Bachelor Engineering Physics – University of Minho



Bachelor Engineering Physics – University of Minho



Master Engineering Physics with a specialization in Quantum computing – University of Minho



Thesis titled "On Quantum Bayesian Decision Making" FUNDAÇÃO CALOUSTE GULBENKIAN



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PhD in theoretical computer science – Quantum computing - International Iberian Nanotechnology Laboratory & Sorbonne University





Thesis titled "The Interplay between Quantum Foundations and Circuit Complexity"





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Thesis titled "The Interplay between Quantum Foundations and Circuit Complexity"





Research on quantum complexity theory and quantum algorithms

My Master's thesis

We are presented every day with multiple decision to make.



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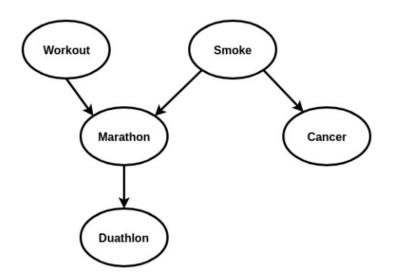


However, we would like to delegate most of them to a kind of personal assistant who can make near-optimal decisions for us.

First, we need a mathematical model that can describe probabilistic systems to handle uncertainty.

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Bayesian networks



Workout	P(Workout)
True	0.7
False	0.3
Smoke	P(Smoke)
True	0.6
False	0.4

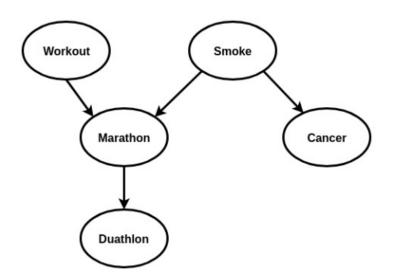
Smoke	Workout	Marathon	P(Marathon Workout,Smoke)
True	True	True	0.2
True	True	False	0.8
True	False	True	0.05
True	False	False	0.95
False	True	True	0.4
False	True	False	0.6
False	False	True	0.1
False	False	False	0.9

Smoke	Lung cancer	P(Lung cancer Somke)
True	True	0.02
True	False	0.98
False	True	0.005
False	False	0.995

Marathon	Duathlon	P(Duathlon Marathon)
True	True	0.7
True	False	0.3
False	True	0
False	False	1

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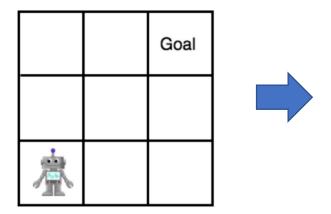
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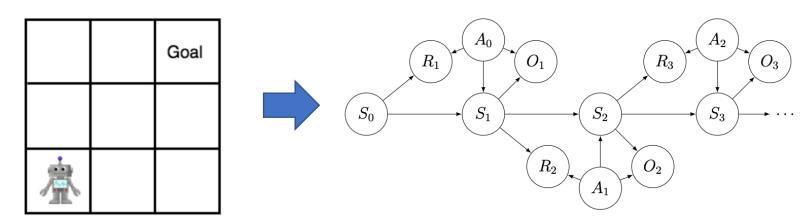
$$P(x_1,...,x_n) = \prod_{i=1}^n P(x_i|Parents(X_i))$$

Problem



Problem

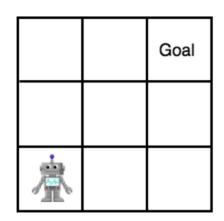
Model/abstraction

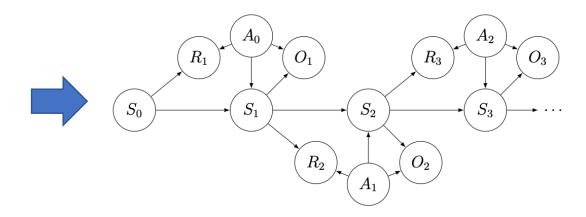


Problem

Model/abstraction

Decision making process





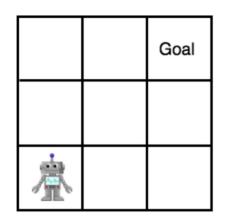




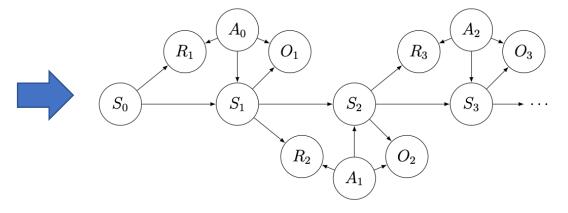
Cumpute A_1 , A_2 , ..., A_n

s.t. $R_T = \sum_{i=1}^n R_i$ is maximal

Problem



Model/abstraction



Decision making process

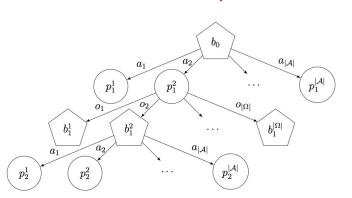


Cumpute A_1 , A_2 , ..., A_n

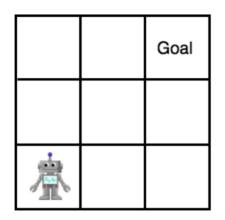
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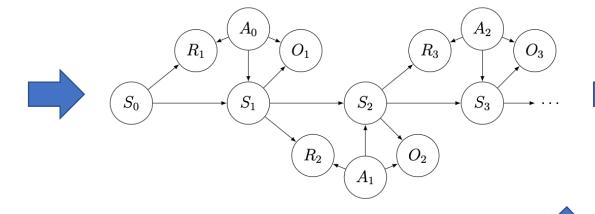
It exponentially harder to interpolate over each additional time step



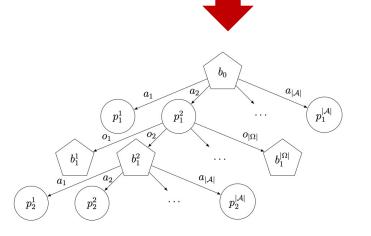
Problem



Model/abstraction



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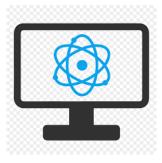


Decision making process



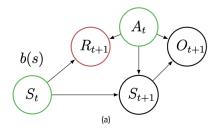
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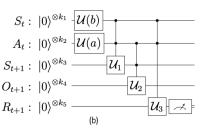
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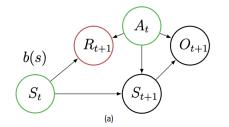
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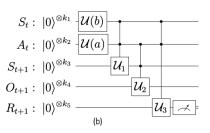
Quantum bayesian network states





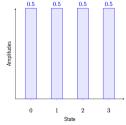
Quantum bayesian network states

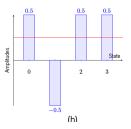


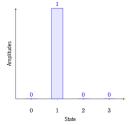




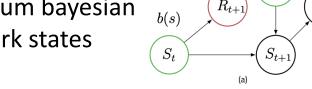
Amplitude amplification algorithm

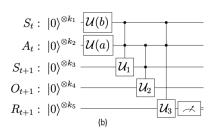






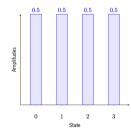
Quantum bayesian network states

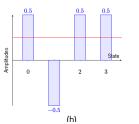


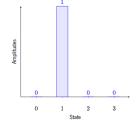




Amplitude amplification algorithm



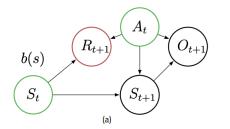


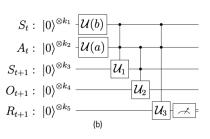




	Classical	Quantum
Complexity	$\mathcal{O}\left(NMP(e)^{-1} ight)$	$\mathcal{O}\left(N2^MP(e)^{-rac{1}{2}} ight)$

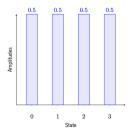
Quantum bayesian network states

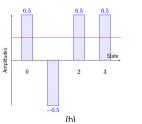


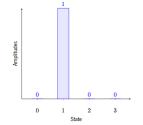




Amplitude amplification algorithm







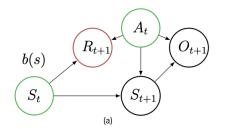


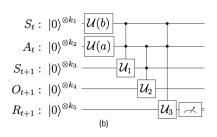
	Classical	Quantum
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$$EU(a|e) = \sum_{r} \underbrace{P(Result = r|a, e)}_{Quantum} * \underbrace{U(r)}_{Classical}$$

$$action = argmax_a EU(a|e)$$

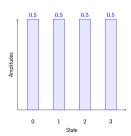
Quantum bayesian network states

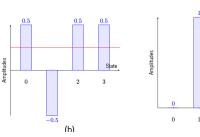






Amplitude amplification algorithm







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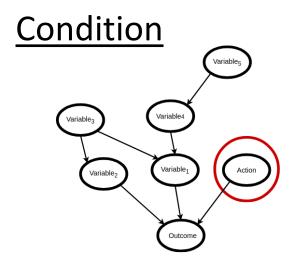
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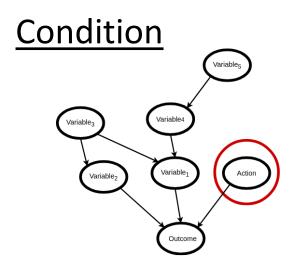
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We obtain subquadratic quantum advantages for sparse Bayesian networks.

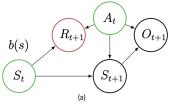
	Goal
慮	

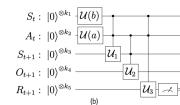




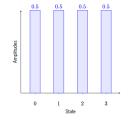
Solution

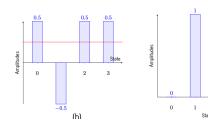
 We encode the BN but do not select an action

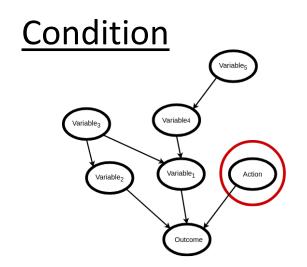




• We amplify the observation v. and the outcome v. based on their utility.

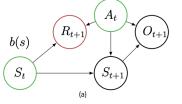


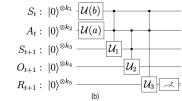




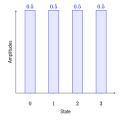
Solution

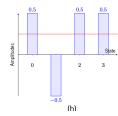
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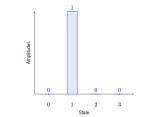




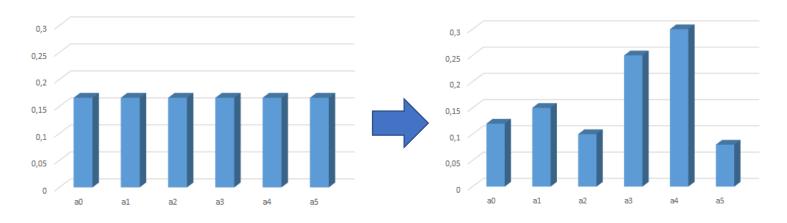
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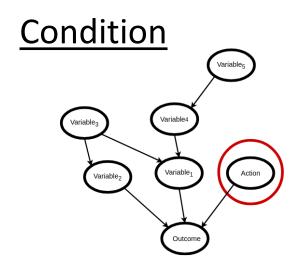






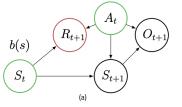
Consequence

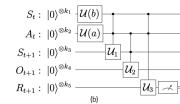




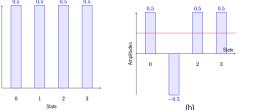
Solution

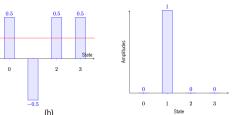
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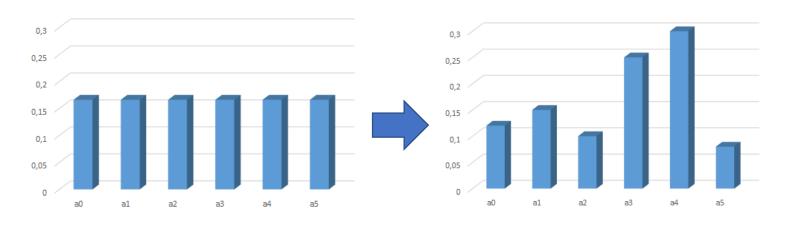


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Consequence



<u>Result</u>

For sparse Bayesian networks, we obtain a subquadratic quantum advantage but a strictly larger advantage than naïve amplitude amplification.

$$\frac{C}{Q_{naive}} = n^{c_1}, \qquad \frac{C}{Q_{new}} = n^{c_2}$$

$$c_1 < c_2$$

This work and its results gave origin to,

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Presentation at an international workshop (Santiago Chile)

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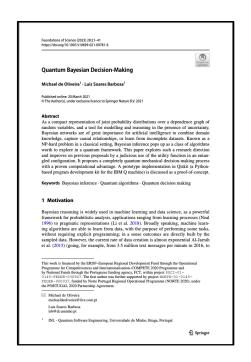


A paper in a scientific journal

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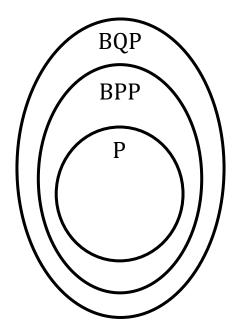


Content of my Gulbenkian research project and master's thesis

My Phd thesis and the quest for realizable quantum computational advantages

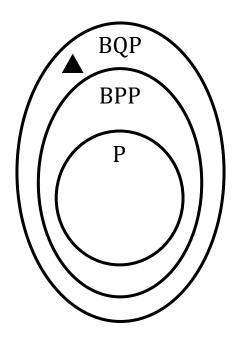
Main interest in quantum comes from the fact that these devices have **computation complexity theoretic implications**.

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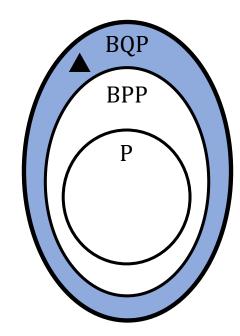
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A Shor's algorithm prime factorization is conjectured to separate BPP from BQP.



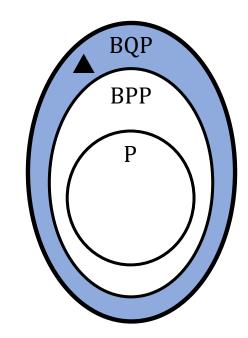
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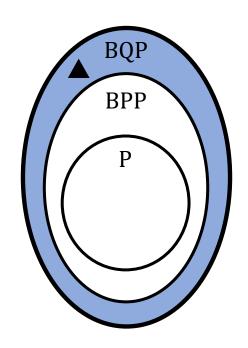
- ▲ Shor's algorithm prime factorization is conjectured to separate BPP from BQP.
 - In simple terms it means that there exist quantum algorithms with **exponential advantages** over any possible classical algorithm.
 - Nevertheless, these quantum algorithms require large-scale fault-tolerant quantum devices (1700 qubits, 10^{36} Toffoli gates).



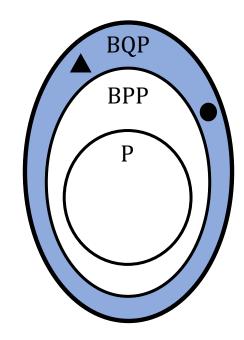
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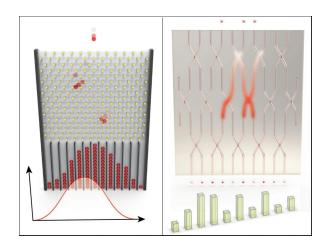


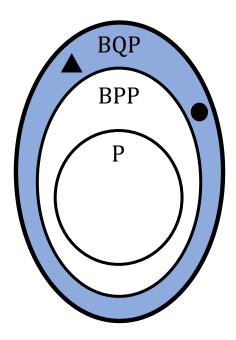


These algorithms (including Grover type search) are not within reach in the next years ...

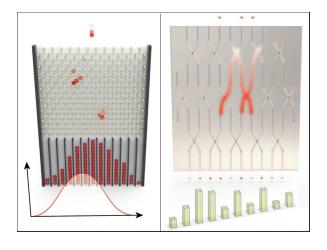


Boson Sampling

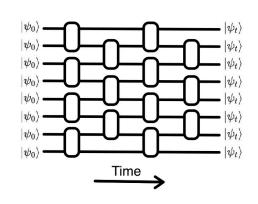




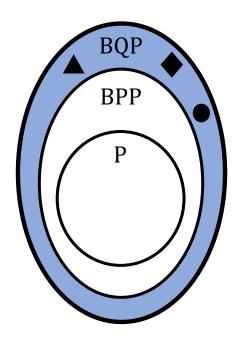
Boson Sampling



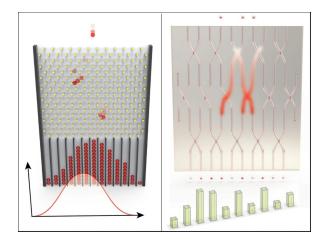
• Random circuit sampling



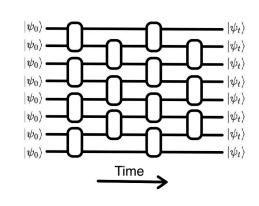




Boson Sampling

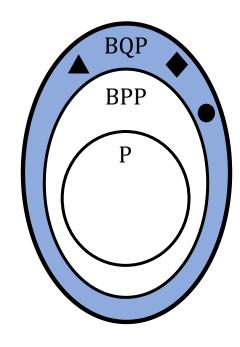


Random circuit sampling

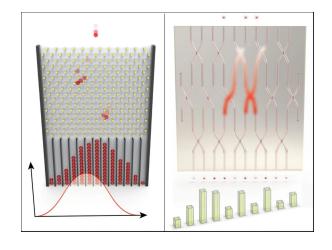




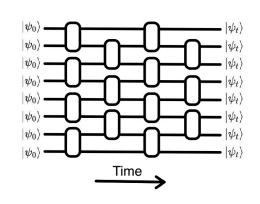




Boson Sampling

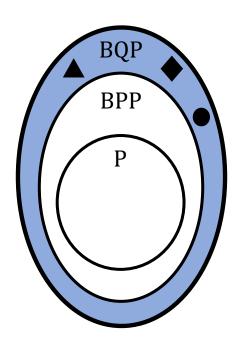


• Random circuit sampling



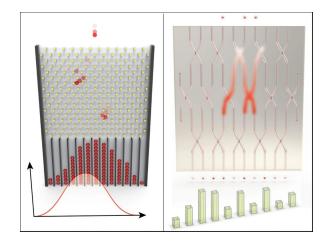




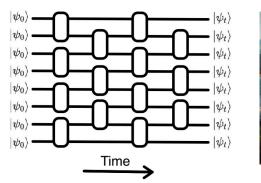


+ Shallow depth small quantum devices.

Boson Sampling

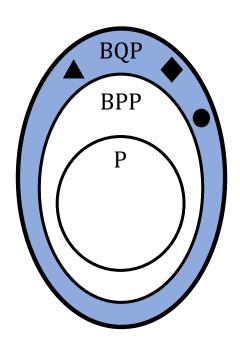


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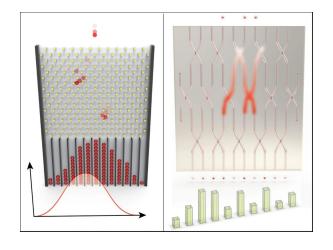




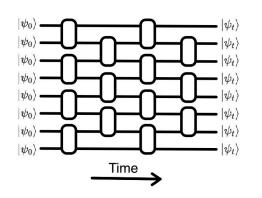


- + Shallow depth small quantum devices.
- Assuming the Polynomial Hierarchy does not collapse to the second level.

Boson Sampling

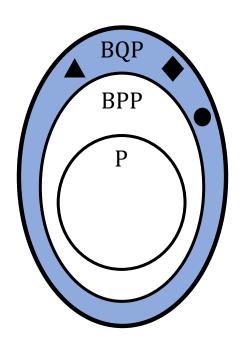


♦ Random circuit sampling









- + Shallow depth small quantum devices.
- Assuming the Polynomial Hierarchy does not collapse to the second level.
- It is hard to verify their correctness in the presence of noise.

Can we prove quantum advantage for any computational problem without relying on computational assumptions?

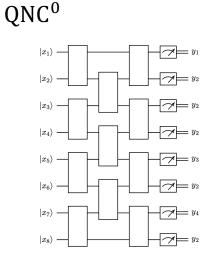
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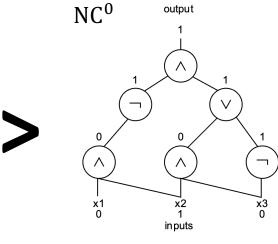
Yes, if we fix time/depth to be constant!!!

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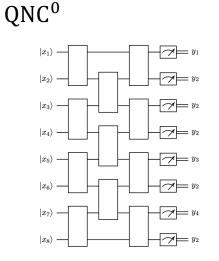


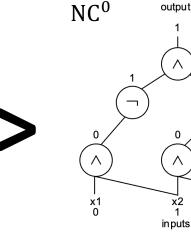


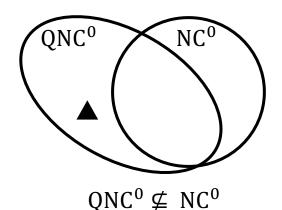
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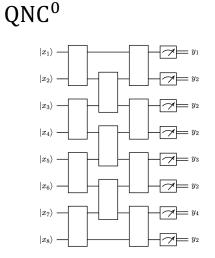
Hidden Linear shift problem

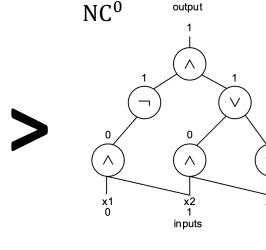
- + Shallow depth small quantum devices.
- + No assumptions.

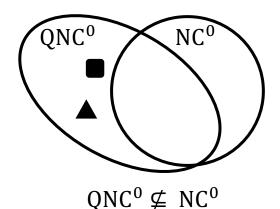
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Hidden Linear shift problem

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1D Magic square problem

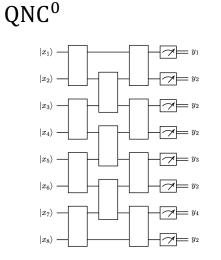
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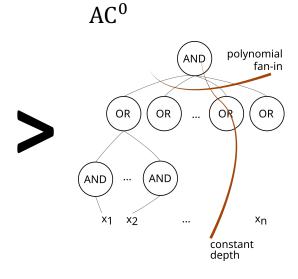
+ Error-robust.

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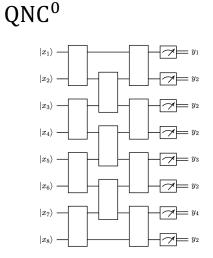


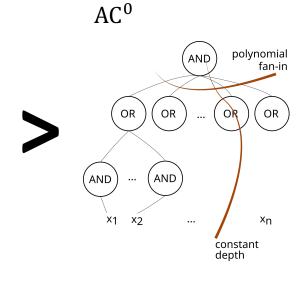


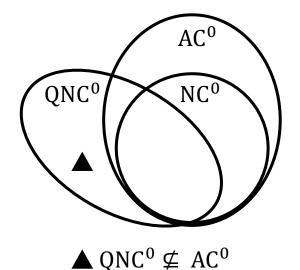
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Parity halving problem

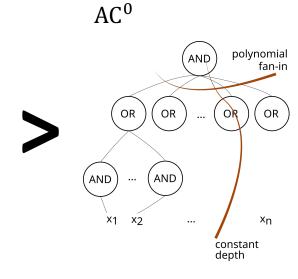
+ Average-case hardness.

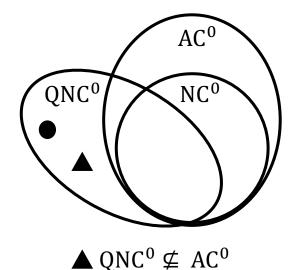
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 $|x_1\rangle \qquad \qquad y_1$ $|x_2\rangle \qquad \qquad y_2$ $|x_3\rangle \qquad \qquad y_2$ $|x_4\rangle \qquad \qquad y_2$ $|x_5\rangle \qquad \qquad y_3$ $|x_6\rangle \qquad \qquad y_2$ $|x_7\rangle \qquad \qquad y_4$ $|x_8\rangle \qquad \qquad y_2$





Parity halving problem

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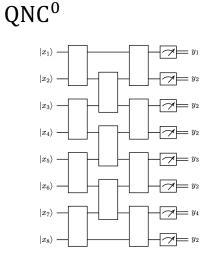
Single-qubit teleportation

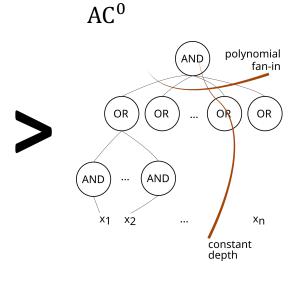
+ Noise resilient.

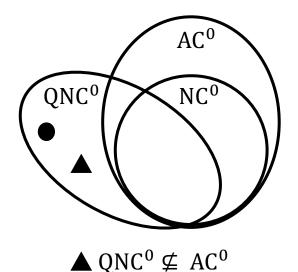
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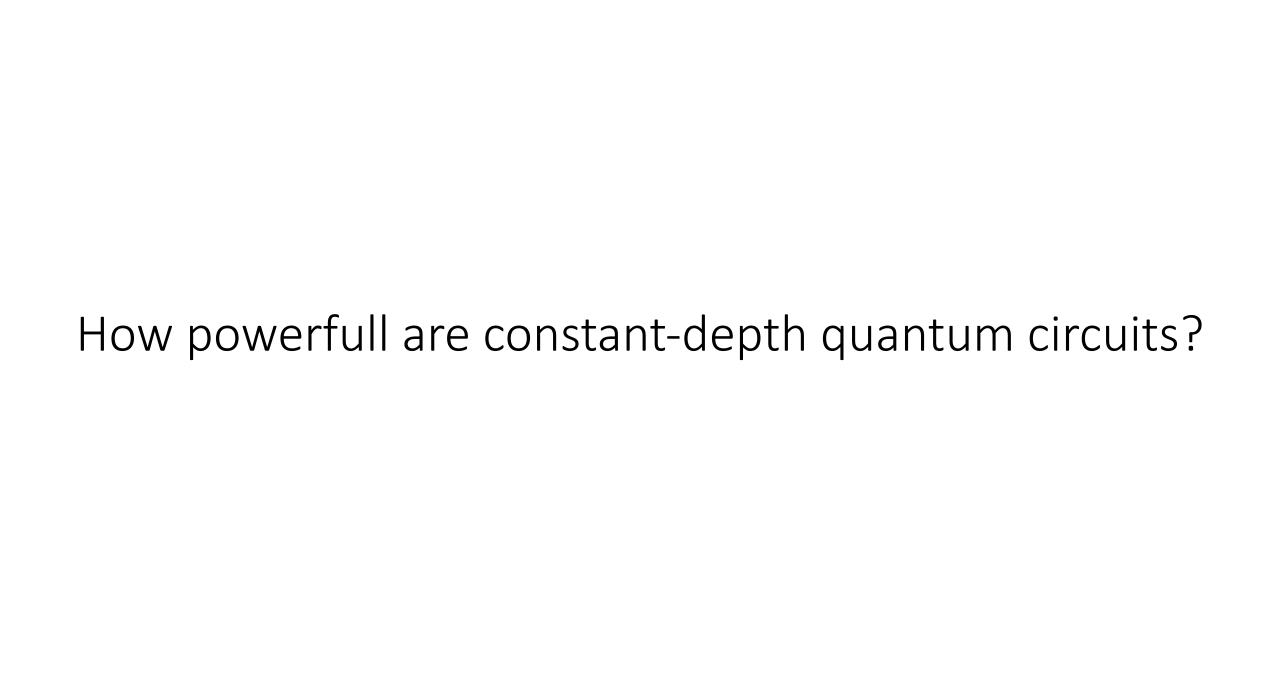


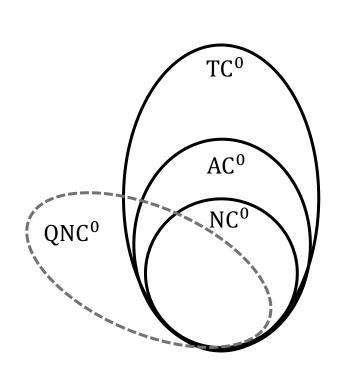
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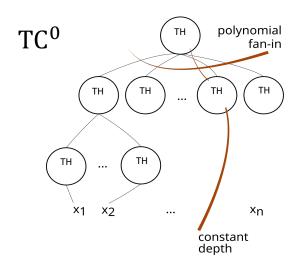
- + Average-case hardness.
- AC⁰ circuit class is of rather small practical use.

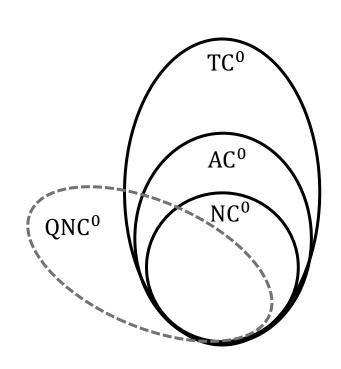
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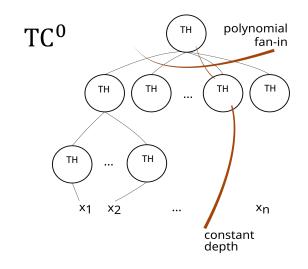
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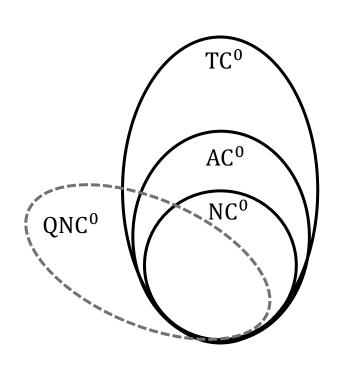


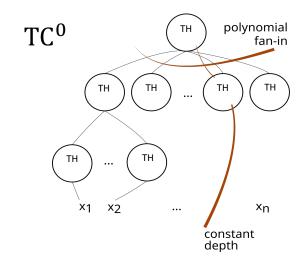




 TC⁰circuits are able a large set of algebraic problems



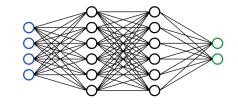


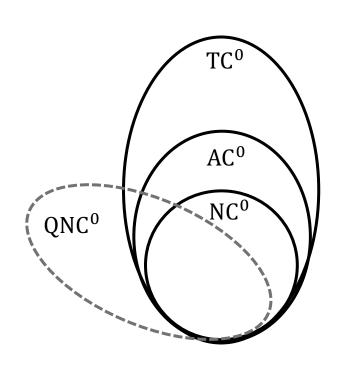


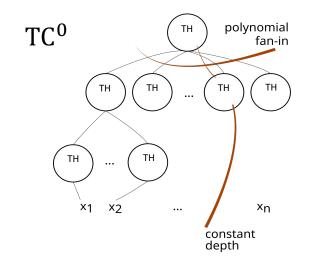
 TC⁰ circuits are able a large set of algebraic problems



 TC⁰circuits serve as the circuit class that models Neural Networks (including LLMs)



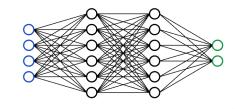




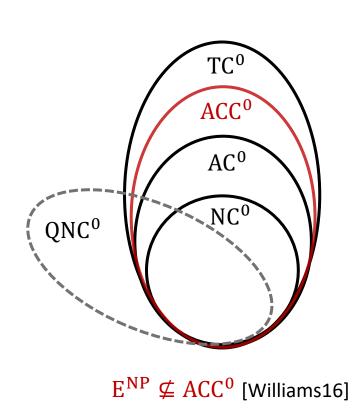
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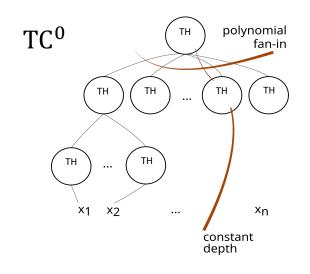


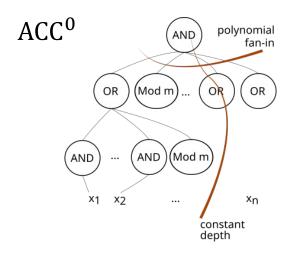
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We do not have any strong lower bound techniques!



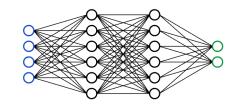




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We do not have any strong lower bound techniques!

• Alternatively can there exists a $bPTF^0[k]$ circuits parameterized by k can interpolate between AC^0 and beyond TC^0 .

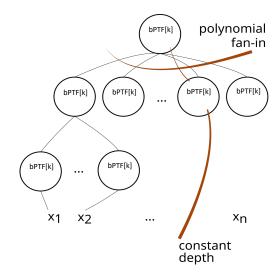
• Alternatively can there exists a $bPTF^0[k]$ circuits parameterized by k can interpolate between AC^0 and beyond TC^0 .

Definition (bPTF $^0[k]$).

Constant-depth circuits with unbounded fan-in bPTF[k] gates defined as follows,

$$f_{Or}(x) = \begin{cases} P(x), & \sum_{i=0}^{n} x_i \le k \\ 1, & \sum_{i=0}^{n} x_i > k \end{cases}$$

with $P: F_2^n \to F_2$ a polynomial over $F_2 = \{0,1\}$.



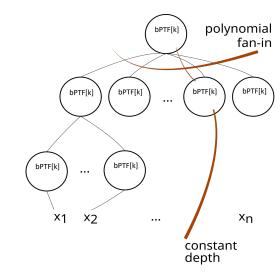
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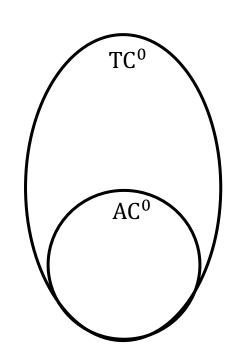
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<u>Example</u>

bPTF $^0[k]$ includes k-maj k-

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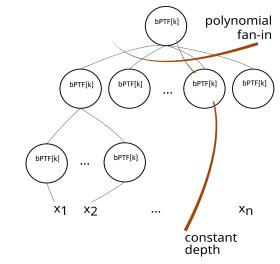


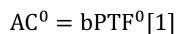
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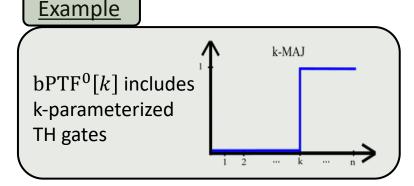
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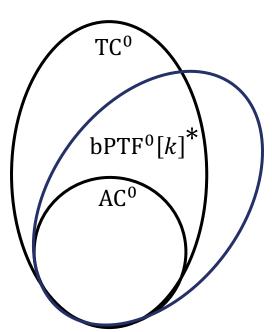
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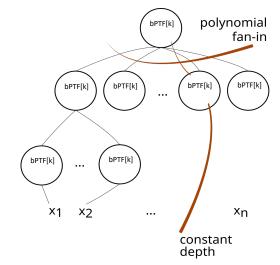


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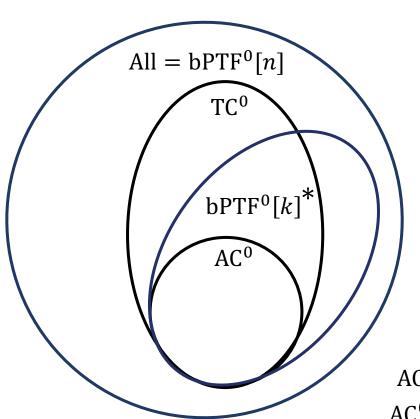
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 $\begin{array}{c} \text{bPTF}^0[k] \text{ includes} \\ \text{k-parameterized} \\ \text{TH gates} \end{array}$

$$AC^0 = bPTF^0[1]$$

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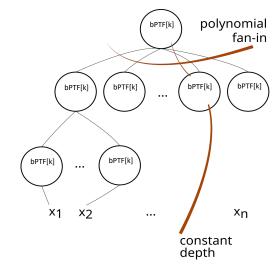


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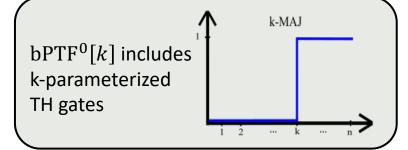
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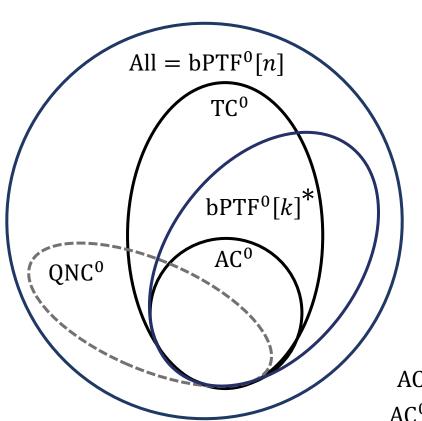


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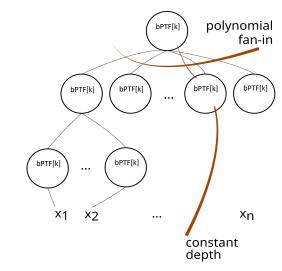


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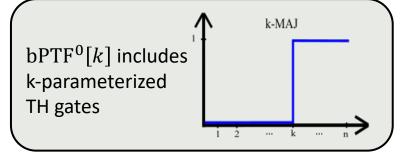
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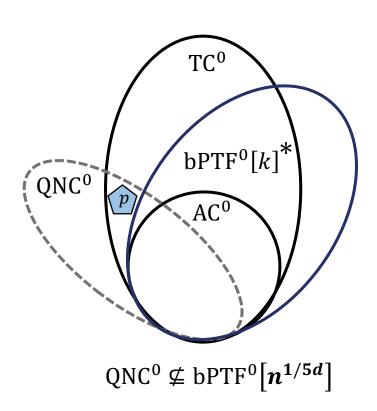


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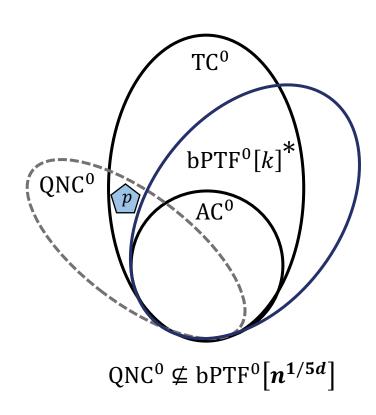


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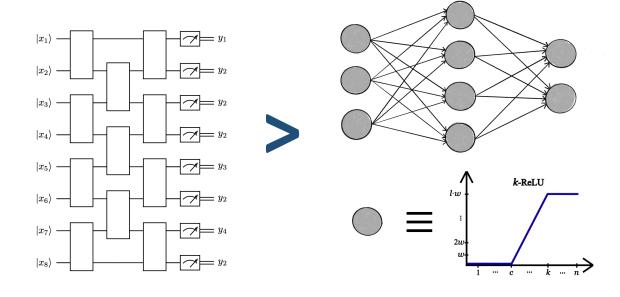
New Quantum-Classical unconditional separations



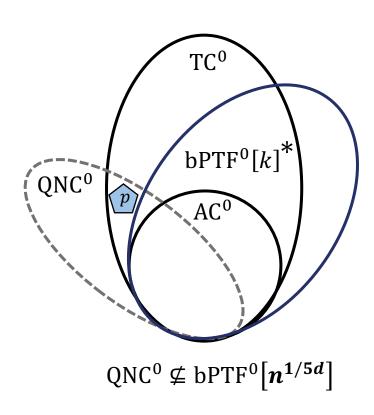
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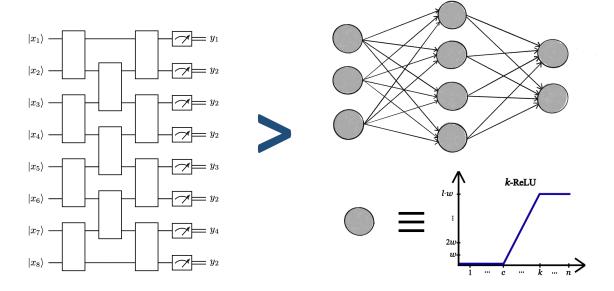
 We proved that parallel quantum computation unconditionally outperforms larger classical circuits classes, including biased Neural Networks.



New Quantum-Classical unconditional separations



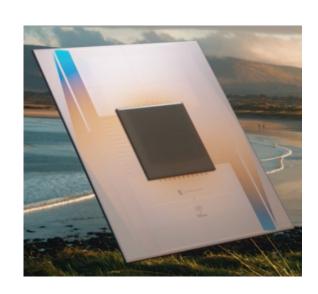
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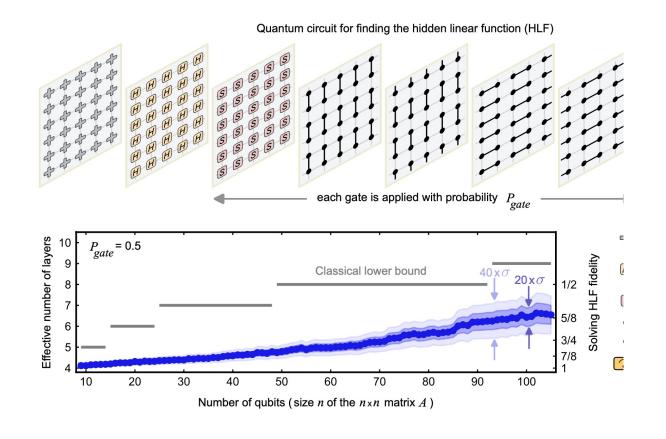


This advantage is robust to noise and operates on qudit quantum devices.

Solving the Hidden Linear shift problem on quantum devices

This week, the Google team released a paper on the realization of HLF on their Willow chip.

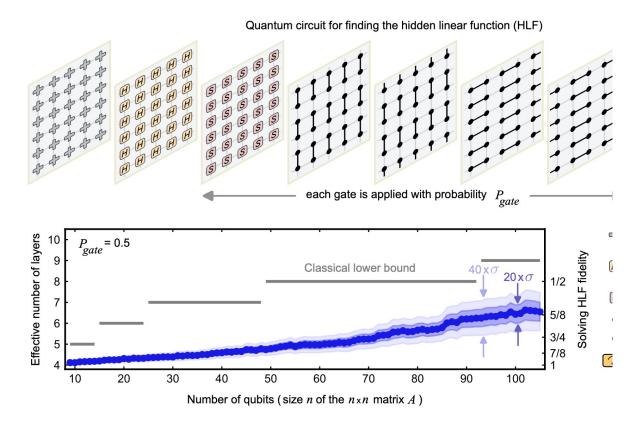




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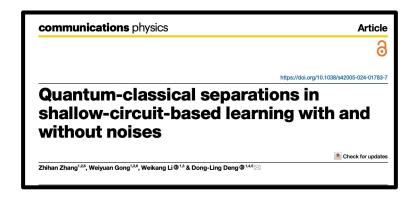
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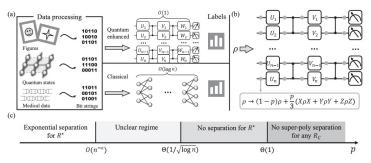




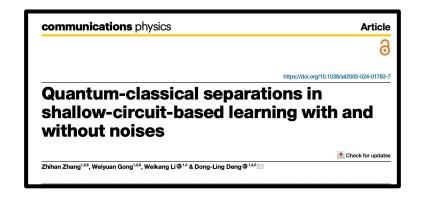
^{*}If they could scale the number of qubits, they could realize our new quantum advantages.

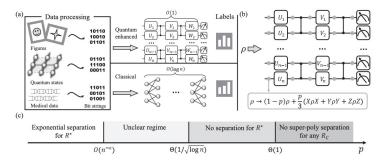
New Practically Motivated Quantum-Classical unconditional separations





New Practically Motivated Quantum-Classical unconditional separations

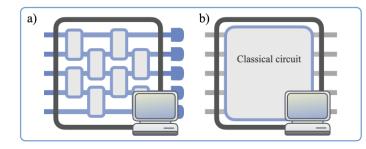


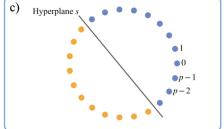


An unconditional distribution learning advantage with shallow quantum circuits

N. Pirnay, S. Jerbi, J.-P. Seifert, and J. Eisert, and J. Eisert,

¹Electrical Engineering and Computer Science, Technische Universität Berlin, Berlin, 10587, Germany
²Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany
³Fraunhofer Heinrich Hertz Institute, 10587 Berlin, Germany
⁴Helmholtz-Zentrum Berlin für Materialien und Energie, 14109 Berlin, Germany





The Quantum Benchmarking problem

Quantum Benchmarks which test quantum devices should be:

- Algorithm agnostic

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- Algorithm agnostic
- Predictive

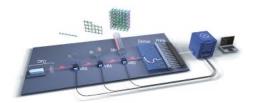
Quantum Benchmarks which test quantum devices should be:

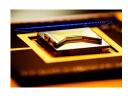
- Algorithm agnostic
- Predictive
- Scalable

Quantum Benchmarks which test quantum devices should be:

- Algorithm agnostic
- Predictive
- Scalable
- Hardware agnostic



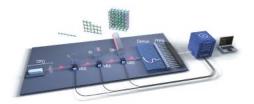


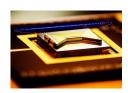


Quantum Benchmarks which test quantum devices should be:

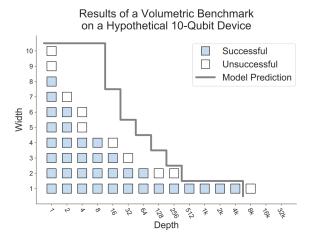
- Algorithm agnostic
- Predictive
- Scalable
- Hardware agnostic







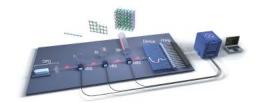
Quantum Volume

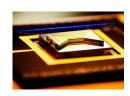


Quantum Benchmarks which test quantum devices should be:

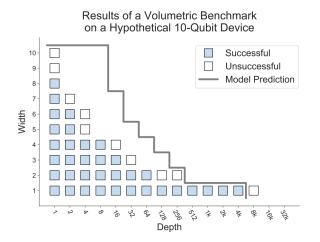
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Quantum Volume



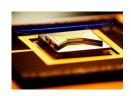
Misses to be predictive and scalable

Quantum Benchmarks which test quantum devices should be:

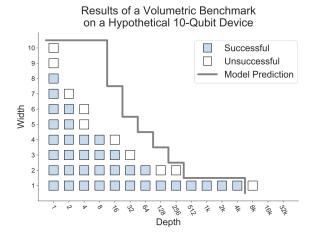
- Algorithm agnostic
- Predictive
- Scalable
- Hardware agnostic







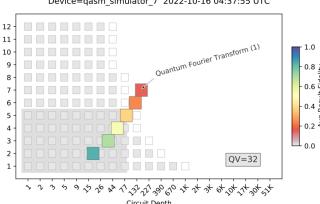
Quantum Volume



Misses to be predictive and scalable

Application-oriented benchmarks

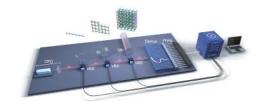
Volumetric Positioning - All Applications (Merged)
Device=qasm simulator 7 2022-10-16 04:37:55 UTC

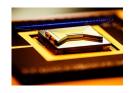


Quantum Benchmarks which test quantum devices should be:

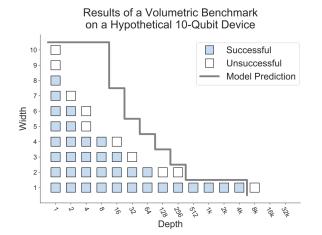
- Algorithm agnostic
- Predictive
- Scalable
- Hardware agnostic





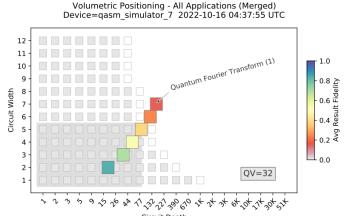


Quantum Volume

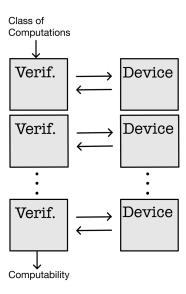


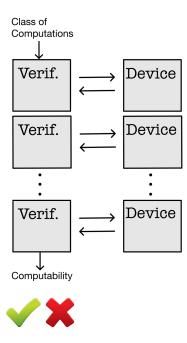
Misses to be predictive and scalable

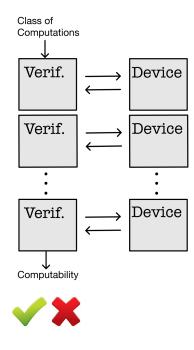
Application-oriented benchmarks

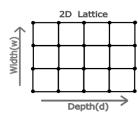


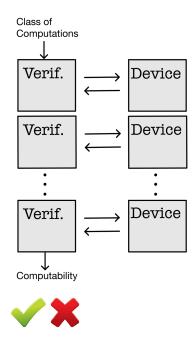
Misses to algorithm agnostic and predictive

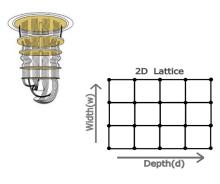


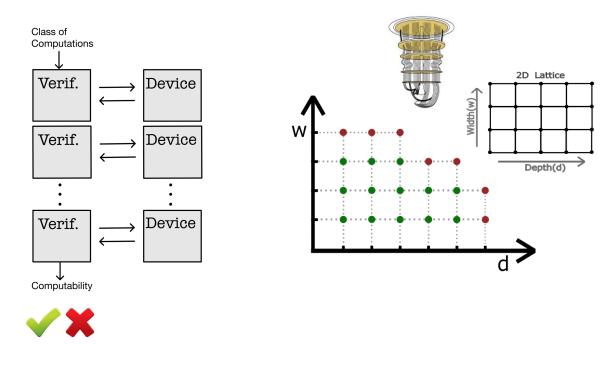




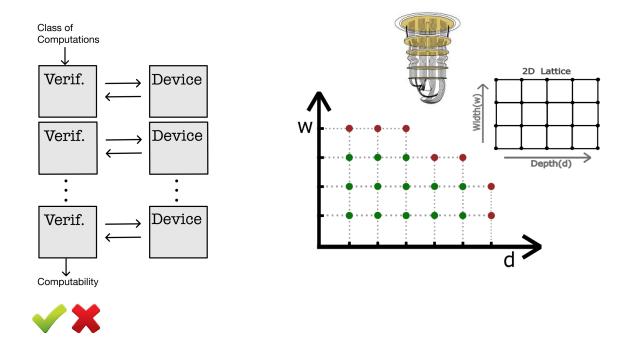




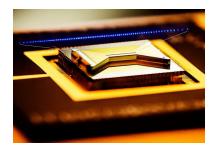




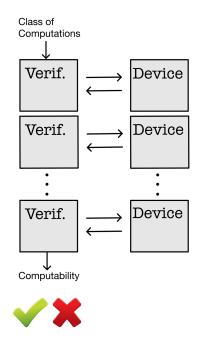
Developed a benchmark to formally certify quantum devices for executing broad classes of circuits.



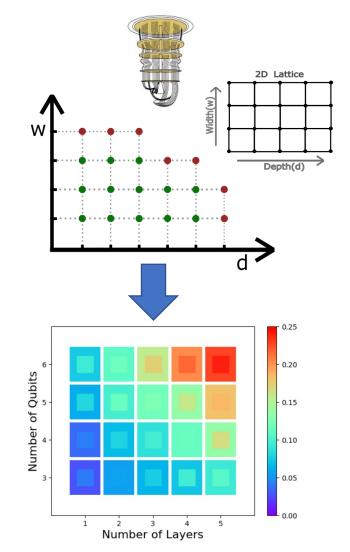
 This benchmark has already been used to test Quantinuum's H1 ion trap quantum processing unit.

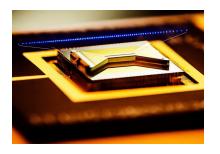


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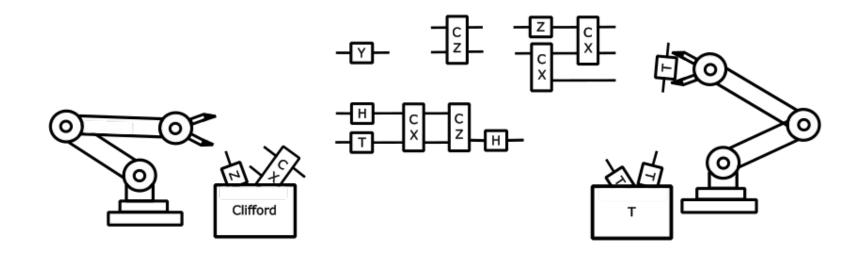


 This benchmark has already been used to test Quantinuum's H1 ion trap quantum processing unit.





The circuit synthesis problem



This work and its results gave origin to,



My PhD thesis

This work and its results gave origin to,



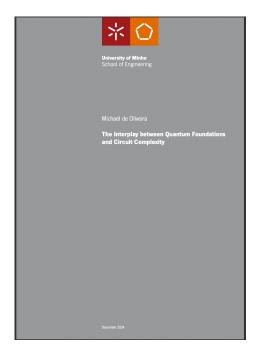
which is composed of



New synthesis techniques for shallow quantum circuits

My PhD thesis

This work and its results gave origin to,



which is composed of



New synthesis techniques for shallow quantum circuits



A new quantum benchmarking protocol

My PhD thesis

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My PhD thesis



New synthesis techniques for shallow quantum circuits





A new quantum benchmarking protocol

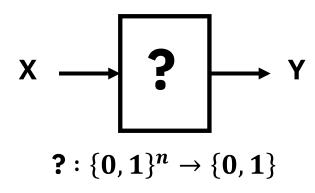


Unconditional quantum advantages with shallow depth circuits

Thank you!

@IBM - A New Quantum Learning Algorithm

Understand how hard it is to learn an unknown process:



Focus:

- Sample complexity
- Accuracy of the learned hypothesis
- Run-time

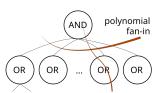
Learning arbitrary Boolean functions is an extremely hard problem!

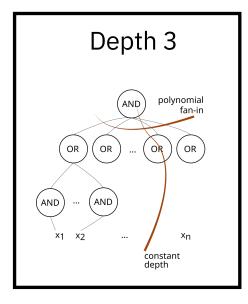
? :

Depth 1



Depth 2





We present a novel, state-ofthe-art quantum learning algorithm for PAC learning depth-3 circuits.

