# Quantum Computing @ MEF

TPC-1

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Figure 1 presents the circuit respective to quantum teleportation. Analyse this circuit and then solve the two exercises below.

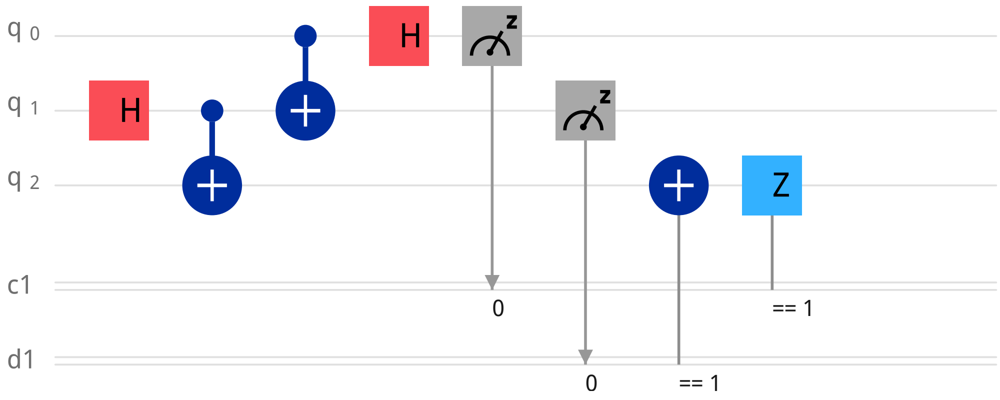


Figure 1: Quantum teleportation circuit.

**Exercise 1.** Write down the mathematical laws and definitions that were used at each step in following calculation.

$$
\begin{aligned}
&(\alpha\,|0\rangle + \beta\,|1\rangle) \otimes \left(\frac{1}{\sqrt{2}}\,|00\rangle + \frac{1}{\sqrt{2}}\,|11\rangle\right)\\
&= \{\ \dots \}\\
&= \frac{1}{\sqrt{2}}\big(\,(\alpha\,|0\rangle + \beta\,|1\rangle) \otimes (|00\rangle + |11\rangle)\,\big)\\
&= \{\ \dots\ \}\\
&= \frac{1}{\sqrt{2}}(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|100\rangle + \beta\,|111\rangle)\\
&\mapsto \{\ \dots\ \}\\
&= \frac{1}{\sqrt{2}}(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|110\rangle + \beta\,|101\rangle)\\
&= \{\ \dots\ \}\\
&= \frac{1}{\sqrt{2}}(|0\rangle \otimes \alpha\,|00\rangle + |0\rangle \otimes \alpha\,|11\rangle + |1\rangle \otimes \beta\,|10\rangle + |1\rangle \otimes \beta\,|01\rangle)\\
&= \{\ \dots\ \}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2}}\big( \left|0\right\rangle \otimes (\alpha \left|00\right\rangle + \alpha \left|11\right\rangle) + \left|1\right\rangle \otimes (\beta \left|10\right\rangle + \beta \left|01\right\rangle)\big) \\
&\mapsto \{ \ \ldots \ \} \\
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(\left|0\right\rangle + \left|1\right\rangle) \otimes (\alpha \left|00\right\rangle + \alpha \left|11\right\rangle) + \frac{1}{\sqrt{2}}(\left|0\right\rangle - \left|1\right\rangle) \otimes (\beta \left|10\right\rangle + \beta \left|01\right\rangle)\right) \\
&= \{ \ \ldots \ \} \\
&= \frac{1}{2}\left((\left|0\right\rangle + \left|1\right\rangle) \otimes (\alpha \left|00\right\rangle + \left|11\right\rangle) + (\left|0\right\rangle - \left|1\right\rangle) \otimes (\beta \left|10\right\rangle + \beta \left|01\right\rangle)\right) \\
&= \{ \ \ldots \ \} \\
&= \frac{1}{2}\left(\alpha \left|000\right\rangle + \alpha \left|011\right\rangle + \alpha \left|100\right\rangle + \alpha \left|111\right\rangle + \beta \left|010\right\rangle + \beta \left|001\right\rangle - \beta \left|110\right\rangle - \beta \left|101\right\rangle\right) \\
&= \{ \ \ldots \ \} \\
&= \frac{1}{2}\big( \left|00\right\rangle \otimes (\alpha \left|0\right\rangle + \beta \left|1\right\rangle) + \left|01\right\rangle \otimes (\beta \left|0\right\rangle + \alpha \left|1\right\rangle) + \left|10\right\rangle \otimes (\alpha \left|0\right\rangle - \beta \left|1\right\rangle) + \left|11\right\rangle \otimes (-\beta \left|0\right\rangle + \alpha \left|1\right\rangle)\big)
\end{aligned}
$$

**Exercise 2.** The quantum teleportation protocol (Fig. 1) starts by putting the qubits shared by Alice and Bob in the entangled state $\frac{1}{\sqrt{2}}(\left|00\right\rangle + \left|11\right\rangle)$. Show that after slight modifications the protocol will work equally well if these two qubits are put instead in the entangled state $\frac{1}{\sqrt{2}}(\left|01\right\rangle + \left|10\right\rangle)$. Present the modified circuit in Qiskit and discuss how you can use the latter to test the circuit.