Quantum Computation 2022/23

TPC-1

Renato Neves

nevrenato@di.uminho.pt

Figure 1 presents the circuit respective to quantum teleportation. Analyse this circuit and then solve the two exercises below.

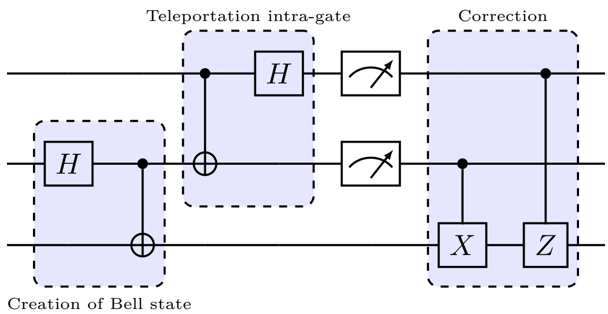

Figure 1: Quantum teleportation circuit.

Exercise 1. Write down the mathematical laws and definitions that were used at each step in the following computation. Note that the first step was already filled-in to serve as an example.

$(\alpha |0\rangle + \beta |1\rangle) \otimes \left(\frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle \right)$

$= \{$ Distributivity of scaling over addition and the tensor law $v \otimes sw = s(v \otimes w)$ $\}$

$= \frac{1}{\sqrt{2}} \big((\alpha |0\rangle + \beta |1\rangle) \otimes (|00\rangle + |11\rangle) \big)$

$= \{ \ \ldots \ \}$

$= \frac{1}{\sqrt{2}} (\alpha |000\rangle + \alpha |011\rangle + \beta |100\rangle + \beta |111\rangle)$

$\mapsto \{ \ \ldots \ \}$

$= \frac{1}{\sqrt{2}} (\alpha |000\rangle + \alpha |011\rangle + \beta |110\rangle + \beta |101\rangle)$

$= \{ \ \ldots \ \}$

$= \frac{1}{\sqrt{2}} (|0\rangle \otimes \alpha |00\rangle + |0\rangle \otimes \alpha |11\rangle + |1\rangle \otimes \beta |10\rangle + |1\rangle \otimes \beta |01\rangle)$

$= \{ \ \ldots \ \}$

$= \frac{1}{\sqrt{2}} \big(|0\rangle \otimes (\alpha |00\rangle + \alpha |11\rangle) + |1\rangle \otimes (\beta |10\rangle + \beta |01\rangle) \big)$

$\mapsto \{ \ \ldots \ \}$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes (\alpha\,|00\rangle + \alpha\,|11\rangle) + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes (\beta\,|10\rangle + \beta\,|01\rangle)\right)$$
$$= \{\ \dots\ \}$$
$$= \frac{1}{2}\left((|0\rangle + |1\rangle) \otimes (\alpha\,|00\rangle + |11\rangle) + (|0\rangle - |1\rangle) \otimes (\beta\,|10\rangle + \beta\,|01\rangle)\right)$$
$$= \{\ \dots\ \}$$
$$= \frac{1}{2}\left(\alpha\,|000\rangle + \alpha\,|011\rangle + \alpha\,|100\rangle + \alpha\,|111\rangle + \beta\,|010\rangle + \beta\,|001\rangle - \beta\,|110\rangle - \beta\,|101\rangle\right)$$
$$= \{\ \dots\ \}$$
$$= \frac{1}{2}\left(\,|00\rangle \otimes (\alpha\,|0\rangle + \beta\,|1\rangle) + |01\rangle \otimes (\beta\,|0\rangle + \alpha\,|1\rangle) + |10\rangle \otimes (\alpha\,|0\rangle - \beta\,|1\rangle) + |11\rangle \otimes (-\beta\,|0\rangle + \alpha\,|1\rangle)\right)$$

Exercise 2. The quantum teleportation protocol (Figure 1) starts by putting the qubits shared by Alice and Bob in the entangled state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Show that after slight modifications the protocol will work equally well if these two qubits are put instead in the entangled state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Present the modified circuit in Qiskit and discuss how you can use the latter to test the circuit.

What to submit: A report in PDF containing the solutions to both exercises. Please send by email (nevrenato@gmail.com) your file named as "QC2223-N.PDF", where N is your student number. The subject of the email should be "QC2223-N".
Deadline: 24th October 2022 @ 23h59