Computação Quântica

Problems on the Quantum Fourier Transform — May 2021

This exercise aims at improving your understanding of the quantum Fourier transform, a most relevant component in several quantum algorithms.

Recall the definition of QFT on K basis states:

$$\mathsf{QFT}_\mathsf{K}(|x\rangle) \;=\; \frac{1}{\sqrt{K}}\sum_{y=0}^{K-1} e^{2\pi i(\frac{x}{K})y}|y\rangle$$

- Compute $\mathsf{QFT}_\mathsf{K}(|00\cdots0\rangle)$.
- The following equality

  $$\mathsf{QFT}_\mathsf{K}(|x_1\cdots x_n\rangle) \;=\; \left(\frac{|0\rangle + e^{2\pi i(0.x_n)}|1\rangle}{\sqrt{2}}\right)\otimes\left(\frac{|0\rangle + e^{2\pi i(0.x_n x_{n-1})}|1\rangle}{\sqrt{2}}\right)\cdots\otimes\cdots\left(\frac{|0\rangle + e^{2\pi i(0.x_1 x_2\cdots x_n)}|1\rangle}{\sqrt{2}}\right)$$

  was used in the lecture slides without proof. Verify it holds indeed.
- One can show, as we did in the lectures, that QFT is a unitary gate by building a unitary quantum circuit for its computation. Give an alternative, direct proof that the linear transformation defined above is unitary.
- Reproduce the circuit for QFT_4 and QFT_8, and compute the corresponding matrices. Give your calculation in detail.