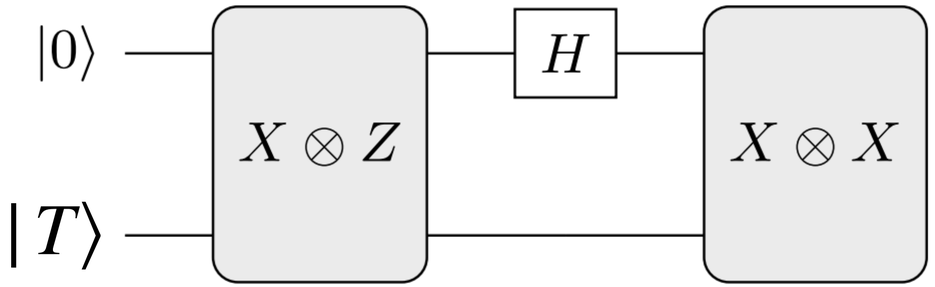


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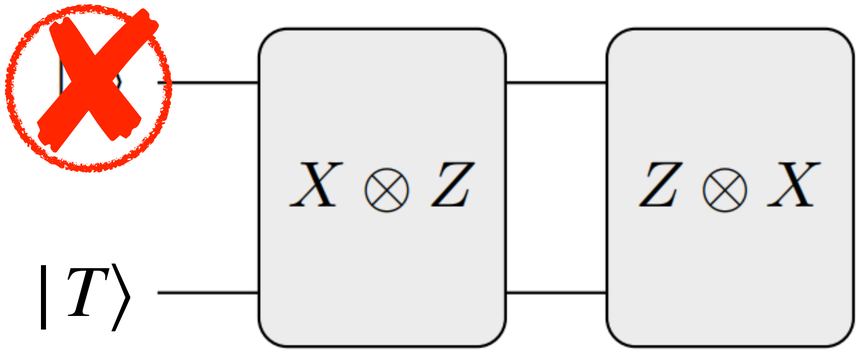


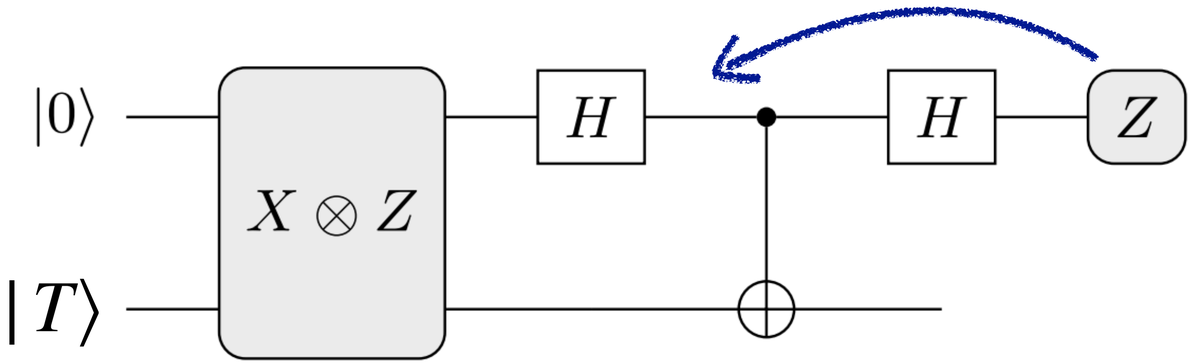


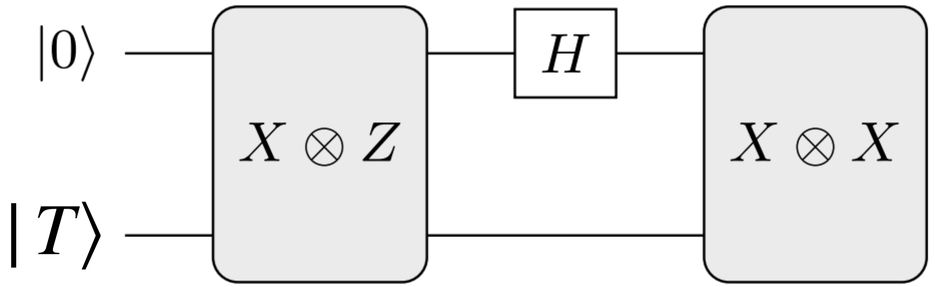


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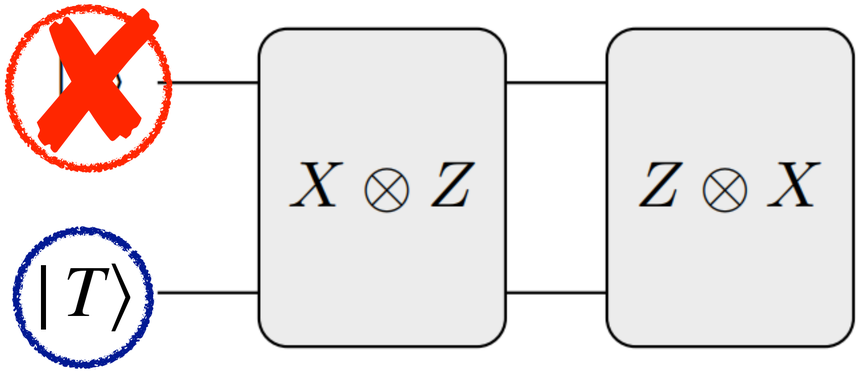






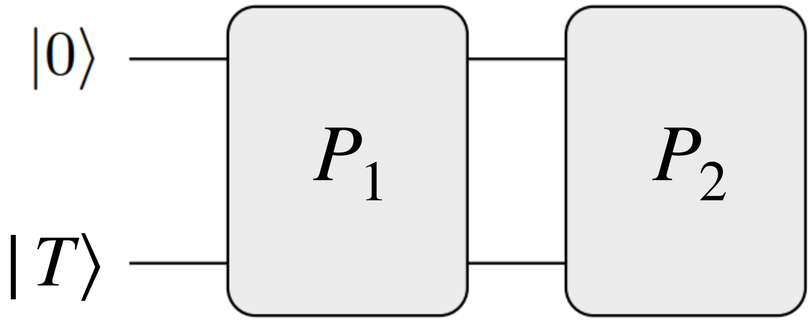
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For the ith measurement:

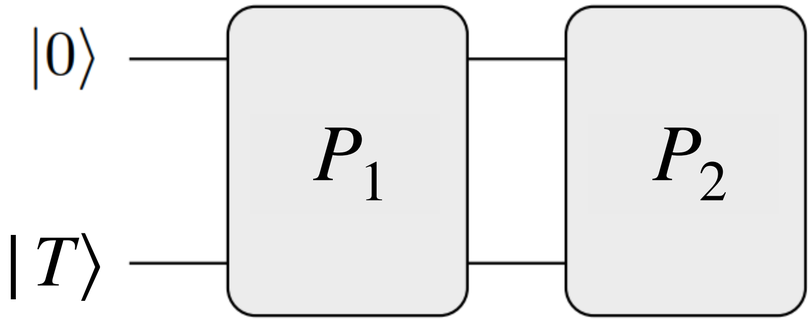
There is a classical procedure that can be used to (efficiently) reduce the computation just to the magic register…



For the ith measurement:

Does P_i commute with all the stabilizers?

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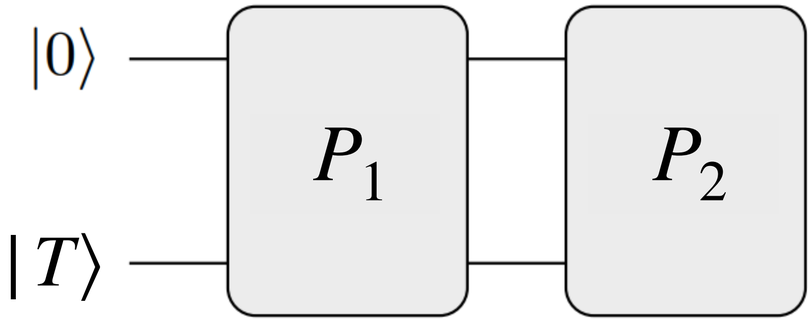


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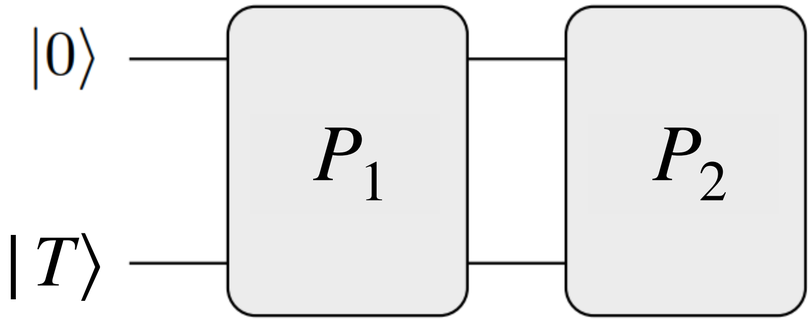
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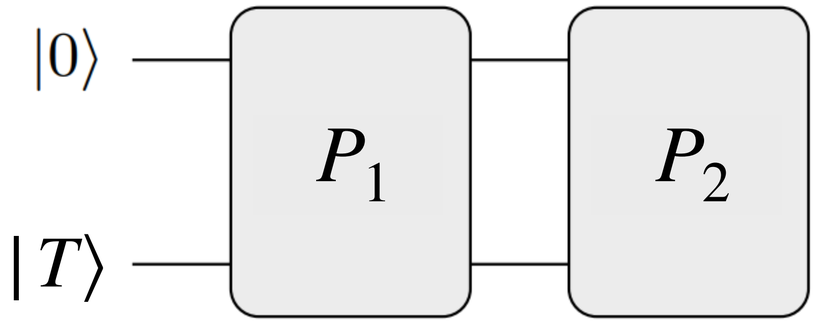
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2. Create the Clifford: $V_i = (S + P_i)/\sqrt{2}$
3. Update upcoming measurements suitably via conjugation by V_i.
4. If P_i is a readout: coin-toss $m_i \in_R \{0,1\}$ and track potential sign flips in upcoming measurements when $m_i = 1$.

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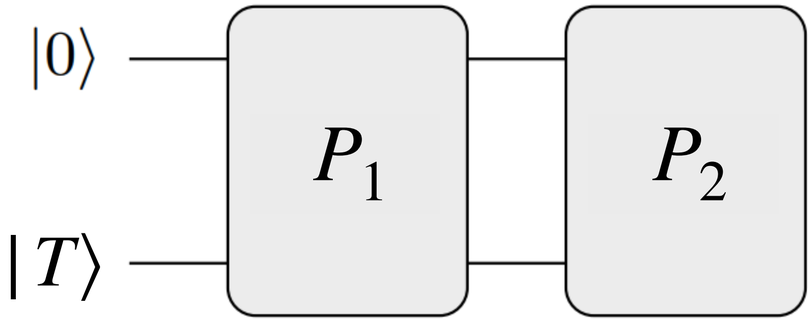
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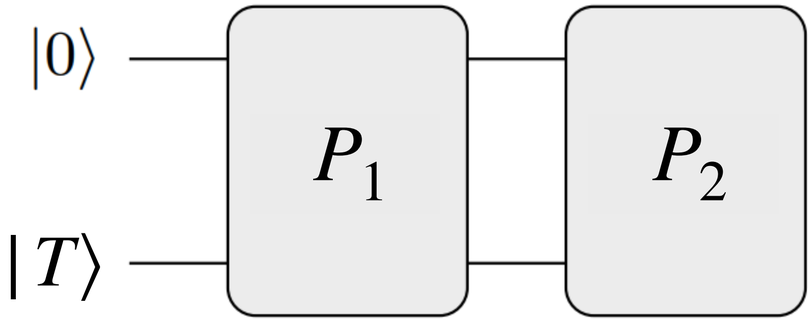
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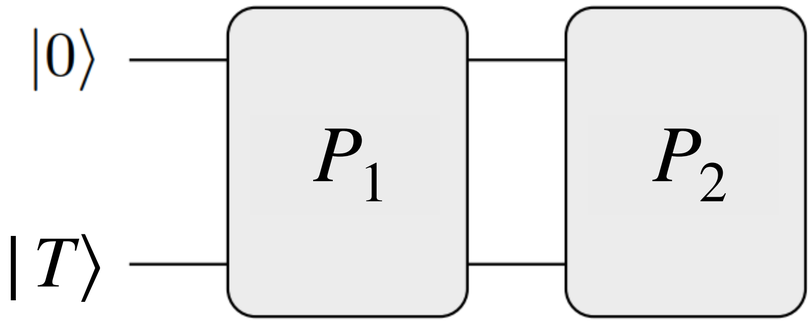
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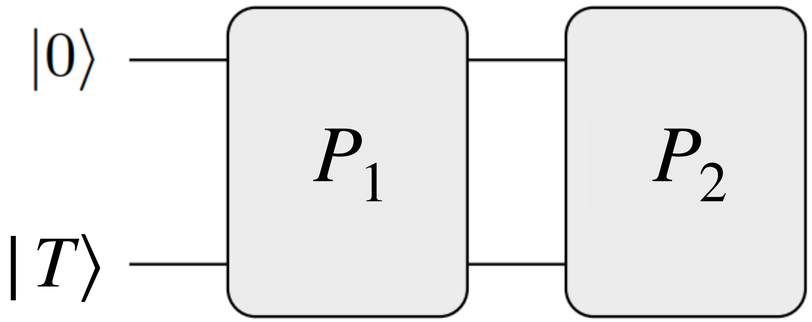
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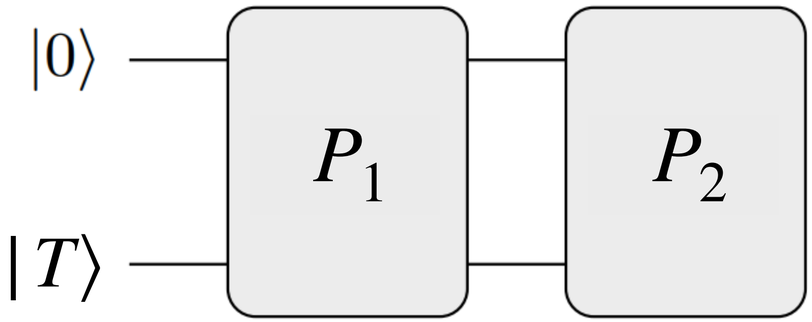
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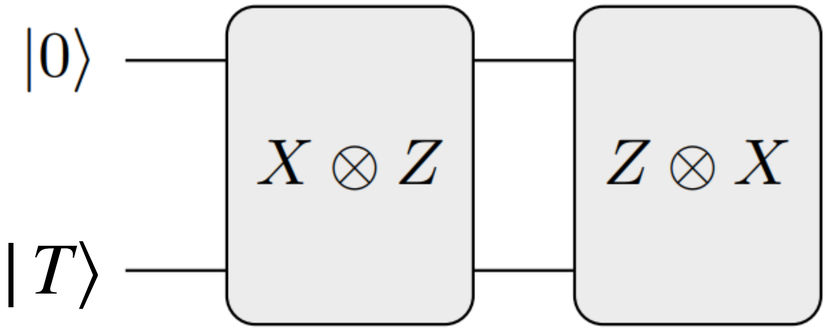
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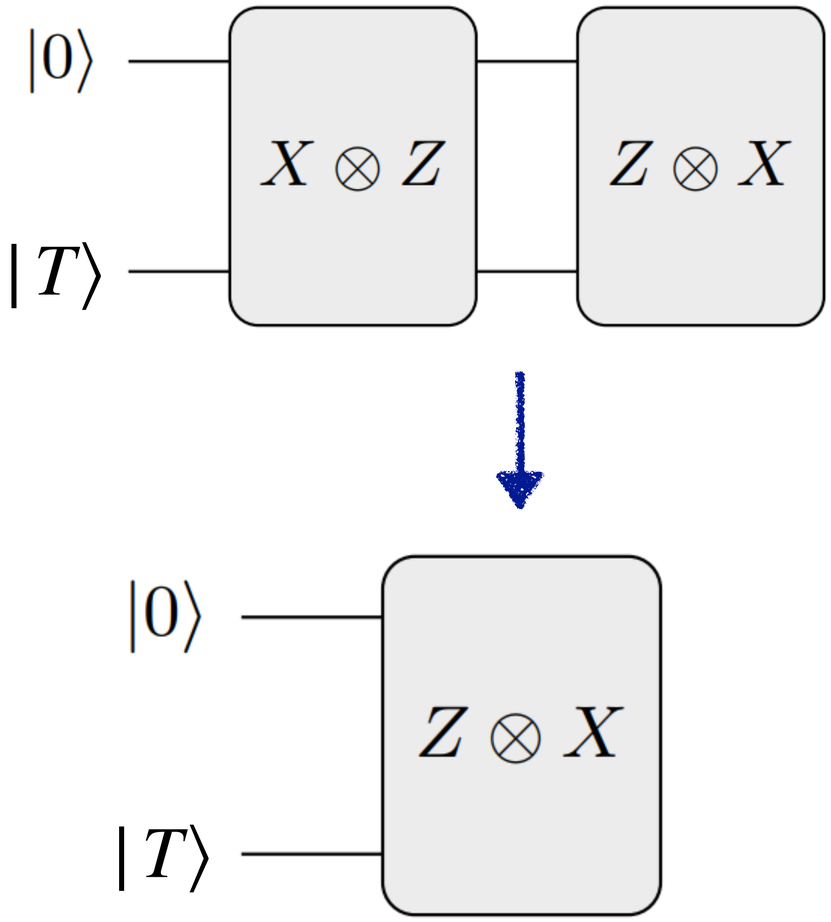
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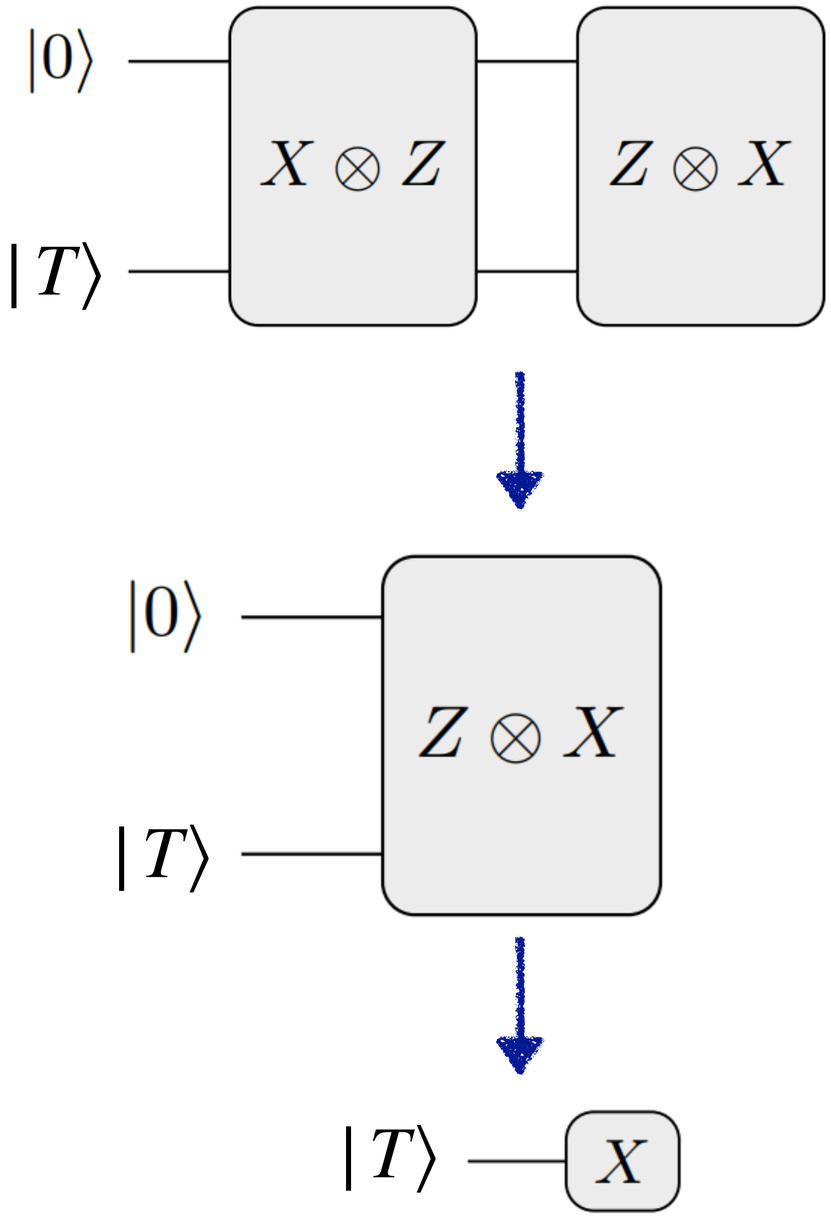
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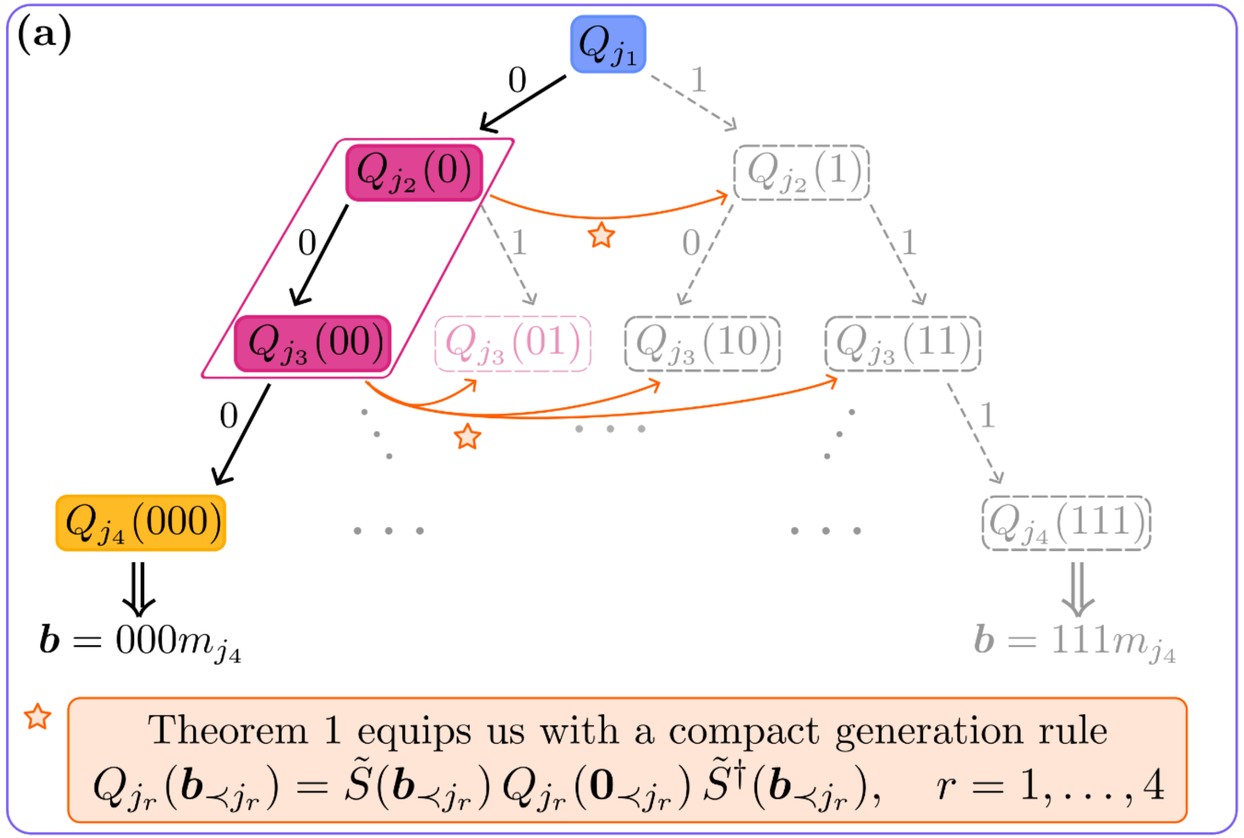
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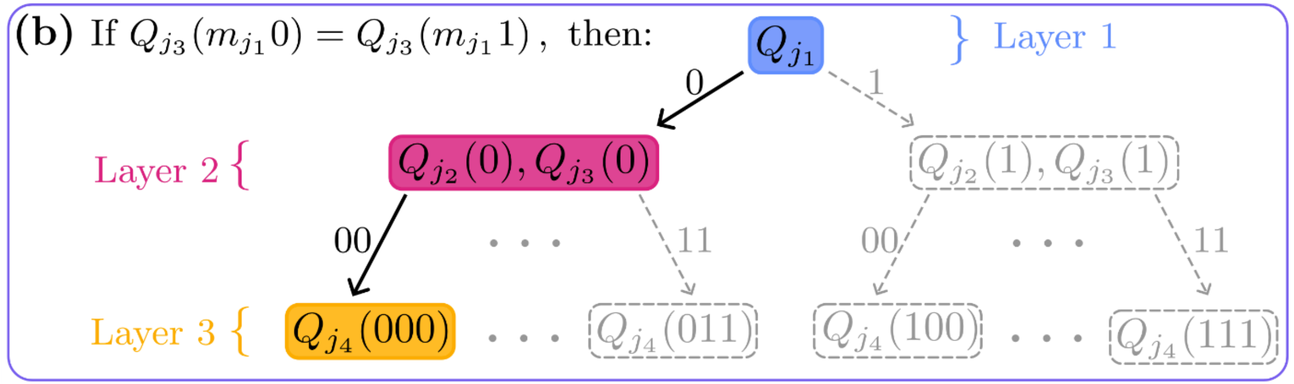
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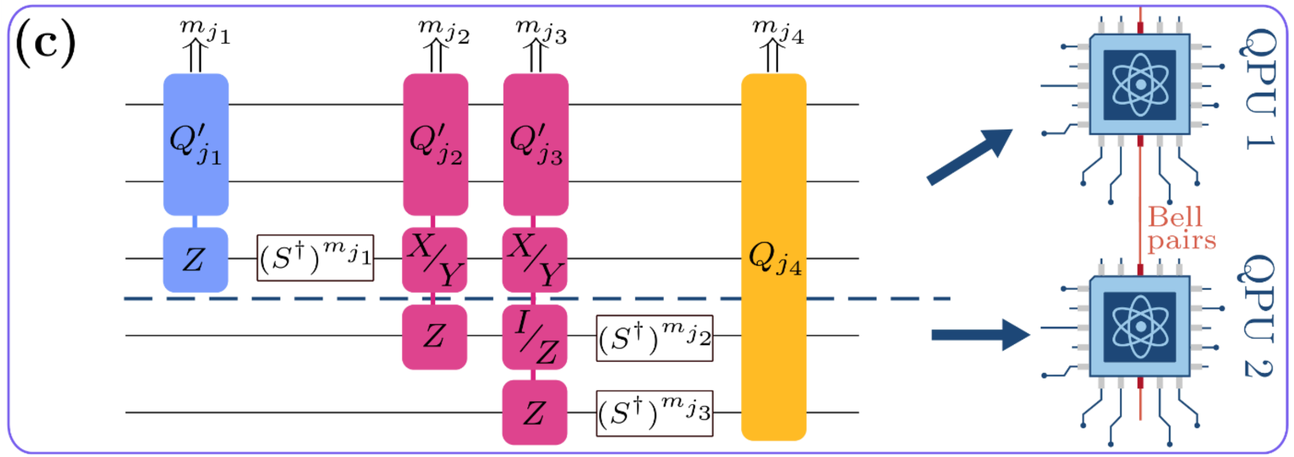
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(a)
Q_{j_1}
$Q_{j_2}(0)$
$Q_{j_2}(1)$
$Q_{j_3}(00)$
$Q_{j_3}(01)$
$Q_{j_3}(10)$
$Q_{j_3}(11)$
$Q_{j_4}(000)$
$Q_{j_4}(111)$
$\boldsymbol{b} = 000 m_{j_4}$
$\boldsymbol{b} = 111 m_{j_4}$
Theorem 1 equips us with a compact generation rule
$Q_{j_r}(\boldsymbol{b}_{\prec j_r}) = \tilde{S}(\boldsymbol{b}_{\prec j_r})\, Q_{j_r}(\boldsymbol{0}_{\prec j_r})\, \tilde{S}^\dagger(\boldsymbol{b}_{\prec j_r}), \quad r = 1, \ldots, 4$


(b) If $Q_{j_3}(m_{j_1}0) = Q_{j_3}(m_{j_1}1)$, then:
Layer 1
Layer 2
Layer 3
Q_{j_1}
$Q_{j_2}(0), Q_{j_3}(0)$
$Q_{j_2}(1), Q_{j_3}(1)$
$Q_{j_4}(000)$
$Q_{j_4}(011)$
$Q_{j_4}(100)$
$Q_{j_4}(111)$


(c)
m_{j_1}
m_{j_2}
m_{j_3}
m_{j_4}
Q'_{j_1}
Q'_{j_2}
Q'_{j_3}
Z
$(S^\dagger)^{m_{j_1}}$
X/Y
I/Z
$(S^\dagger)^{m_{j_2}}$
$(S^\dagger)^{m_{j_3}}$
Q_{j_4}
Bell pairs
QPU 1
QPU 2

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UNIVERSITÄT LINZ





Richard Küng