Grover for satisfiability problems

Exercise 1 - 3-SAT Problem

The 3-Satisfiability Problem (3-SAT) is a classic problem in computational complexity theory and an important example of an NP-complete problem. It asks whether an assignment of boolean variables exists that satisfies a given boolean formula. The formula is expressed in conjunctive normal form (CNF), where each clause contains exactly three literals. A literal is a boolean variable or its negation, and a clause is a disjunction (logical OR) of literals. The entire formula is a conjunction (logical AND) of these clauses. Consider the Boolean formula below:

$$F = (x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2 \lor x_3) \land (x_1 \lor \neg x_2 \lor \neg x_3)$$

In this formula, each clause is enclosed in parentheses and contains exactly three literals combined using the logical OR operator $\lor$. The entire formula F is a conjunction of these clauses combined using the logical AND operator $\land$.

The goal is to determine whether an assignment of truth values (true or false) exists to the variables x_1, x_2, x_3 such that the entire formula F evaluates to true. This involves checking all possible combinations of truth values for the variables and identifying those that satisfy all the clauses simultaneously.

Tasks:

        1) Propose a solvable 3-SAT boolean formula.

        2) Implement Grover's algorithm to find a solution.

        3) Assess the algorithm's efficiency in finding a solution.

Exercise 2 - 2x2 binary sudoku

Sudoku is a popular logic-based number-placement puzzle. The objective is to fill a grid with numbers so that each row, column, and designated subgrid contains all numbers without repetition. For instance, in a 4x4 grid, there are 16 values to be assigned, with each position in the grid taking values 0 to 15. In a 2x2 sudoku, the problem is much simpler, and the rules are reduced to:

        1) No column may contain the same value twice
        2) No row may contain the same value twice
        3) Assignments are binary.

As you should convince yourself, there are only two possible binary assignments to solve the problem.

Tasks:

        1) Implement Grover's algorithm to find a solution to the problem.

        2) Assess the algorithm's efficiency and complexity.

        3) Propose a generalization for 3x3 sudoku and discuss potential limitations and resources required.