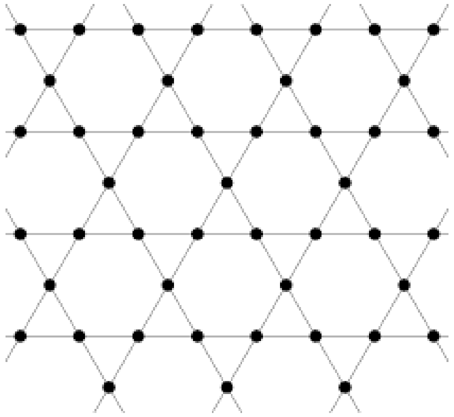
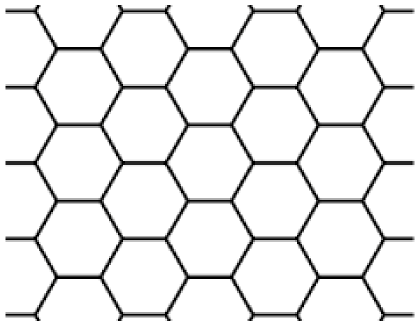
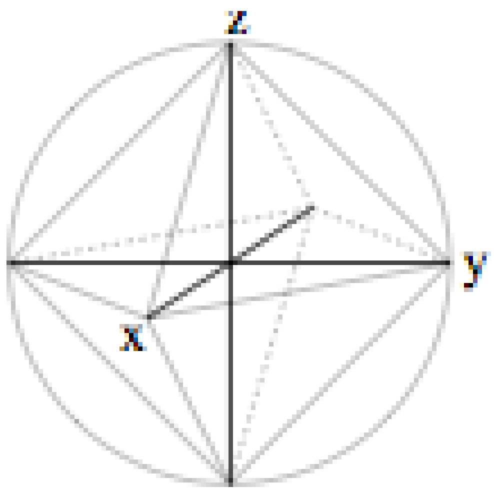
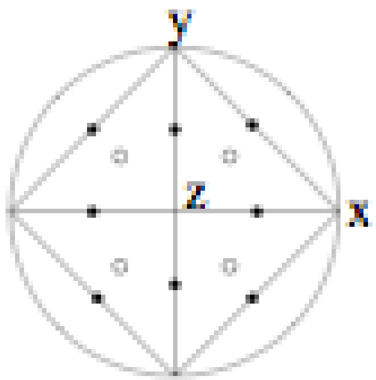


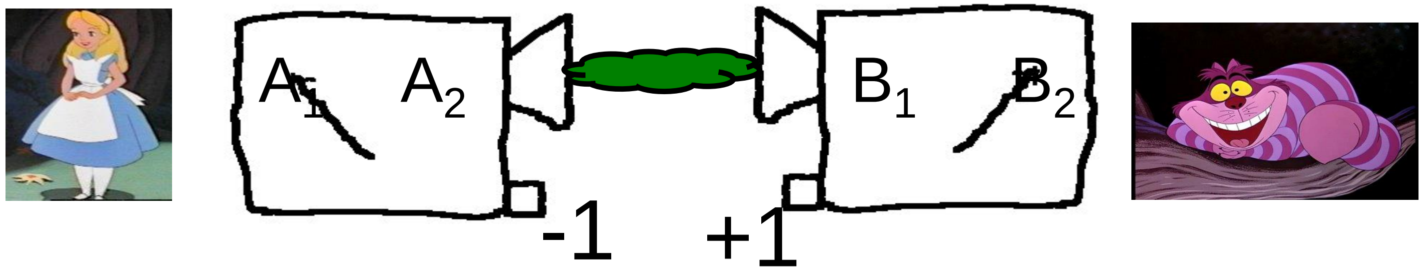
Introduction to measurement-based quantum computation



Ernesto F. Galvão (INL)







Aula convidada, Mestrado em Eng. Física, UMinho. 7/5/2020

Quantum and Linear-Optical Computation group - INL

Group leader: Ernesto Galvão
(started in July 2019)
Previously:
Universidade Federal Fluminense (Brazil)
(on leave)

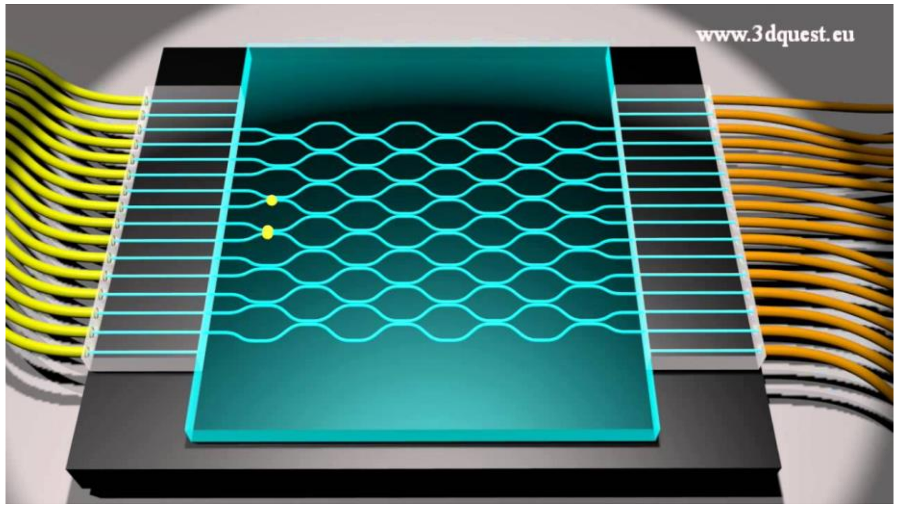
Staff Researcher: Rui Soares Barbosa
(Staff Researcher from February 2020)

Staff Researcher
(on-going selection process)

Quantum and Linear-Optical Computation group - INL

Research:

- Foundations of quantum computation
  - Contextuality as a resource
  - Measurement-based and topological quantum computation
- Photonic quantum computation
  - Characterization of bosonic indistinguishability
  - Implementation of complex multimode linear optical interferometers for computation



Boson sampling devices use quantum interference for quantum computational advantage

<u>Publication highlights</u>

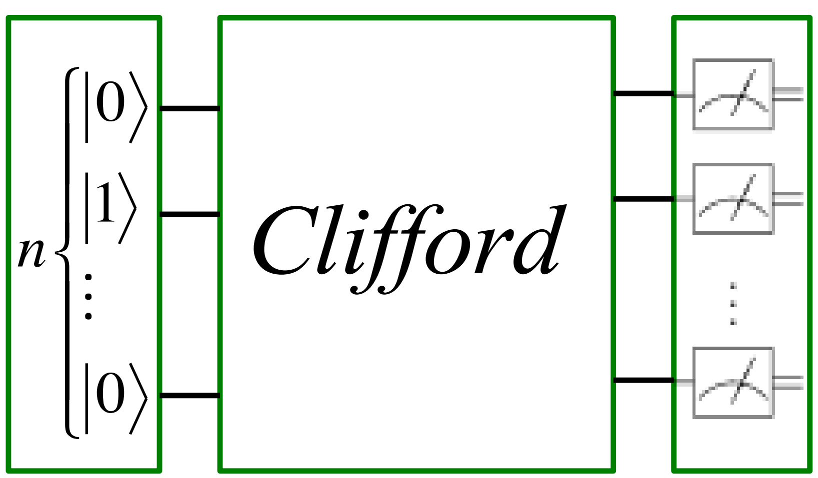
Photonic implementation of boson sampling: a review. *Advanced Photonics* **1** (3), 034001 (2019).
Witnessing genuine multiphoton indistinguishability. *Phys. Rev. Lett.* **122**, 063602 (2019).
Contextual Fraction as a Measure of Contextuality. *Phys. Rev. Lett.* **119**, 050504 (2017).
Experimental scattershot boson sampling. *Science Advances* **1** (3), e1400255 (2015).
Experimental validation of photonic boson sampling. *Nature Photonics* **8**, 615 (2014).
General rules for bosonic bunching in multimode interferometers. *Phys. Rev. Lett.* **111**, 130503 (2013).
Integrated multimode interferometers with arbitrary designs for photonic boson sampling. *Nature Photonics* **7**, 545–549 (2013).

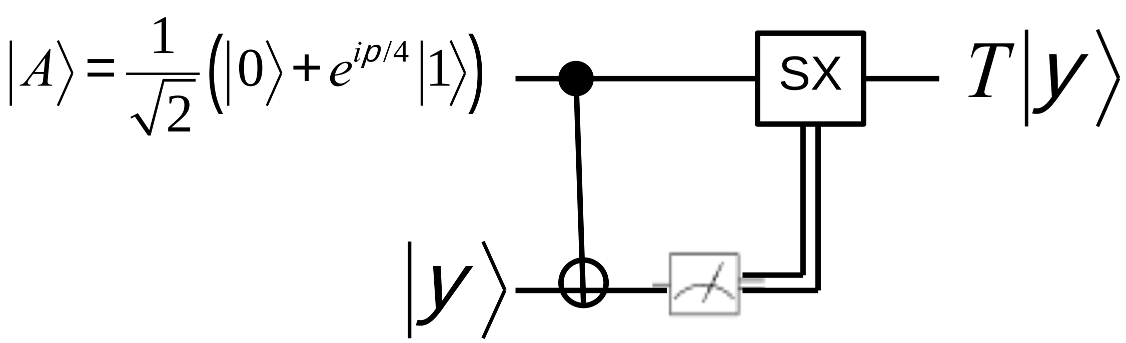
Introduction to measurement-based quantum computation

Outline:

- Clifford circuits
  - Pauli and Clifford groups
  - Simulability of Clifford circuits
  - Upgrading Clifford circuits to universal QC
- How MBQC works
  - One-bit teleportation circuit
  - Gate teleportation
  - Concatenating MBQC gates
- Resources for MBQC: graph and cluster states
- Experimental implementations
- Resources for MBQC: contextuality and non-locality

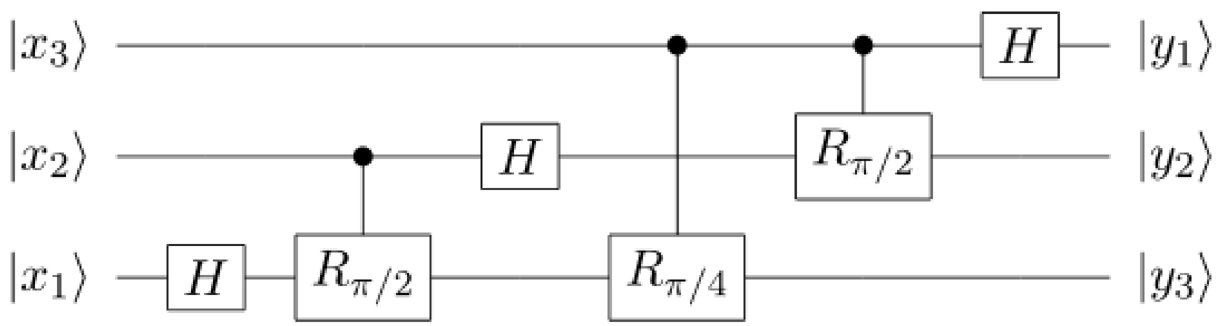
Clifford circuits



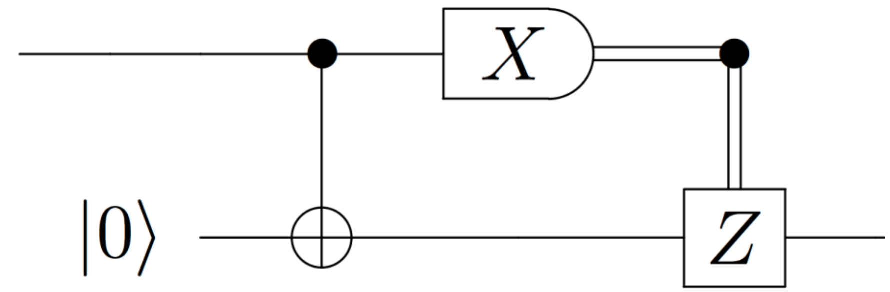


Basics of the circuit model

- The most well-known model for quantum computation is the circuit model, obtained in analogy with classical circuits

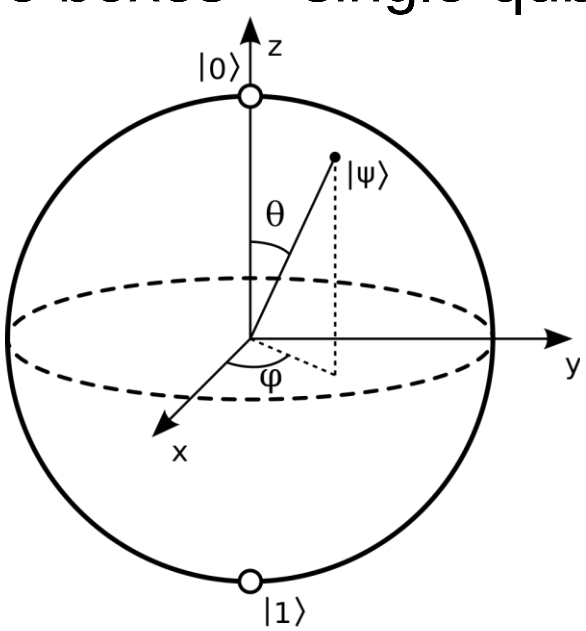


3-qubit QFT

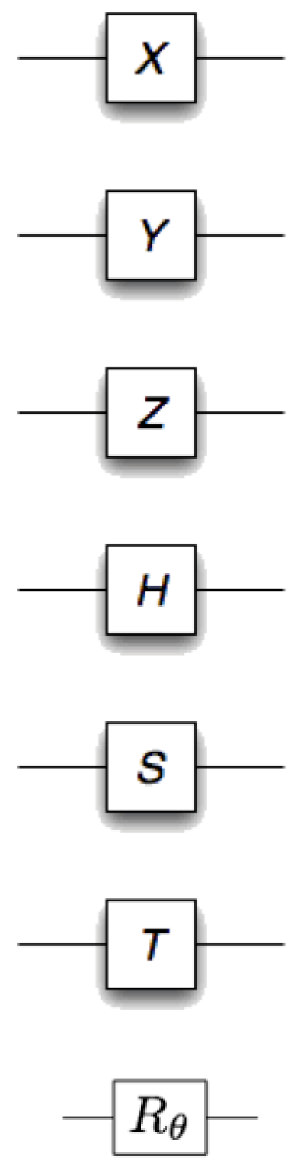


1-bit Z teleportation

- wires = qubits (i.e. 2-level systems)
- little boxes = single-qubit gates



$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$$



Pauli X (NOT) = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Pauli Y = $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Pauli Z (Phase Flip) = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Hadamard = $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

Phase = $\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

$\pi/8$ = $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

Phase shift = $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

Clifford circuits

- **Pauli group**: tensor products of $\pm I, \pm iI, X, Z$

- example: $-iZ_1 \otimes X_2 \otimes I_3$

Clifford circuits

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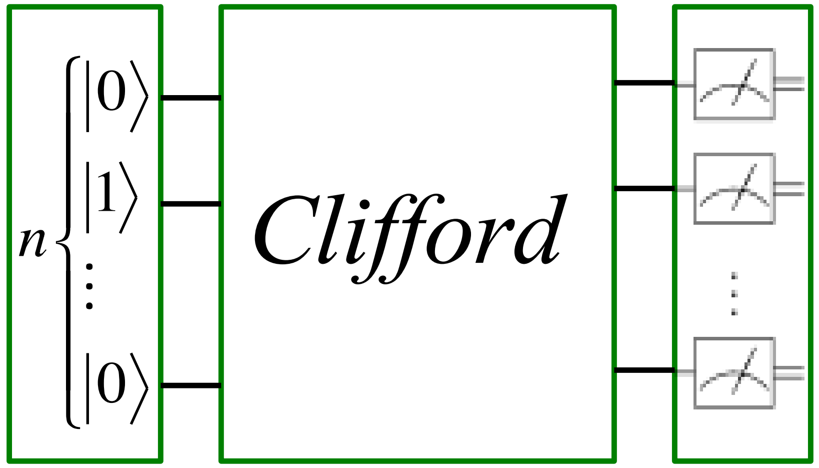
- **Clifford group**: unitaries C that map Paulis into Paulis:

$$CP_iC^+ = P_j \Leftrightarrow CP_i = P_jC$$

- Clifford group is generated by $\{H, P, CNOT\}$

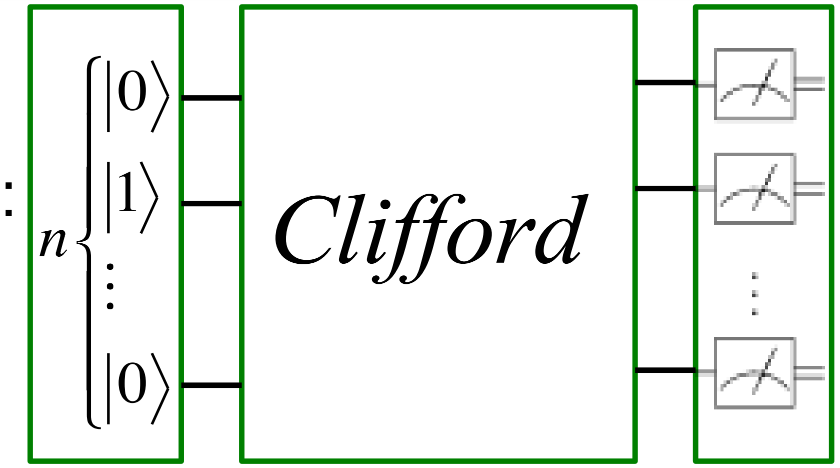


- Clifford circuits create large amounts of entanglement, are useful for teleportation, error correction…

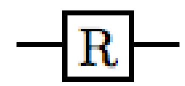
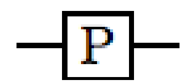
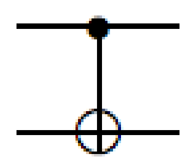
…but are **efficiently simulable**.

Clifford circuits

- **Pauli group**: tensor products of $\pm I, \pm iI, X, Z$
- **Clifford group**: unitaries C that map Paulis into Paulis:



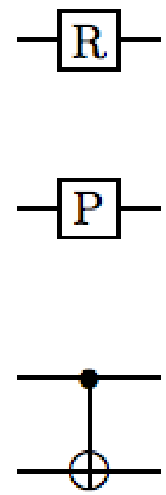
$$CP_iC^{+} = P_j \Leftrightarrow CP_i = P_jC$$

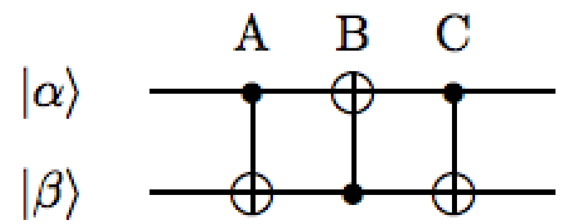
R	$X \rightarrow Z$ $Z \rightarrow X$	
P	$X \rightarrow Y$ $Z \rightarrow Z$	
CNOT	$X \otimes I \rightarrow X \otimes X$ $I \otimes X \rightarrow I \otimes X$ $Z \otimes I \rightarrow Z \otimes I$ $I \otimes Z \rightarrow Z \otimes Z$	

- The key simulation idea is to use Heisenberg picture:
  - initial state is eigenstate of Pauli operator
  - each Clifford gate maps it into a new Pauli (efficient computation)
  - keep track of the Pauli transformation until end, when measurement outcomes can be efficiently computed.
- Clifford circuits are not believed even to be able to do universal classical computation…

Example: Heisenberg simulation of Clifford circuit

R	$X \to Z$ $Z \to X$
P	$X \to Y$ $Z \to Z$
CNOT	$X \otimes I \to X \otimes X$ $I \otimes X \to I \otimes X$ $Z \otimes I \to Z \otimes I$ $I \otimes Z \to Z \otimes Z$





b)

A: CNOT$(1 \to 2)$	$\overline{X}_1$	$X \otimes X$
	$\overline{X}_2$	$I \otimes X$
	$\overline{Z}_1$	$Z \otimes I$
	$\overline{Z}_2$	$Z \otimes Z$
B: CNOT$(2 \to 1)$	$\overline{X}_1$	$I \otimes X$
	$\overline{X}_2$	$X \otimes X$
	$\overline{Z}_1$	$Z \otimes Z$
	$\overline{Z}_2$	$Z \otimes I$
C: CNOT$(1 \to 2)$	$\overline{X}_1$	$I \otimes X$
	$\overline{X}_2$	$X \otimes I$
	$\overline{Z}_1$	$I \otimes Z$
	$\overline{Z}_2$	$Z \otimes I$

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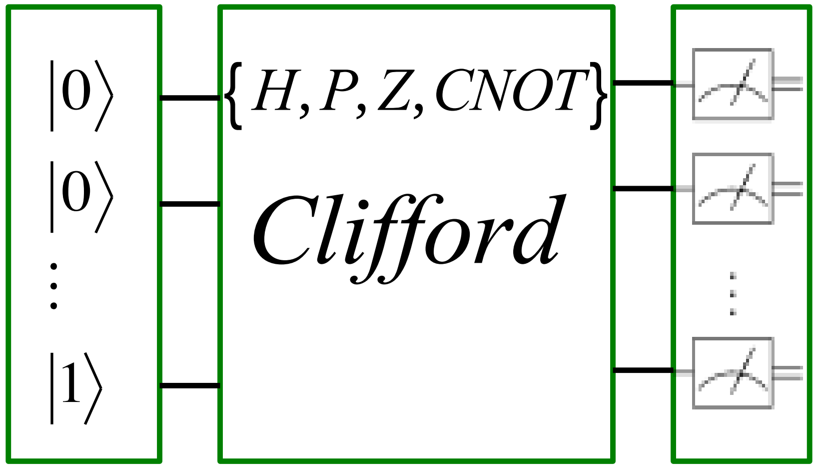
"Upgrading" a Clifford computer

- Clifford: $\{H, P, Z, CNOT\}$, all that's missing is *T* gate



“Upgrading” a Clifford computer

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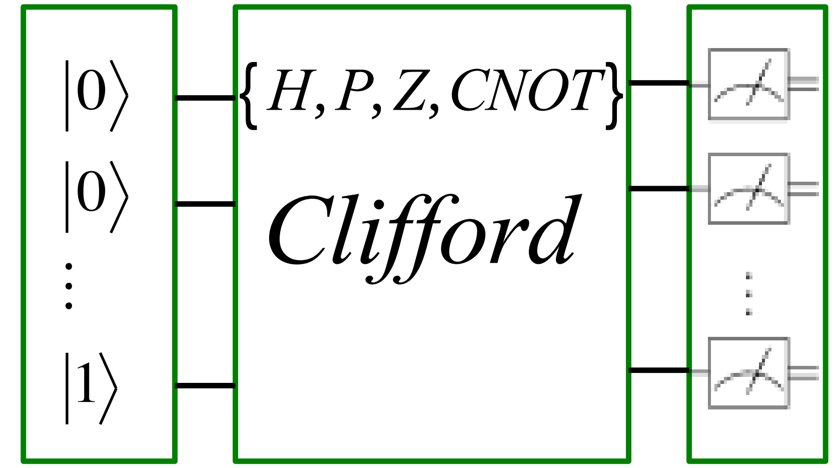
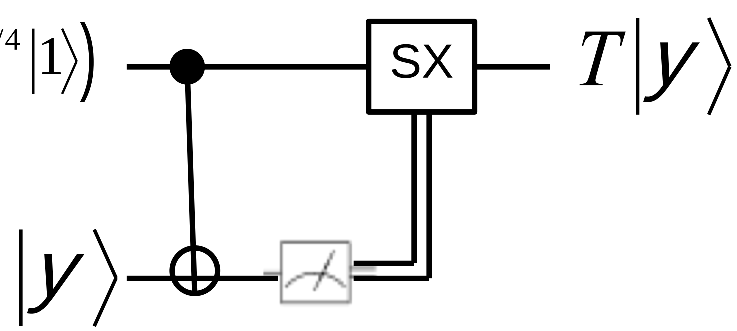


- There’s a work-around using:
  - **magic input states** and
  - **adaptativity**

  [Bravyi, Kitaev PRA 71, 022136 (2005)]

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{ip/4}|1\rangle\right)$$



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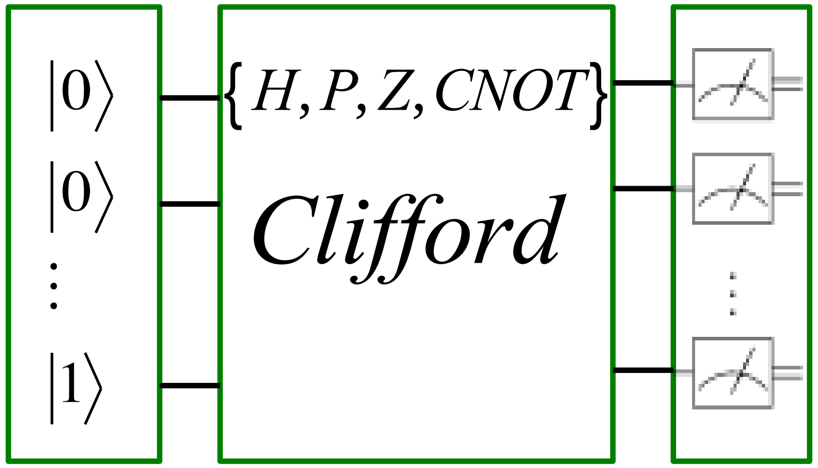

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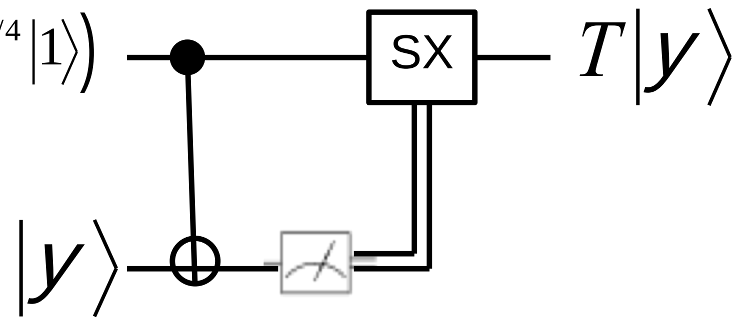

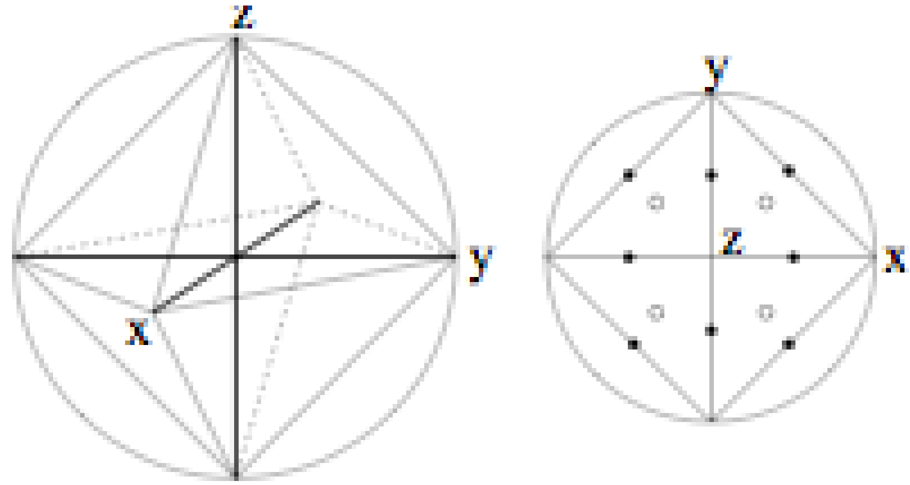

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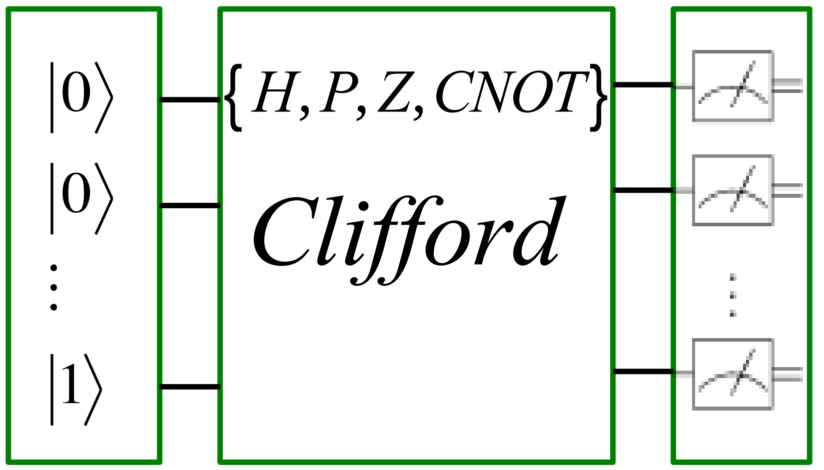

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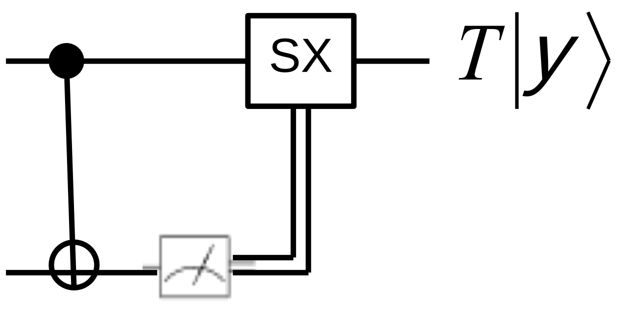
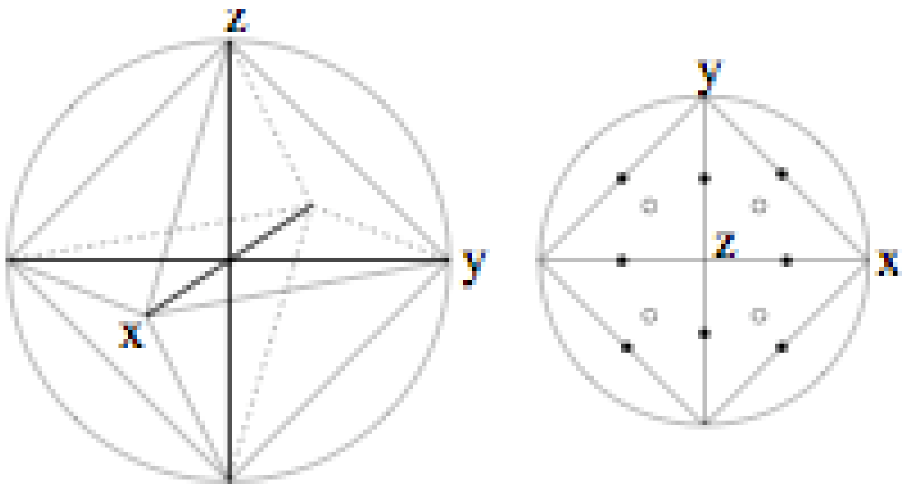
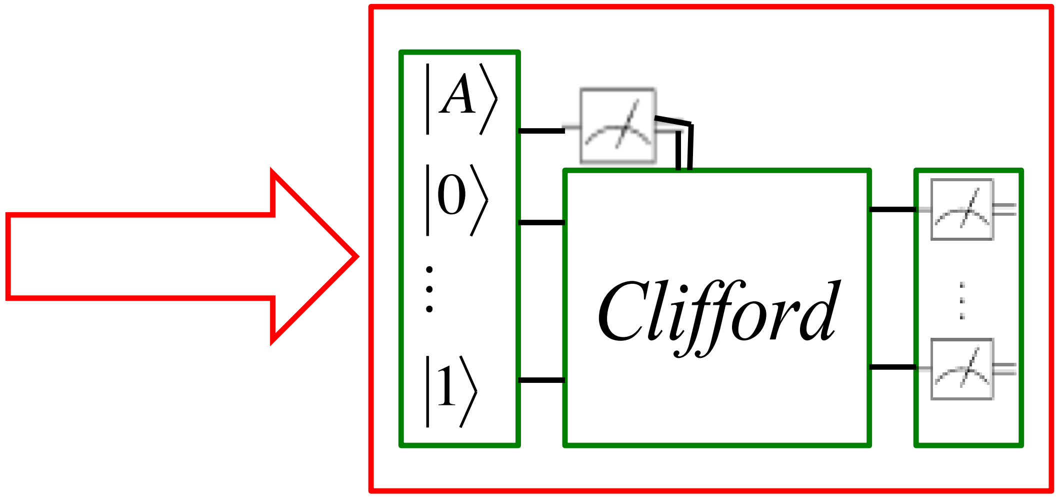


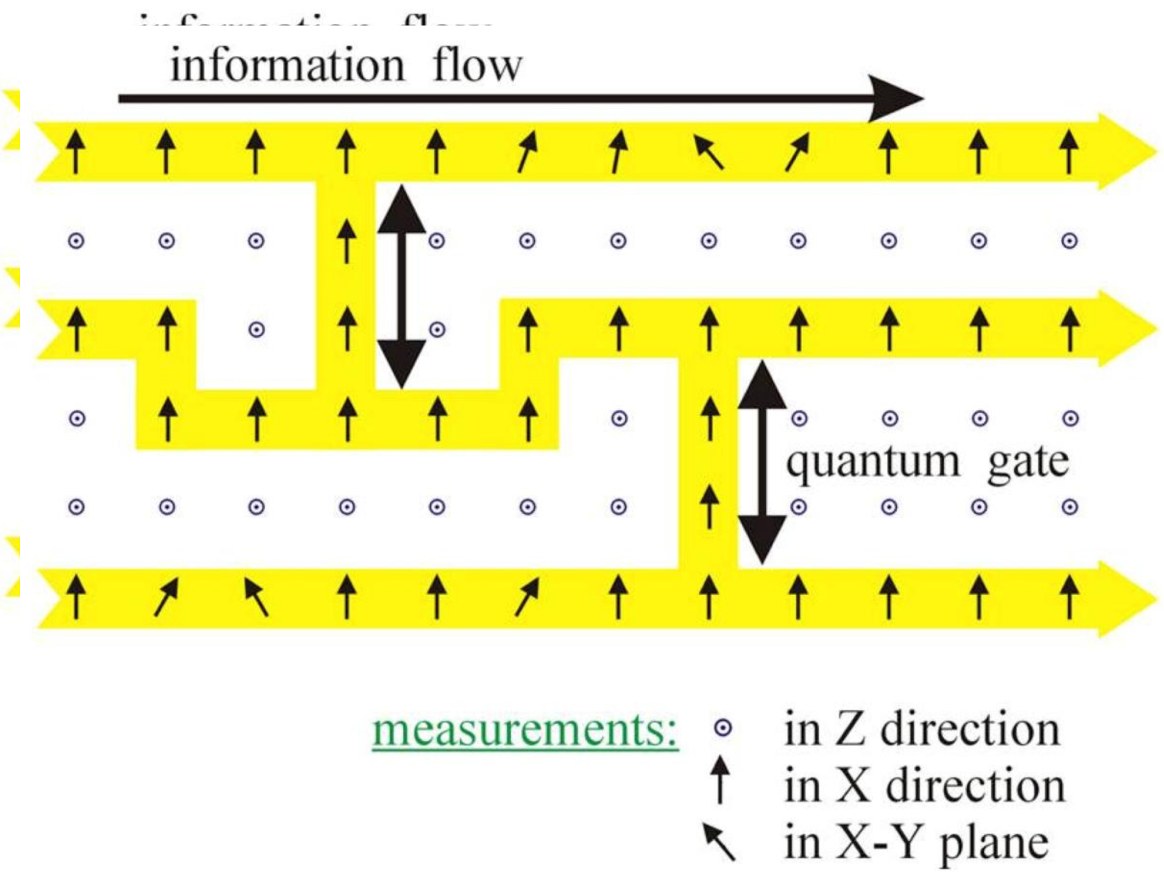


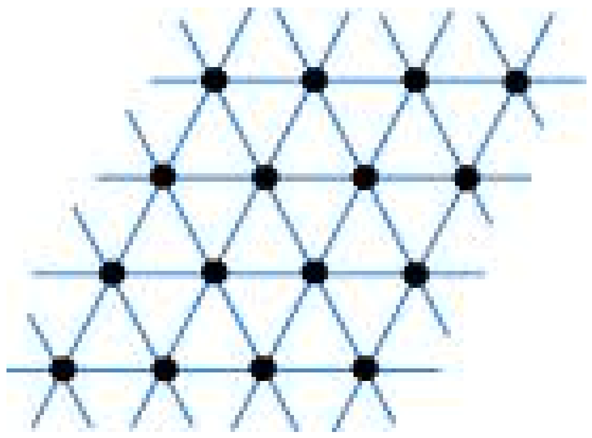


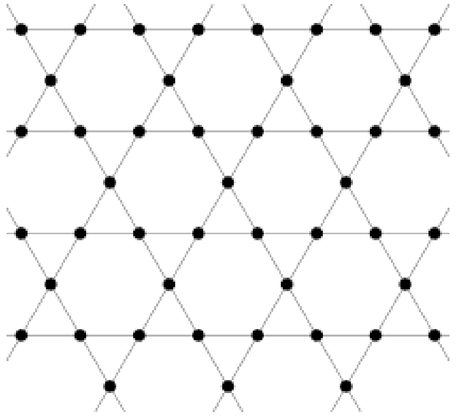
is universal for QC

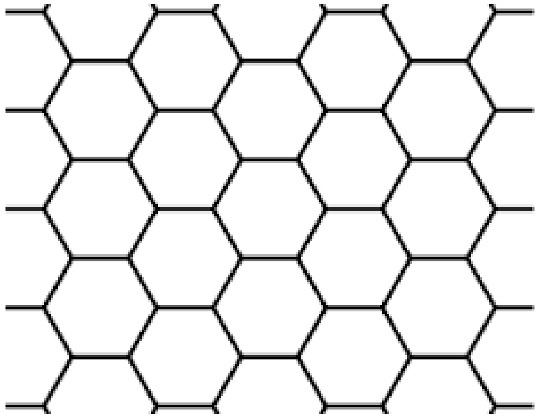
- Relevant for topological quantum computation with anyons, as for example Ising model implements Clifford operations in a topologically protected way

Measurement-based quantum computation (MBQC)



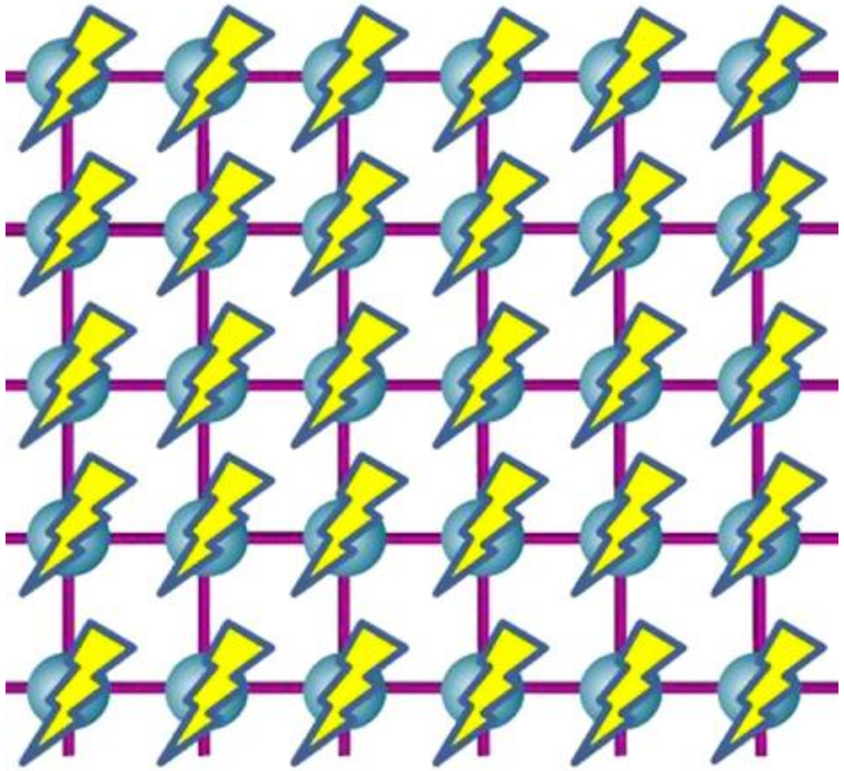






MBQC: basic ingredients

- Class of QC models where the computation is driven by measurements on previously entangled states



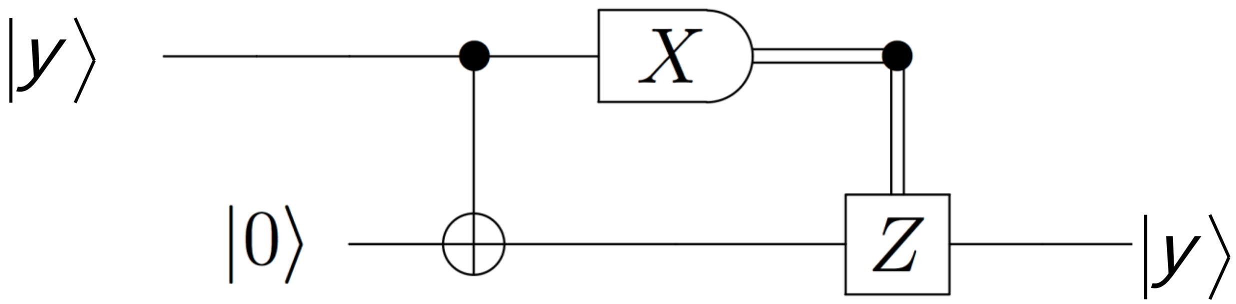
1- Initialization by CZ gates on $|+\rangle$ states;

2- Sequence of single-qubit, adaptive measurements.

- Origin: gate teleportation idea [Gottesman, Chuang, Nature 402, 390 (1999)]
- Most well-know variant is the one-way model (1WQC)[Raussendorf, Briegel PRL 86, 5188 (2001)]
- Brief introduction to MBQC based on McKague's paper "Interactive proofs for BQP via self-tested graph states" arxiv:1309.5675 (2013)

MBQC: step-by-step

3 versions of the "1-bit *Z* teleportation" circuit:



- X measurement result controls Z gate
- Direct calculation shows this works

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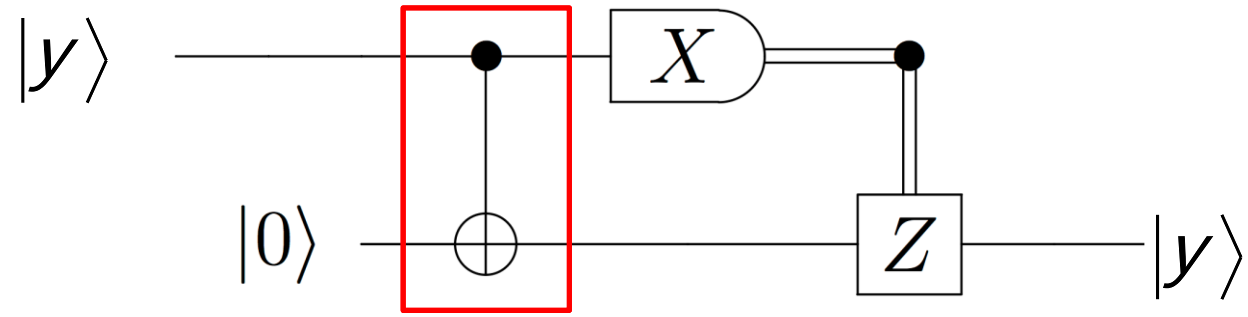

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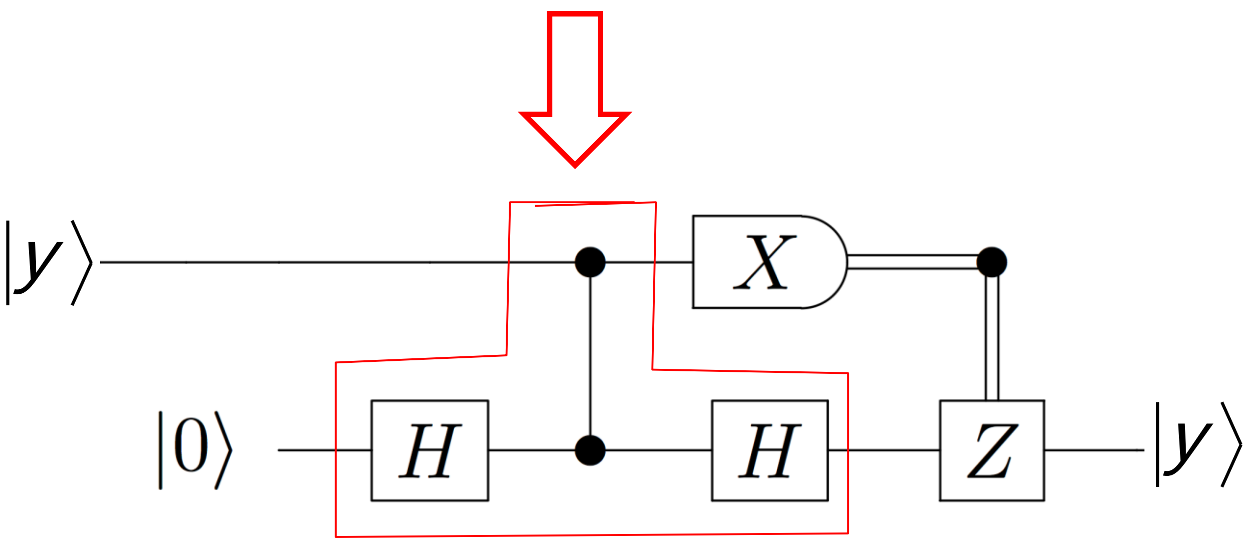

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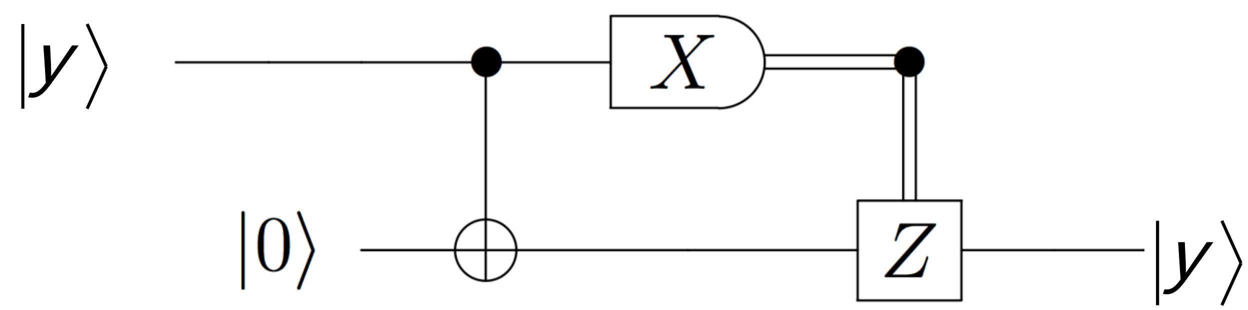

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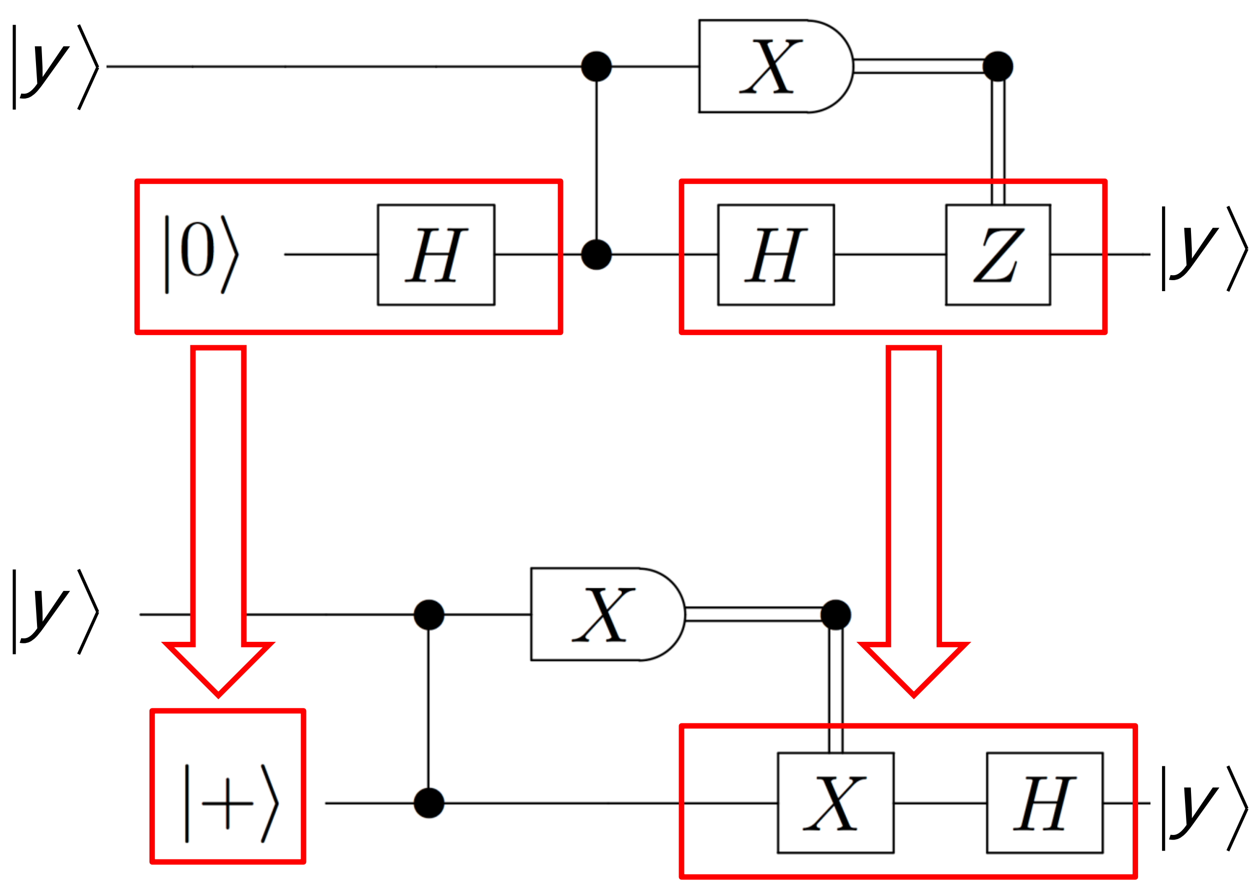

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- Left H incorporated in input $|+\rangle$
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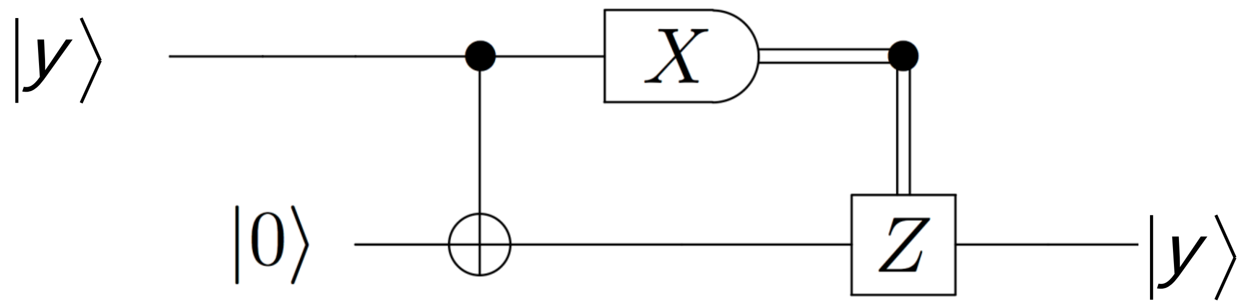

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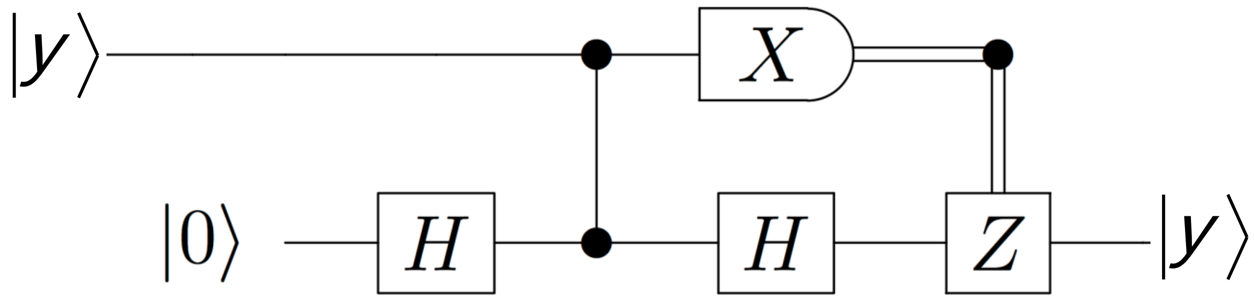

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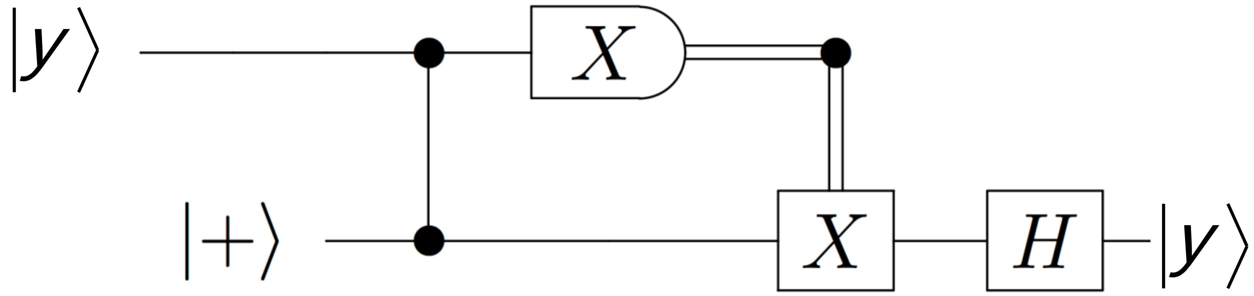
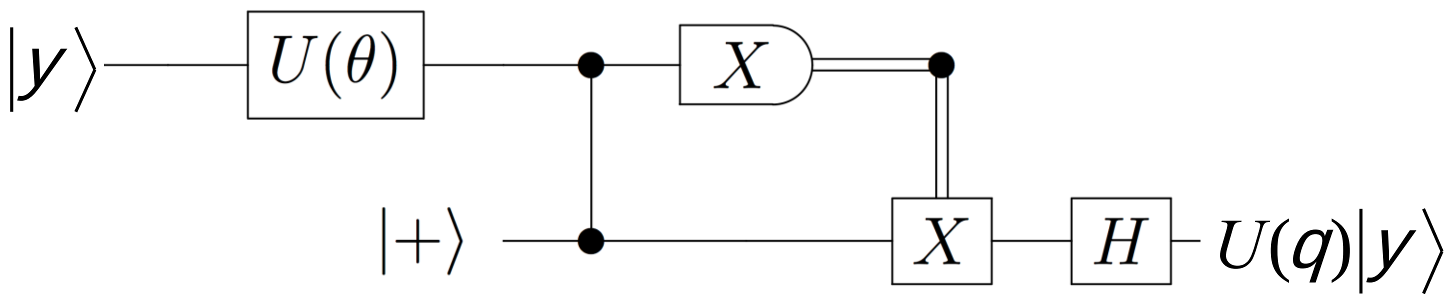


- Left H incorporated in input $|+\rangle$
- HZ = XH identity

So far: no computation, but: ancilla initialized in $|+\rangle$ state; CZ gate creates entanglement

MBQC: step-by-step

Now let's teleport the unitary $U(q) = \exp(iqZ/2)$:



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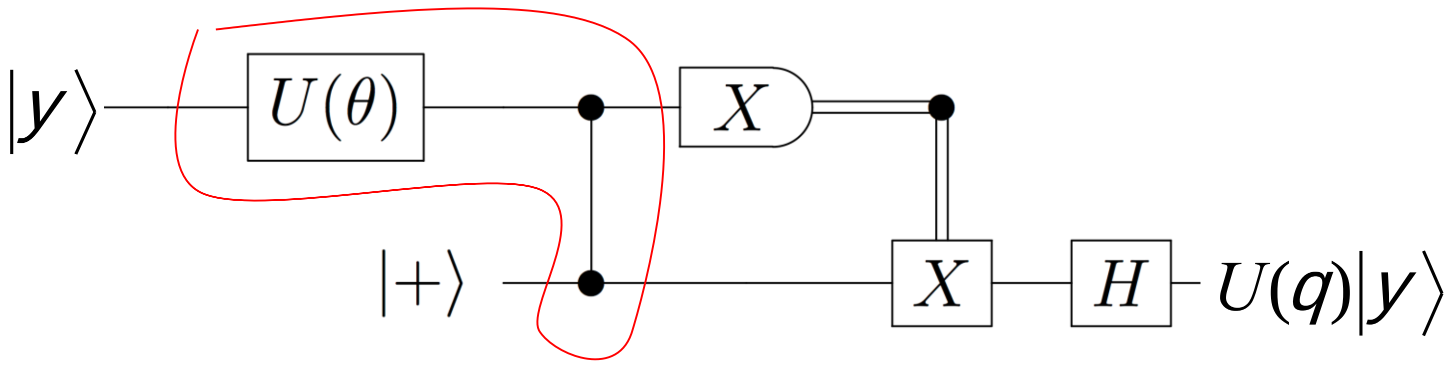

- U commutes with CZ

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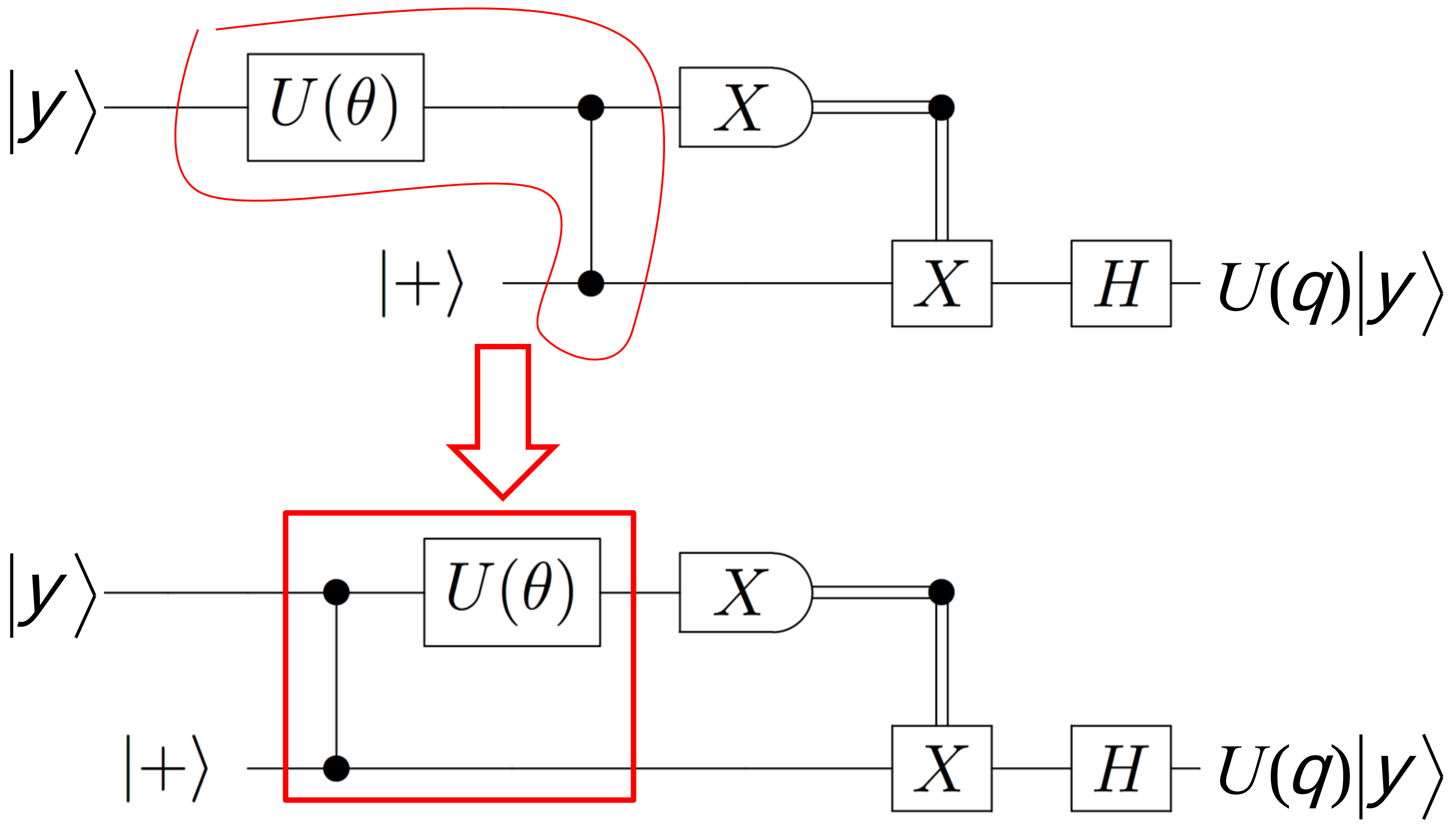

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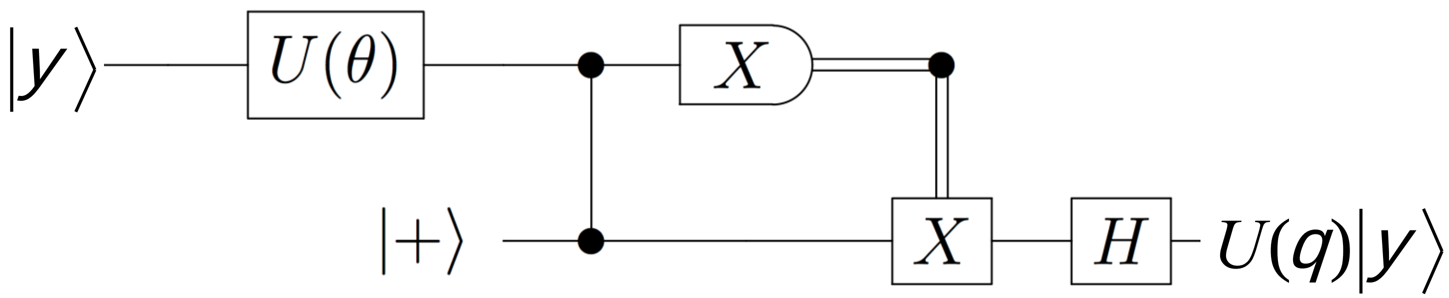

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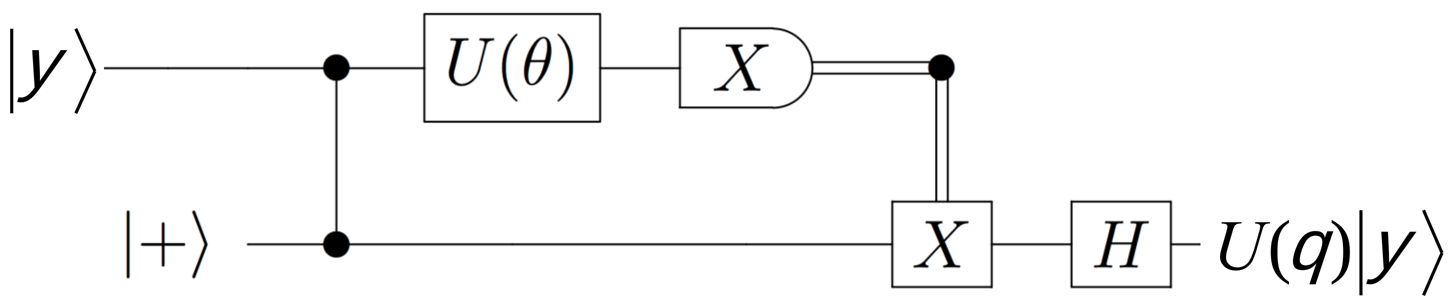
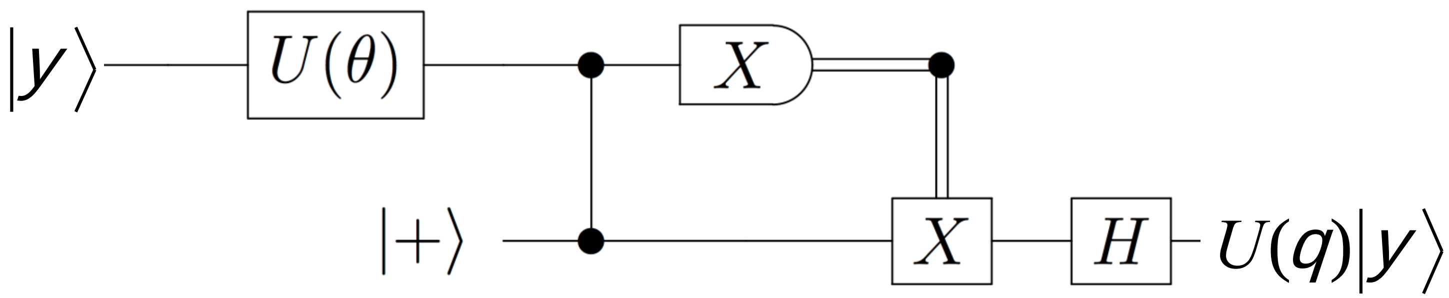


MBQC: step-by-step

Now let's teleport the unitary $U(\theta) = \exp(i\theta Z/2)$**:**



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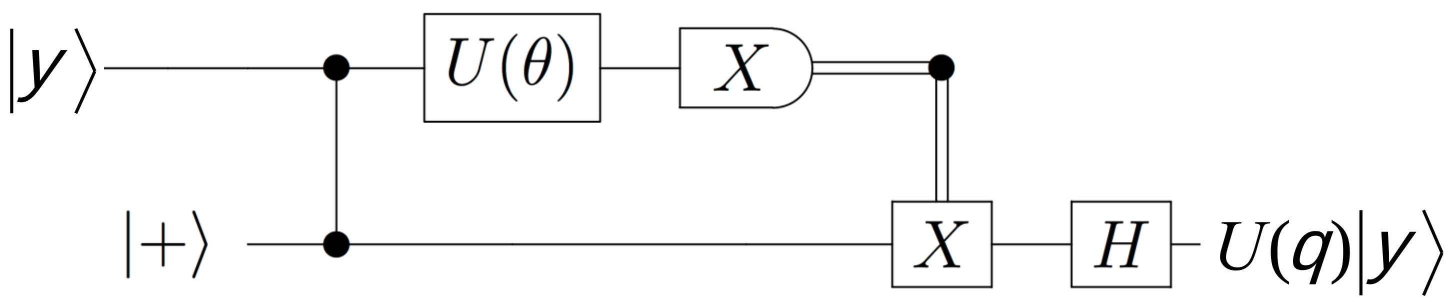
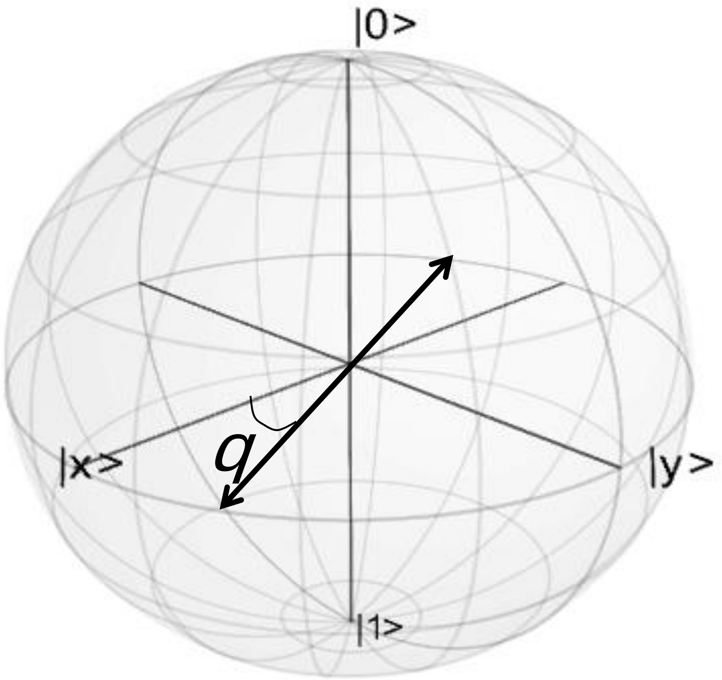


- U followed by X-measurement = measurement in *x-y* plane of Bloch sphere:

$$U^{+}XU = R(\theta) = \cos(\theta)X + \sin(\theta)Y$$



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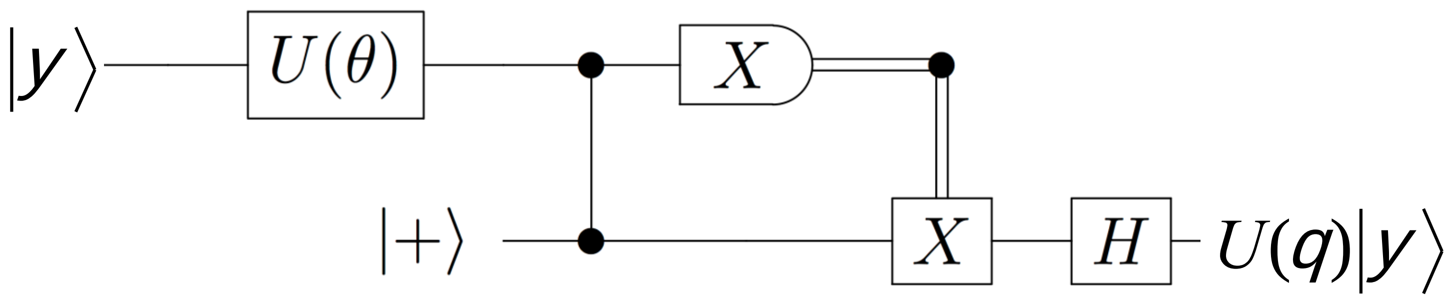
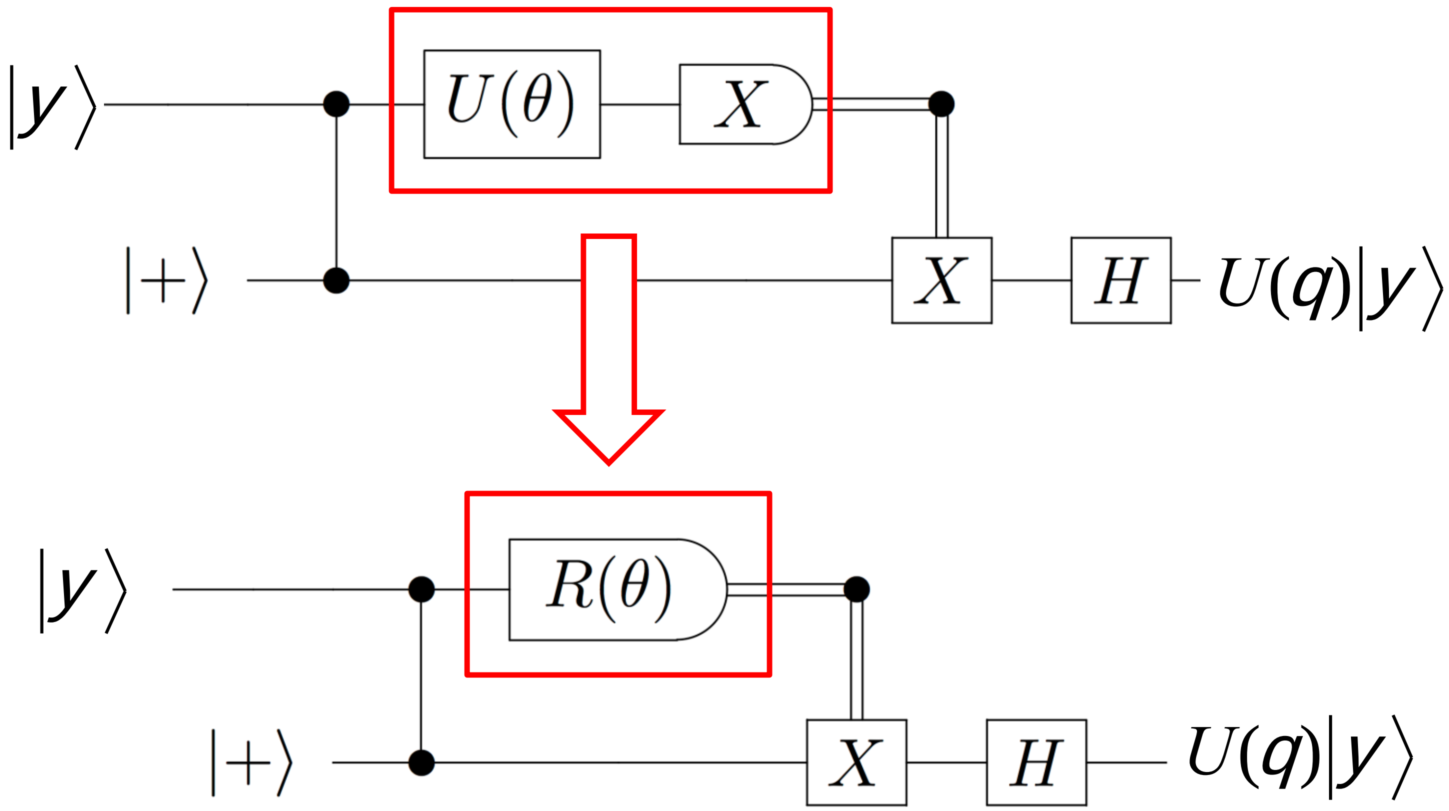
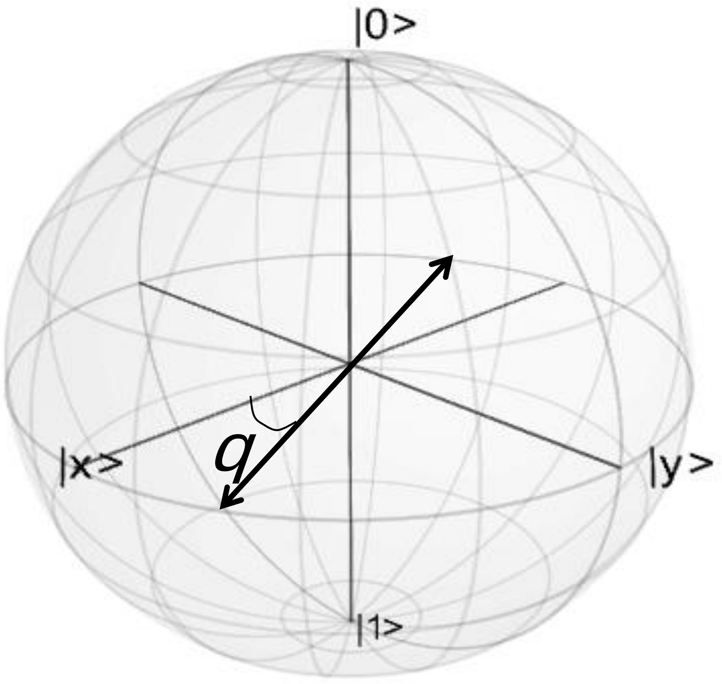


- U commutes with CZ



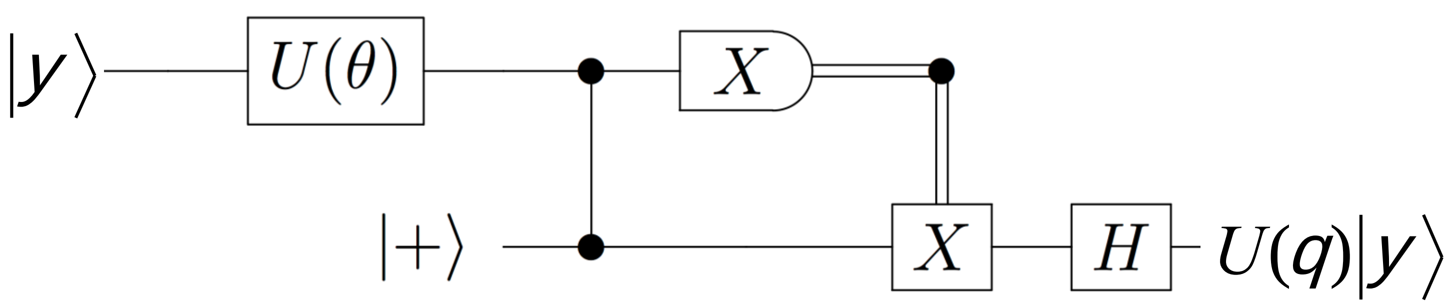
- U followed by X-measurement = measurement in *x*-*y* plane of Bloch sphere:

$$U^{+}XU = R(\theta) = \cos(\theta)X + \sin(\theta)Y$$



MBQC: step-by-step

Now let's teleport the unitary $U(q) = \exp(iqZ/2)$**:**



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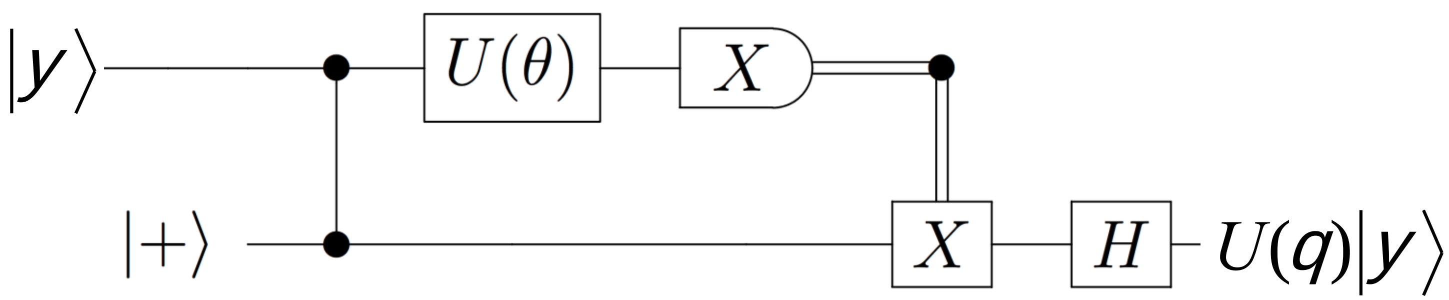

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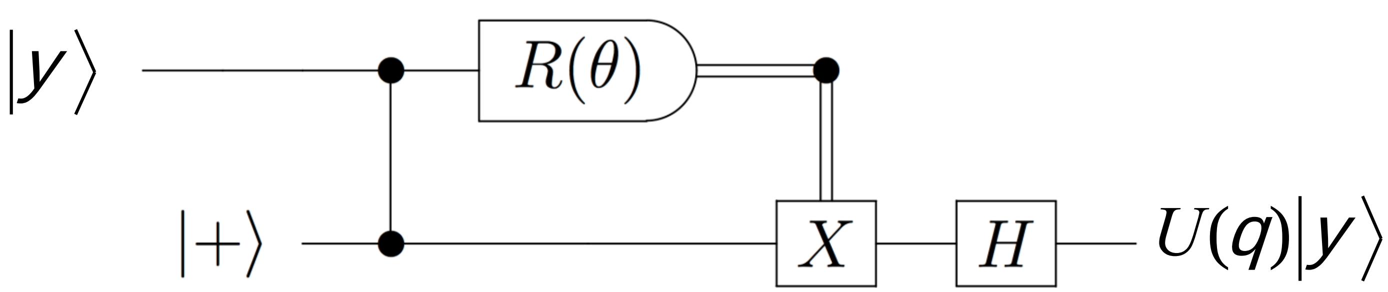
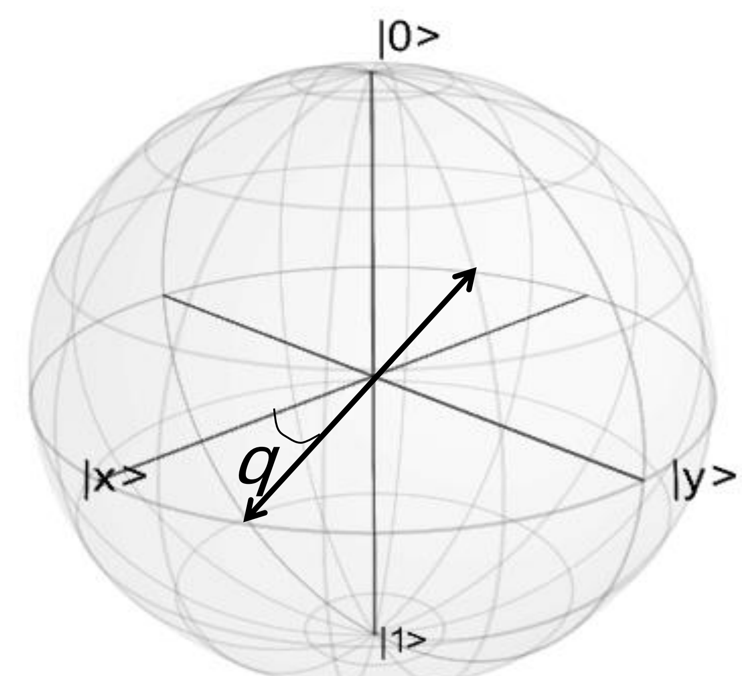


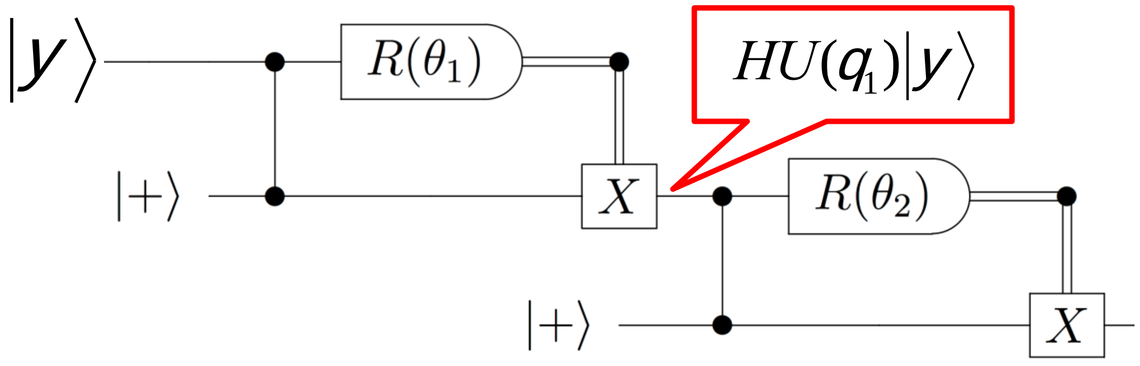
Evolved state $U(q)|y\rangle$ is teleported, via entanglement and right choice of measurement basis of top qubit

(*gate teleportation* idea of Gottesman and Chuang)



MBQC: step-by-step

Now two different unitaries in sequence:



- Two gate teleportations, without final H gates, result in final state

$$HU(q_2)HU(q_1)|y\rangle$$

MBQC: step-by-step

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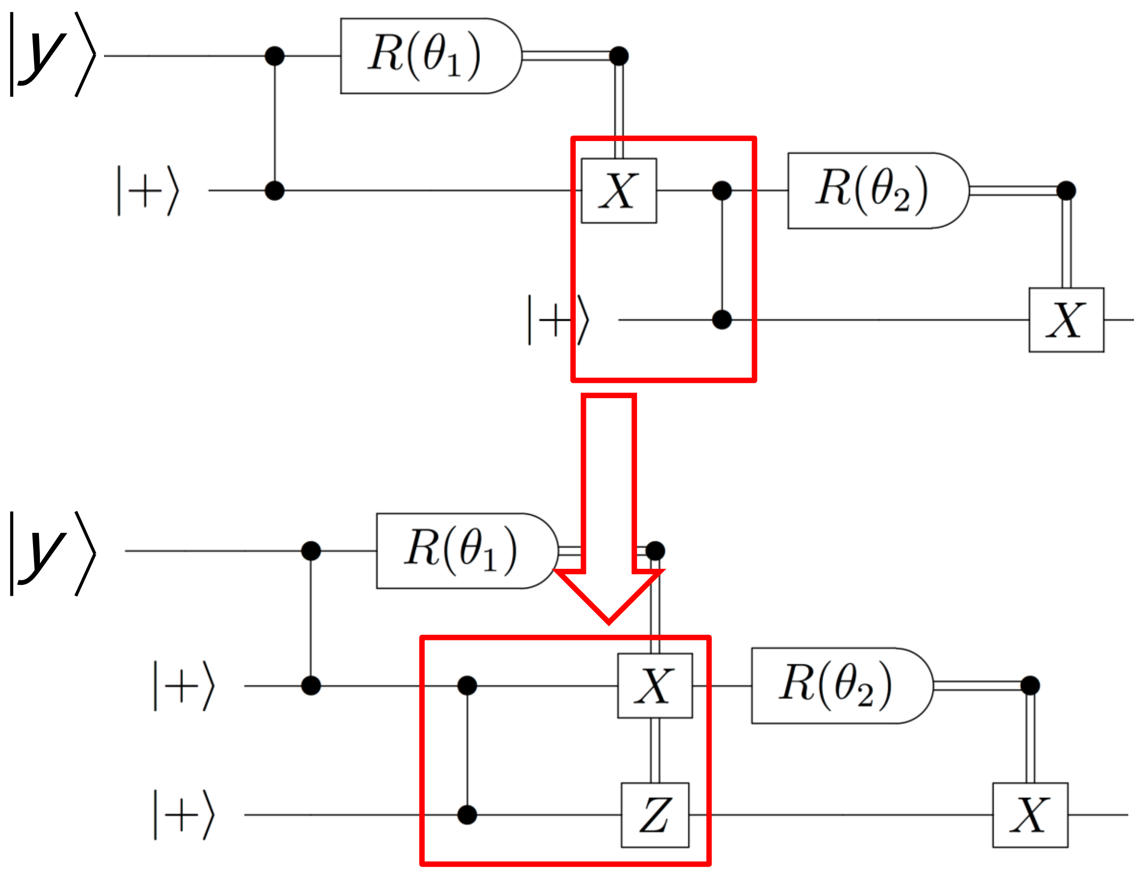

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MBQC: step-by-step

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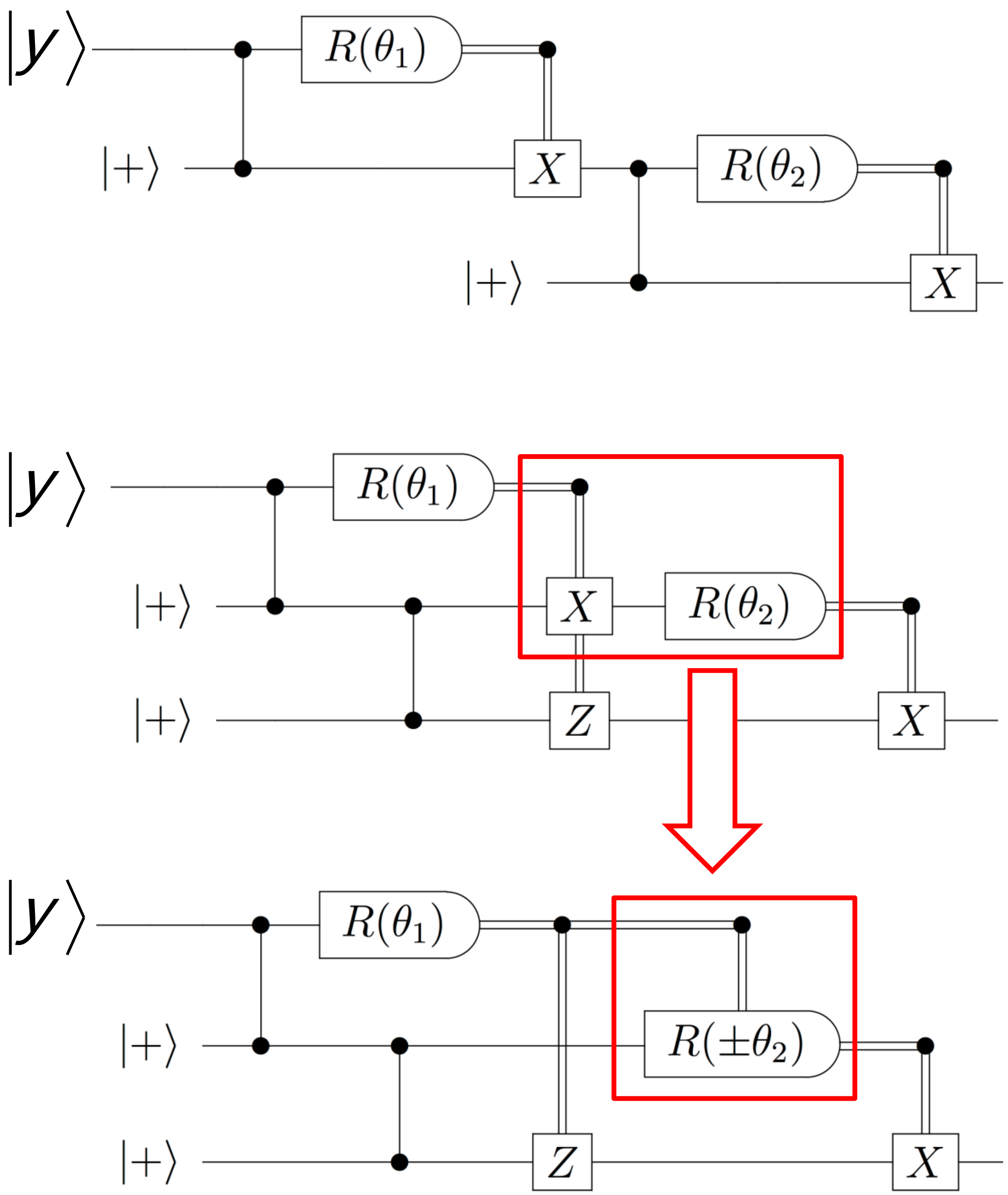

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- Incorporate X correction into measurement angle of 2. When X is applied because:

$$q_2 \rightarrow -q_2 \quad XR(q)X = R(-q)$$

MBQC: step-by-step

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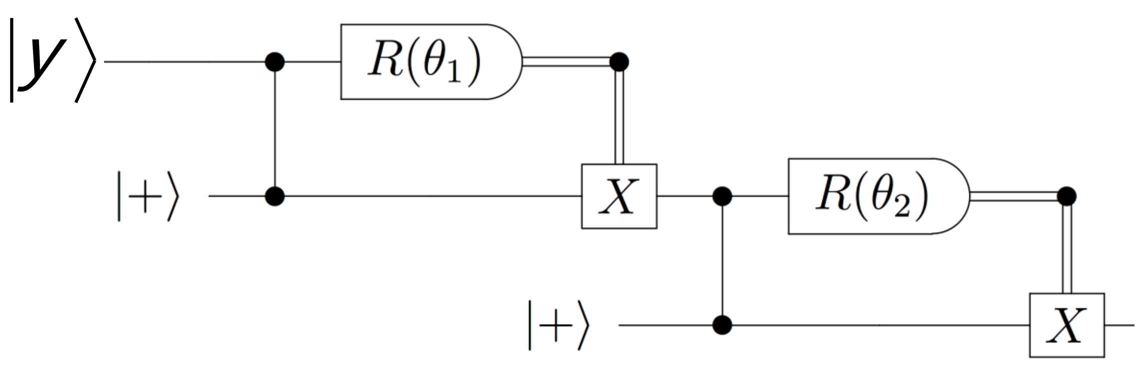

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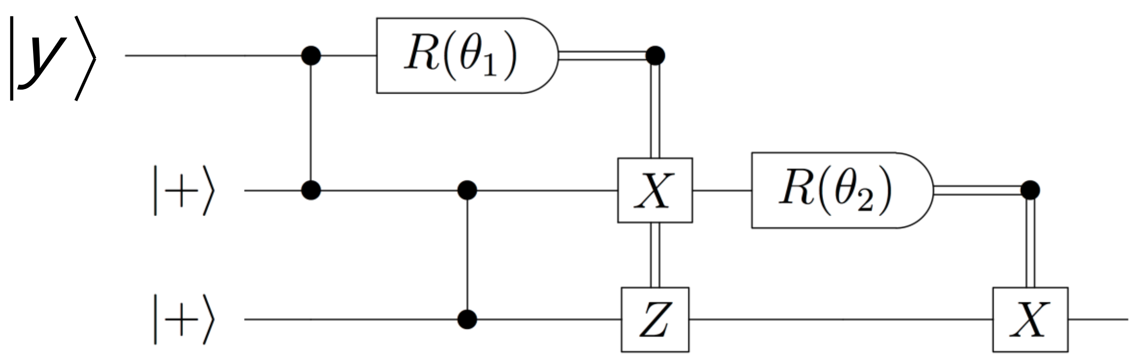

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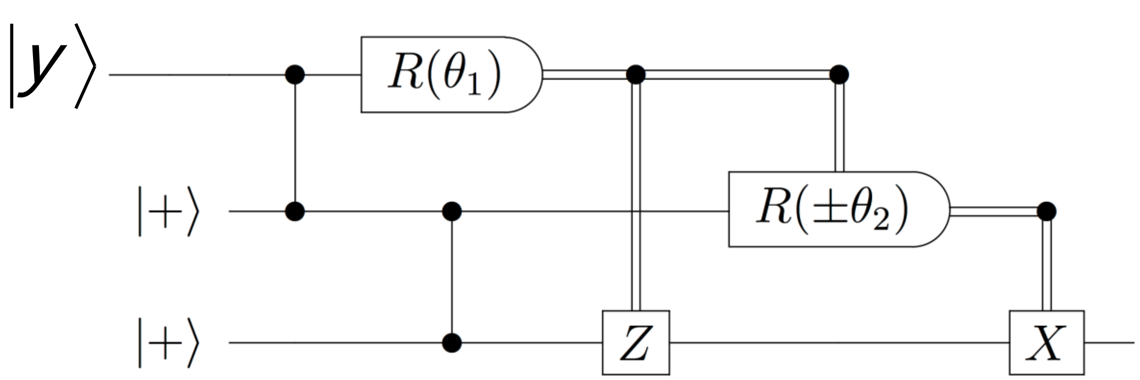


- By adapting measurement 2 according to outcome of 1, we can apply

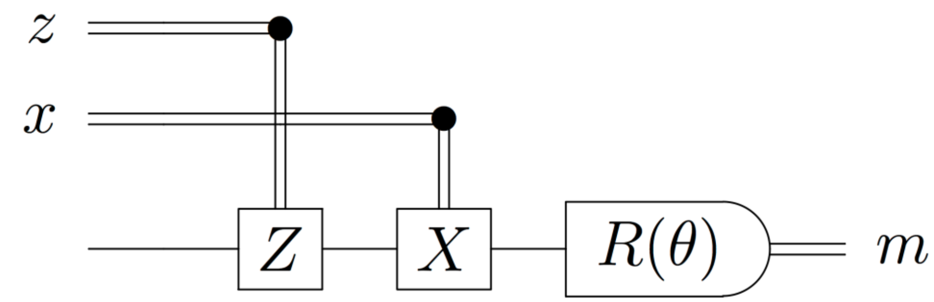
$$HU(\theta_2)HU(\theta_1)|\psi\rangle$$

- Easy to extend to multiple single-qubit unitaries, and $\{HU(\theta)\}$ is universal set for 1 qubit

Adaptativity allows for any single-qubit unitary to be implemented in the one-way model
CZ gates can be implemented similarly, propagation to beginning induces extra corrections

MBQC: step-by-step

- How do corrections affect future measurements? We can have both X and Z corrections:
  Outcomes of previous measurements:
  $$z, x \text{ Î } \{-1,1\}$$



- As $XR(q)X = R(-q)$, X corrections turn $q \to -q$

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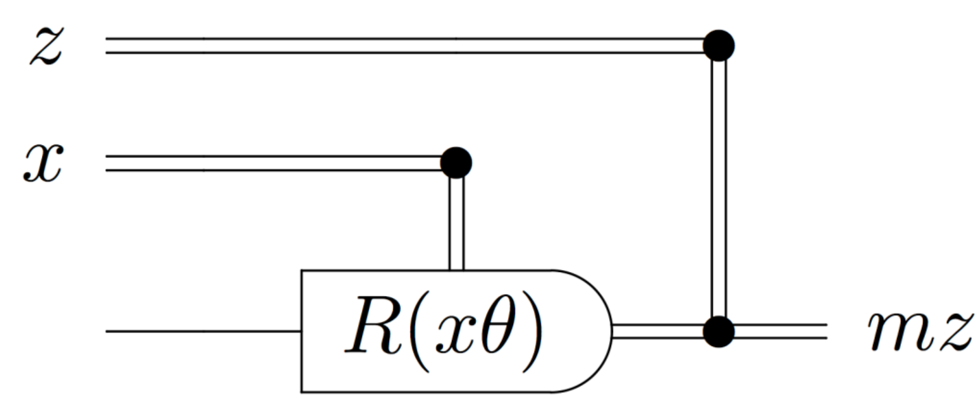

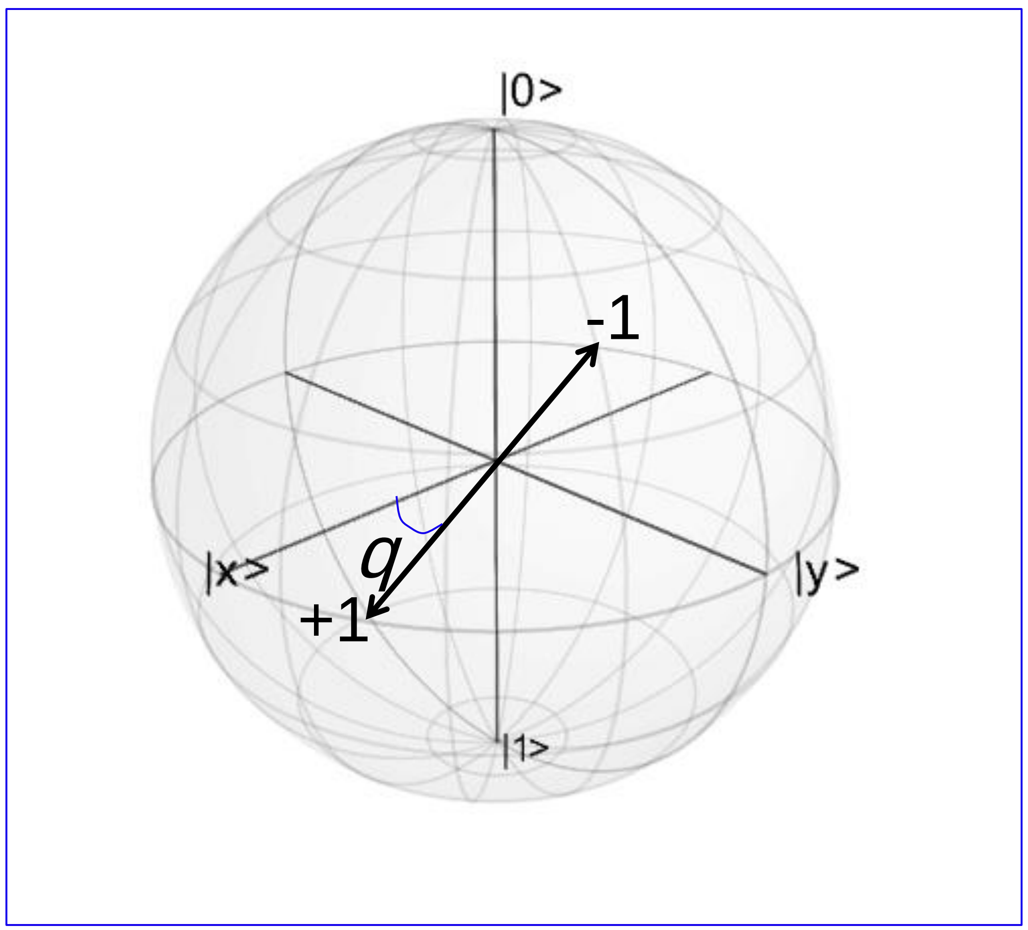

MBQC: step-by-step

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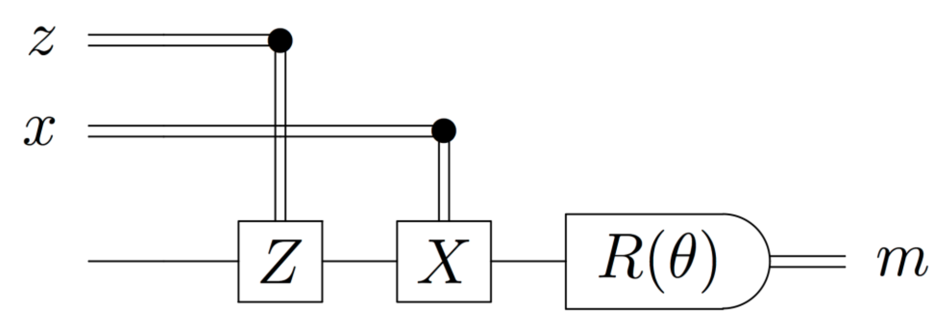

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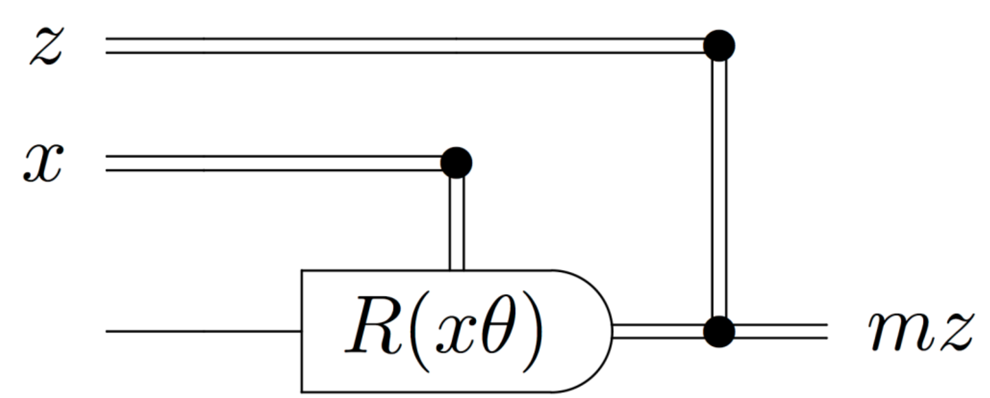
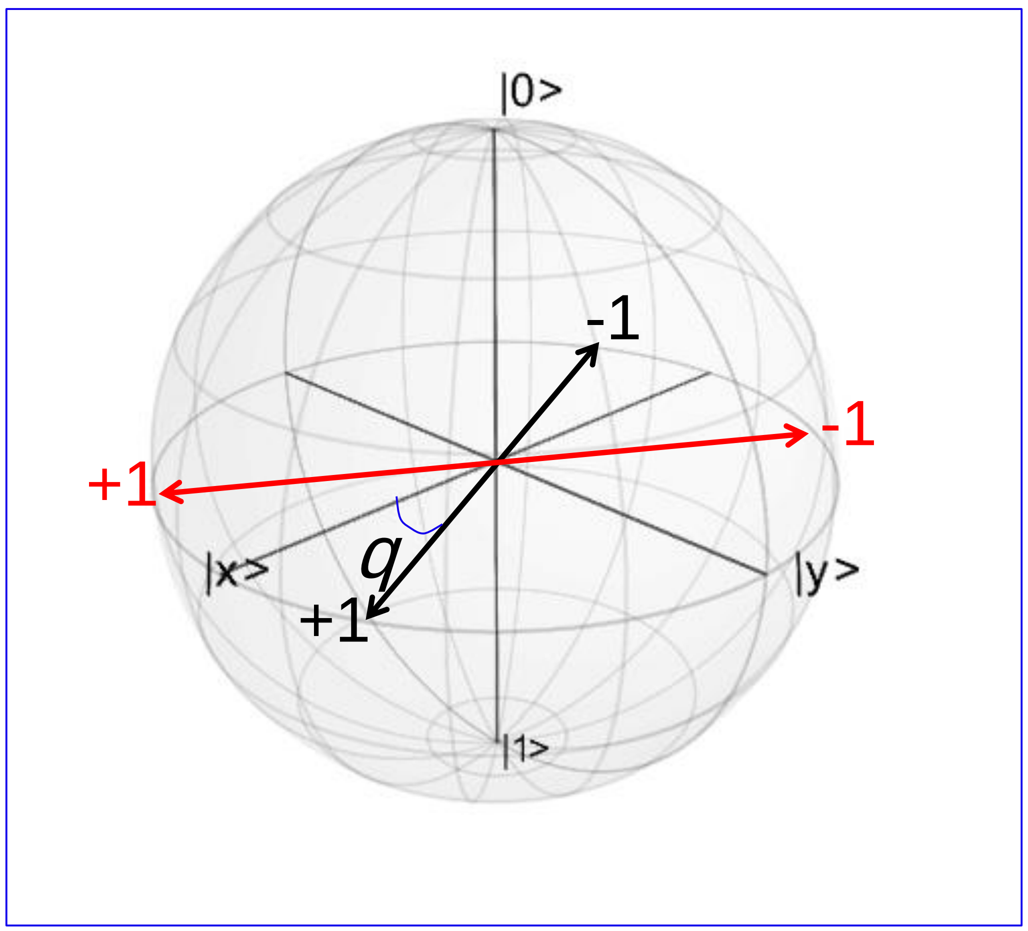


X correction



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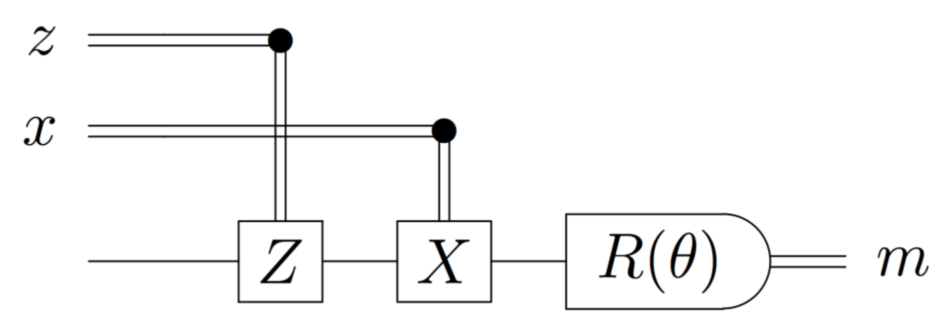

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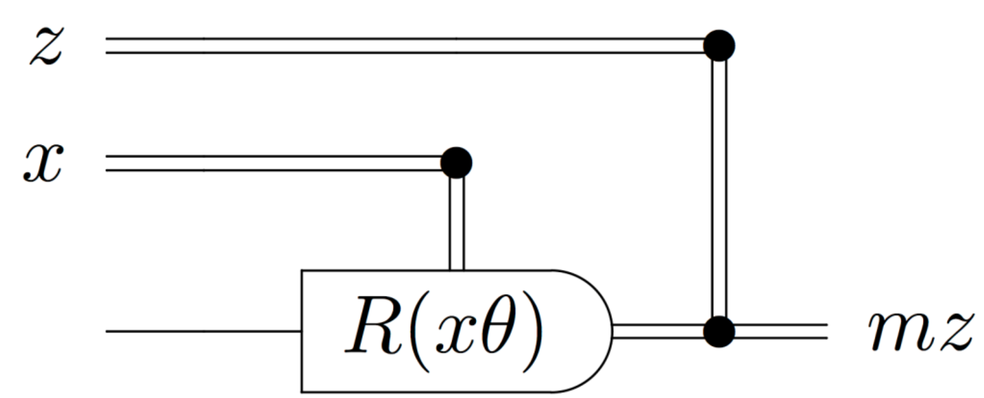
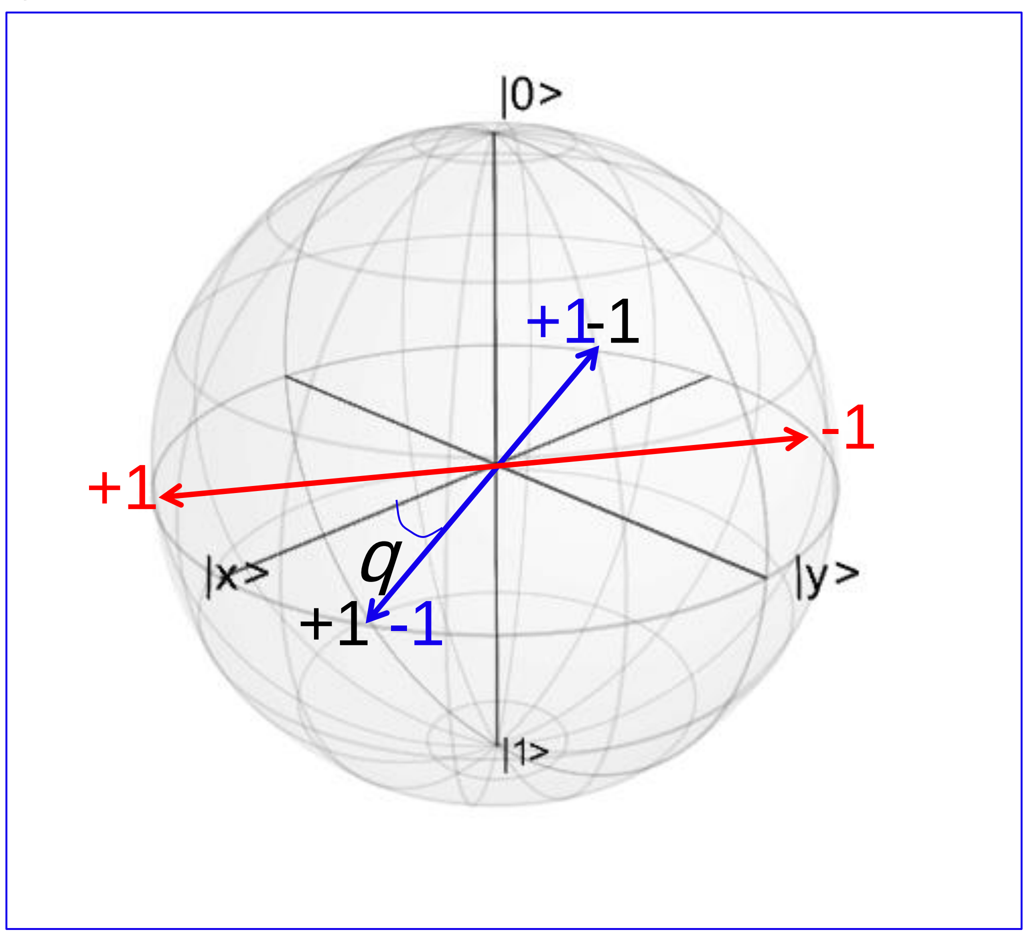


X correction
Z correction



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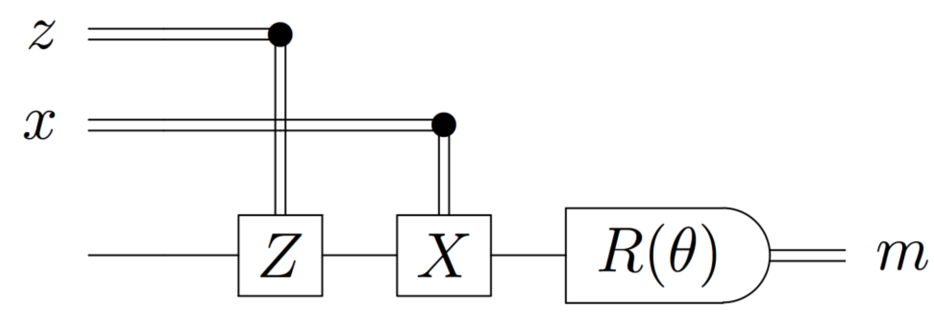

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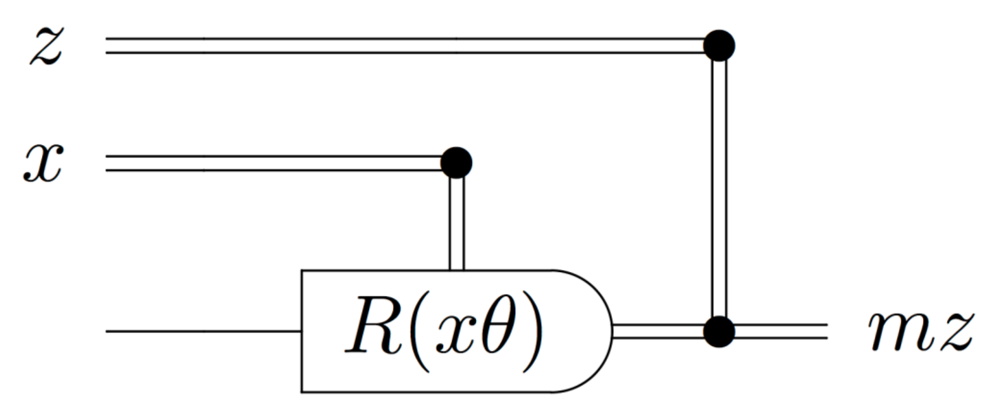

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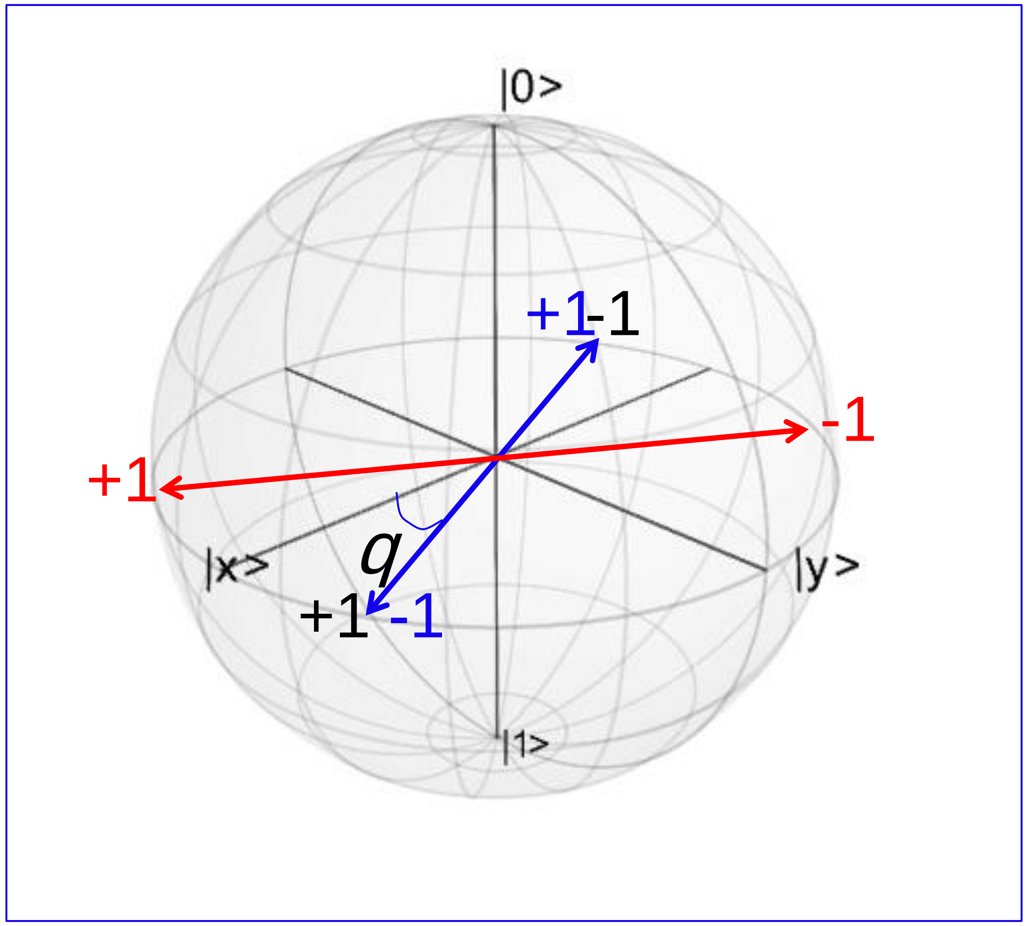


Classical control computer needs only store&update **sum modulo 2** of X and Z corrections of each qubit

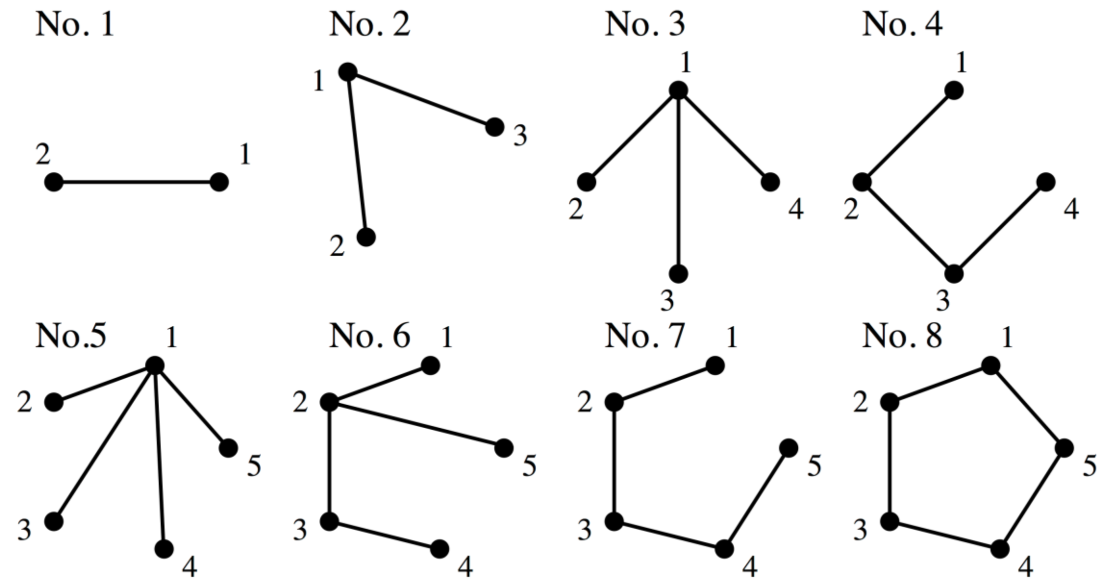
This **parity computer** is quite simple, but together with the quantum resource yields universal QC

Entanglement resources for MBQC

- **Graph states**: class of states obtainable by

  1. Initialization of a set of qubits in $|+\rangle$ states
  2. CZ gates between neighboring vertices in a graph

- Examples:



- No. 7 (5 qubits): sufficient for any single qubit unitary
- No. 3 (4 qubits): sufficient for CNOT

Entanglement resources for MBQC

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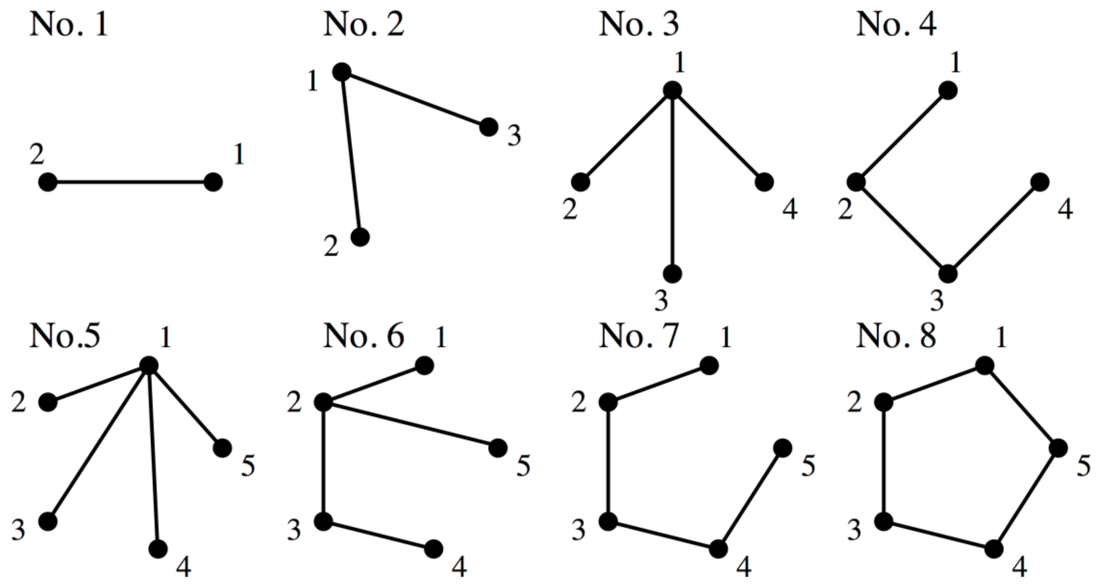
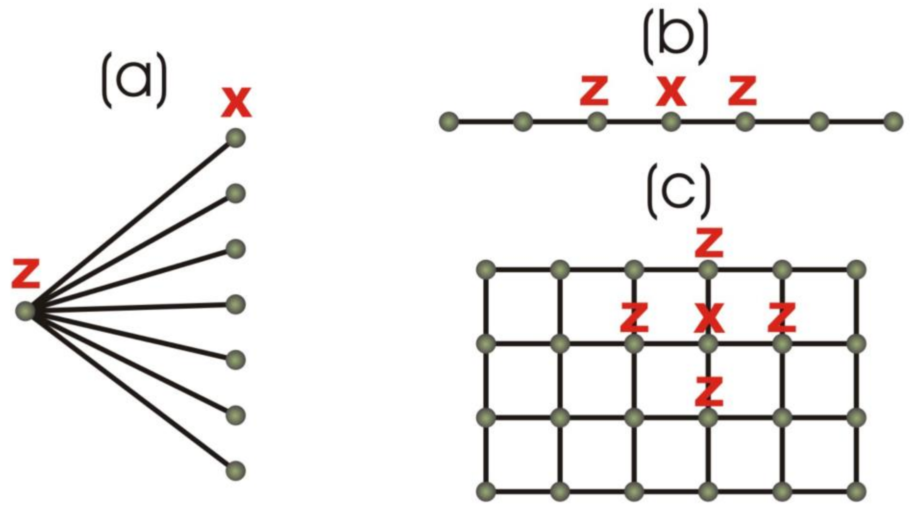


- Alternative characterization of graph states:

- Unique state which is simultaneous eigenstate (with eigenvalue 1) of set of operators

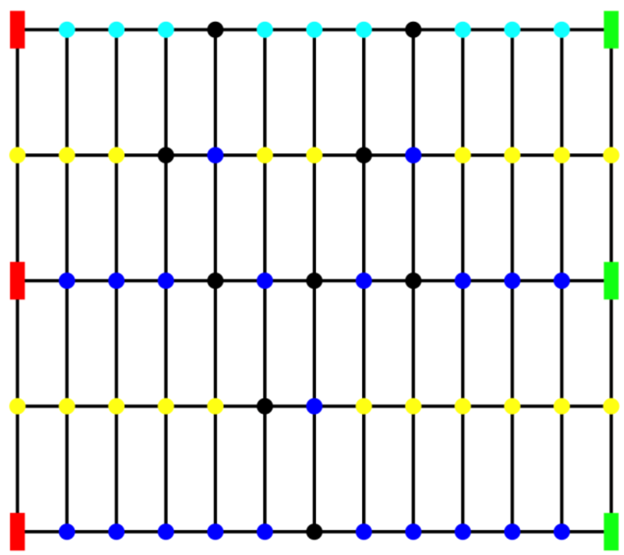
$$\left\{ K_i = X_i \bigotimes_{j \text{ neighbor of i}} Z_j \right\}$$

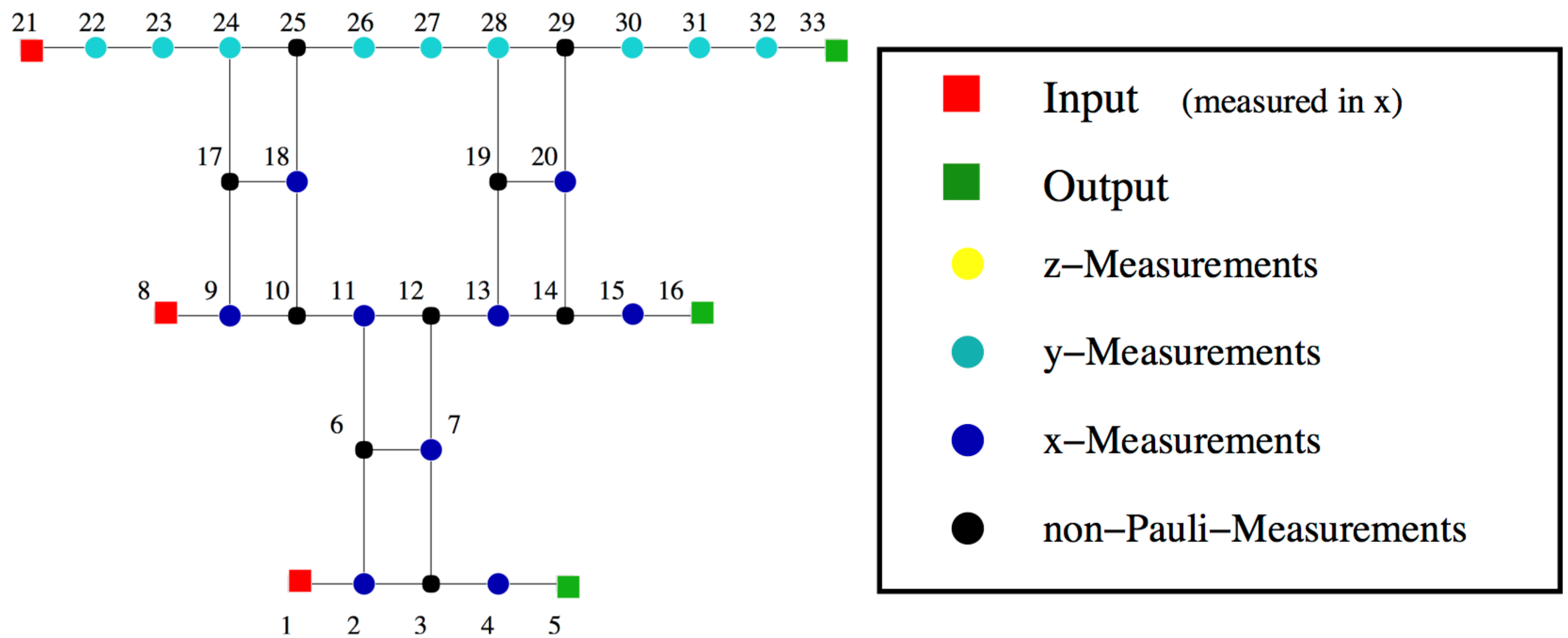


- Are there families of graph states which are universal for QC?

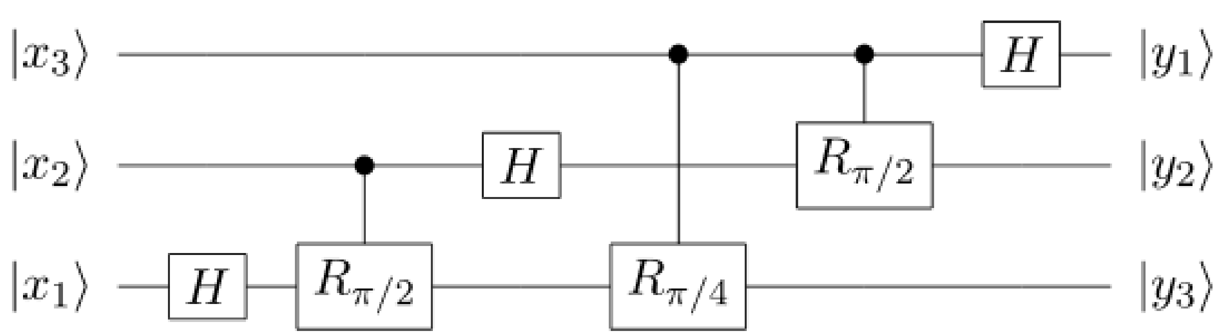
Entanglement resources for MBQC

M. HEIN, W. DÜR, J. EISERT, R. RAUSSENDORF, M. VAN DEN NEST and H.-J. BRIEGEL





from: Proc. Int. School of Physics "Enrico Fermi" on "Quantum Computers, Algorithms and Chaos", Varenna, Italy (2005)

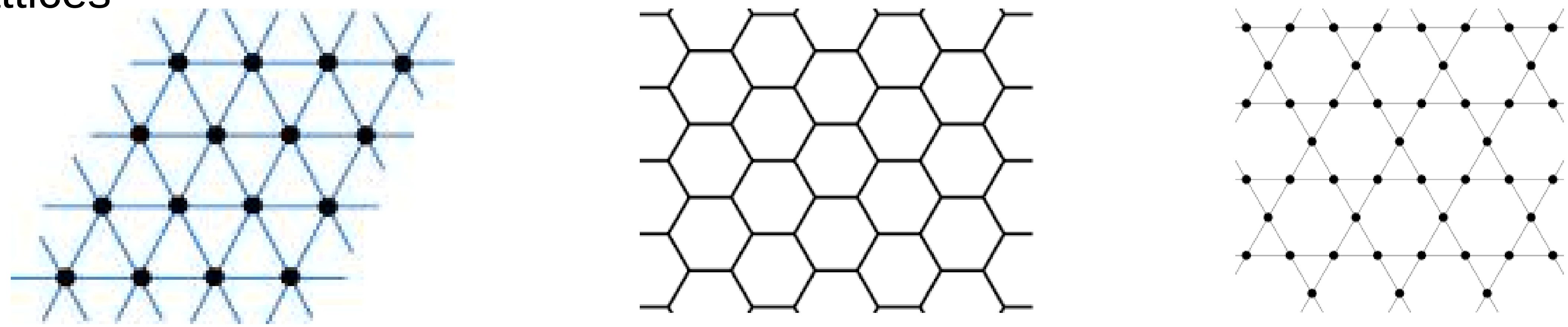


3-qubit QFT

- Example of universal graph: 2D square lattice (called **cluster state**)
  - Above: MBQC implementation of 3-qubit discrete Fourier Transform
  - “Unwanted” vertices deleted by Z-measurements; resulting corrections must be taken into account

Entanglement resources for MBQC

- Some known universal resources for MBQC: 2D triangular, hexagonal, Kagome lattices



  - These resources are "universal state preparators" = strong notion of universality

- Other resource states enable simulation of classical measurement statistics of any universal quantum computer = weaker notion of universality
  - Some of these require a universal classical computer (instead of a parity computer)

[Gross *et al.*, PRA 76, 052315 (2007)]

- Universality also for ground state of 2D Affleck-Kennedy-Lieb-Tasaki (AKLT) model

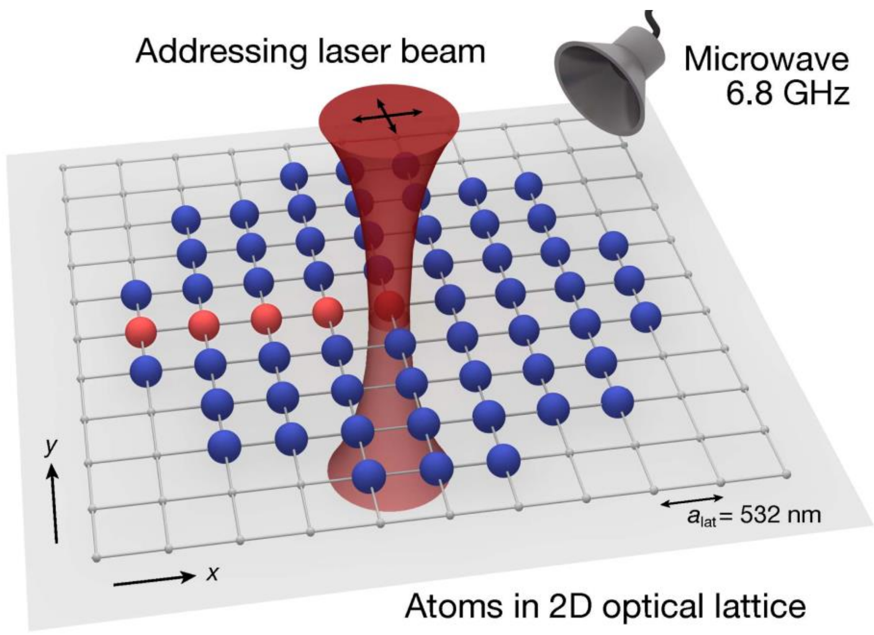
[Wei, Affleck, Raussendorf PRL 106, 070501 (2011)]

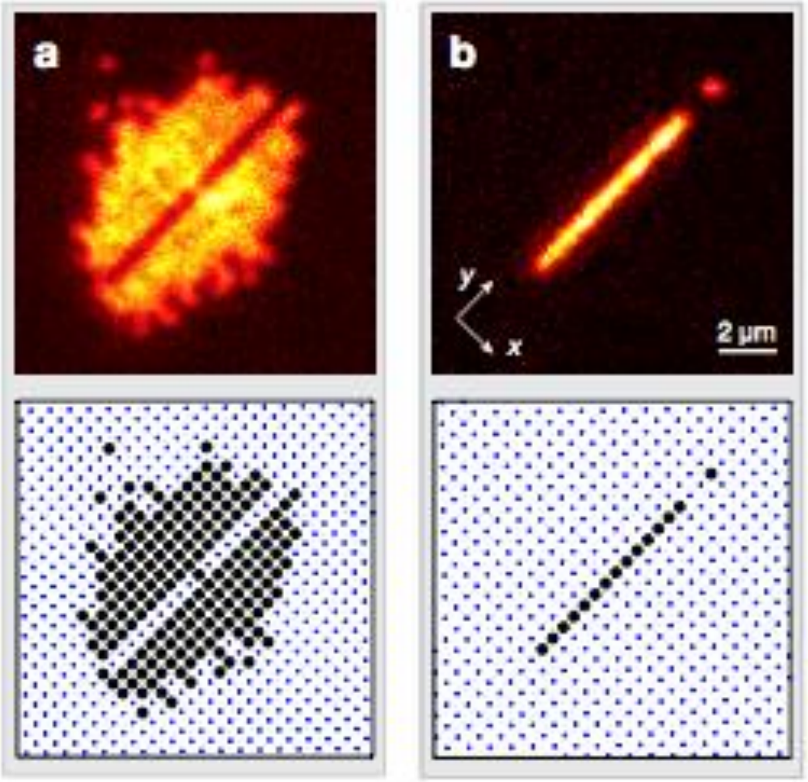
- MBQC on some resource states is known to be simulable, e.g. on 1D chain

[Markov, Shi, SIAM J. Comput. 38, 963 (2008)]

MBQC - implementations

- Optical lattices – counter-propagating laser beams trap cold neutral atoms
  - Challenge: single-site addressing

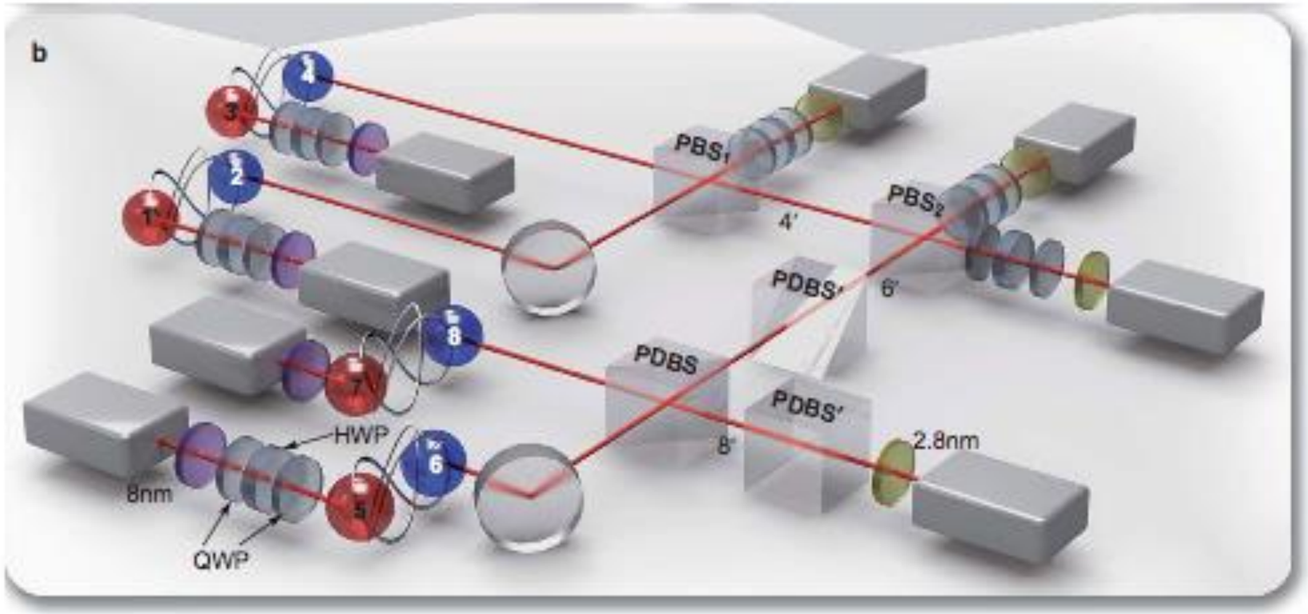




from: Weintenberg et al., *Nature* 471, 319 (2011)

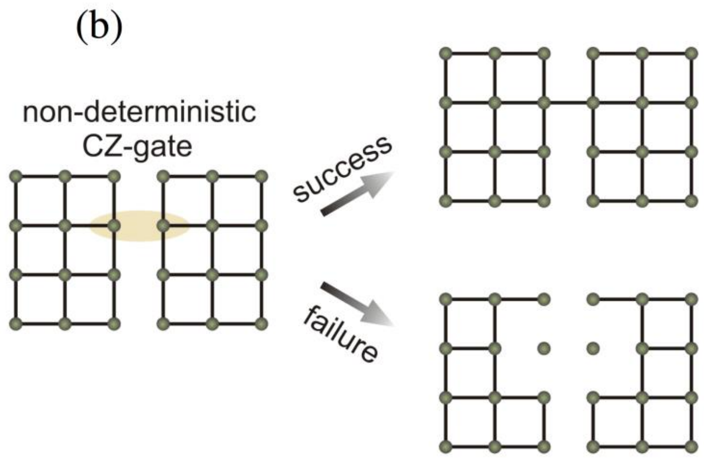
- Proof-of-principle implementations using photons
  - Topological error-correction using eight-photon cluster states

from: Yao et al., *Nature* 482, 489 (2012)

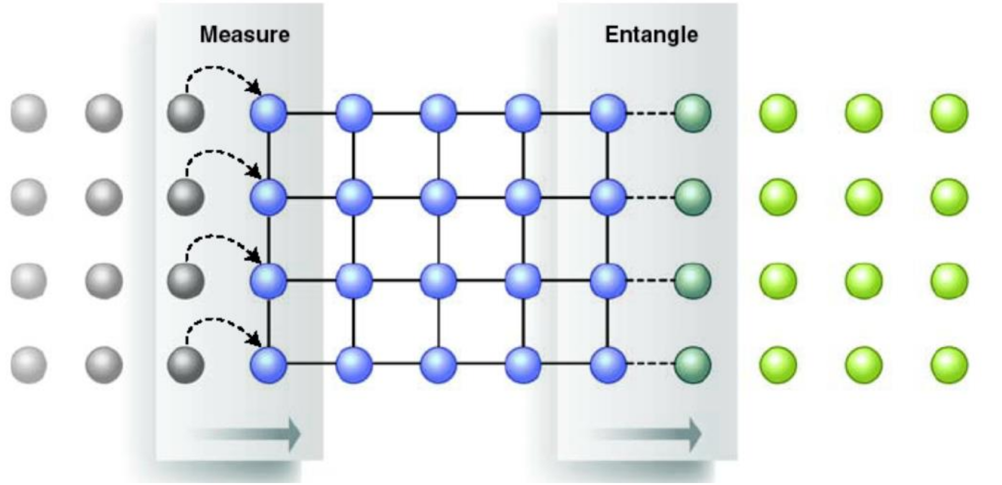


MBQC - implementations

- Using one-way model to advantage: building large resource states from probabilistic operations; at once or on the go



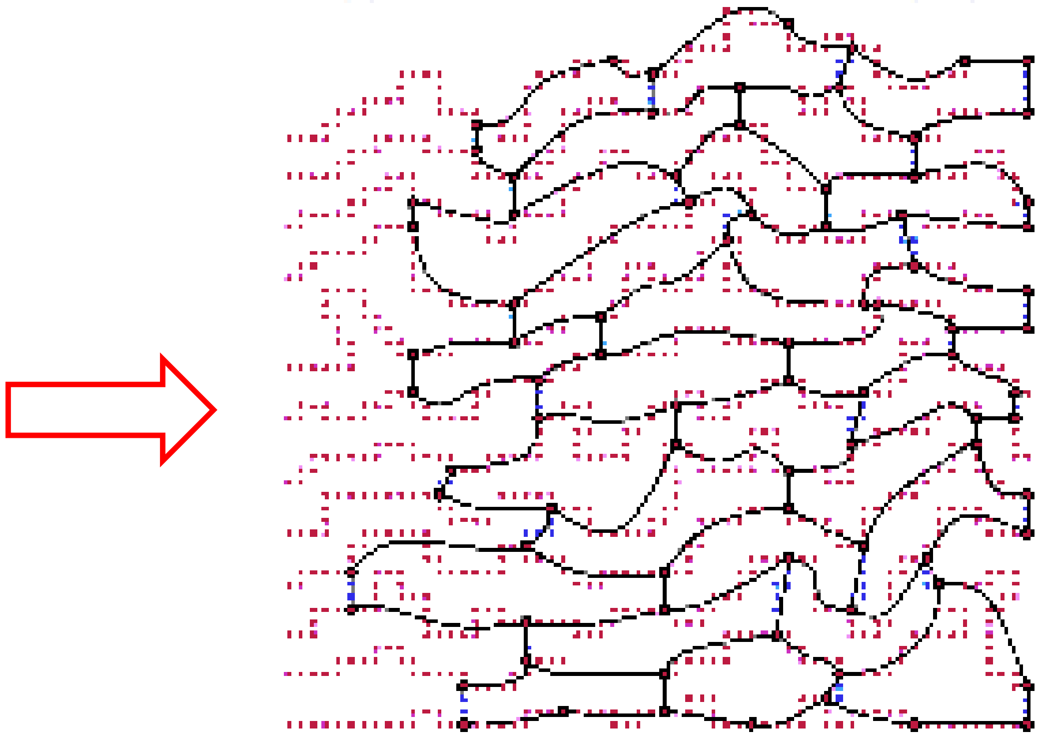
from: Briegel *et al., Nat. Phys.* 5 (1), 19 (2009)



from: O'Brien, Science 318, 1467 (2007)

- Schemes for adapting imperfect clusters for MBQC

(a) initial faulty square lattice

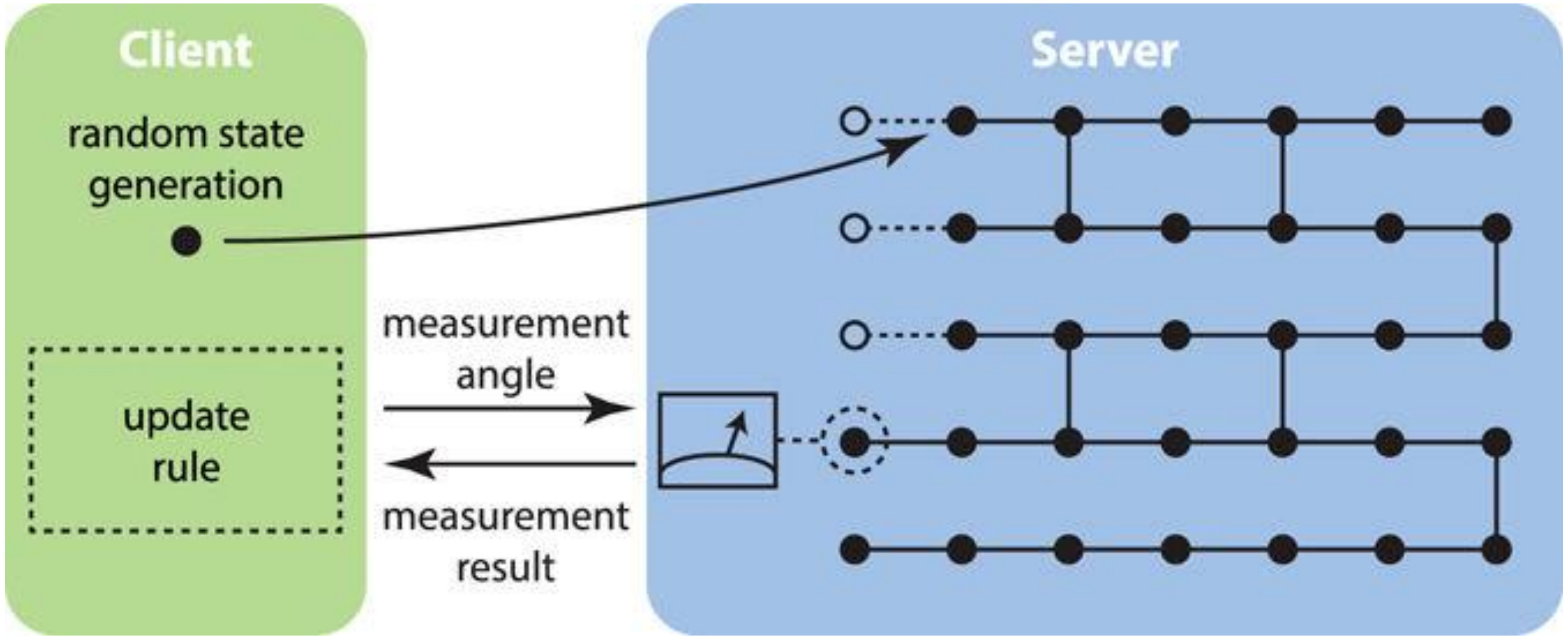


(f) deletion and contraction (Q.1 & Q.2)

from: Browne *et al., New J. Phys.* 10, 023010 (2008)

Application: blind quantum computation

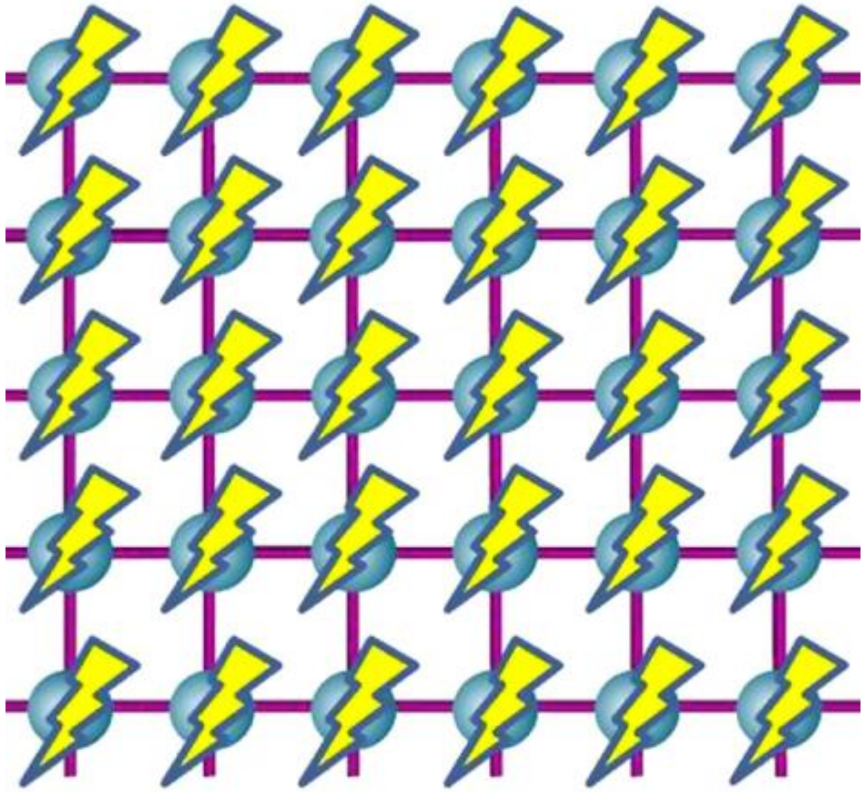
- Classical/quantum separation in MBQC allow for implementation of novel protocols – such as blind quantum computation
- Here, client has limited quantum capabilities, and uses a server to do computation for her.
- Blind = server doesn't know what's being computed.



Broadbent, Fitzsimons, Kashefi, axiv:0807.4154 [quant-ph]

Which resource gives MBQC its power?

- Clearly, the correlations in the resource state.



- Analysis of MBQC protocols in terms of Bell inequalities:
  - Anders/Browne PRL 102, 050502 (2009)
  - Hoban et al., New J. Phys. 13, 023014 (2011)

- …but measurements are usually not space-like separated:
  ⇒ quantum contextuality

  - Raussendorf, PRA 88, 022322 (2013)

Quantum contextuality

- Context of an observable A = set of commuting observables measured together with A
- Non-contextuality hypothesis: outcomes of observables are context-independent
- Violated by quantum mechanics!

- Famously proved by Kochen and Specker (1967). Let's see a proof by Mermin (1990).

$\mathbb{1} \otimes \sigma_z$	$\sigma_z \otimes \mathbb{1}$	$\sigma_z \otimes \sigma_z$
$\sigma_x \otimes \mathbb{1}$	$\mathbb{1} \otimes \sigma_x$	$\sigma_x \otimes \sigma_x$
$\sigma_x \otimes \sigma_z$	$\sigma_z \otimes \sigma_x$	$\sigma_y \otimes \sigma_y$

- Operators in each row and column commute;

Moreover, they are the product of the other two in same row/column

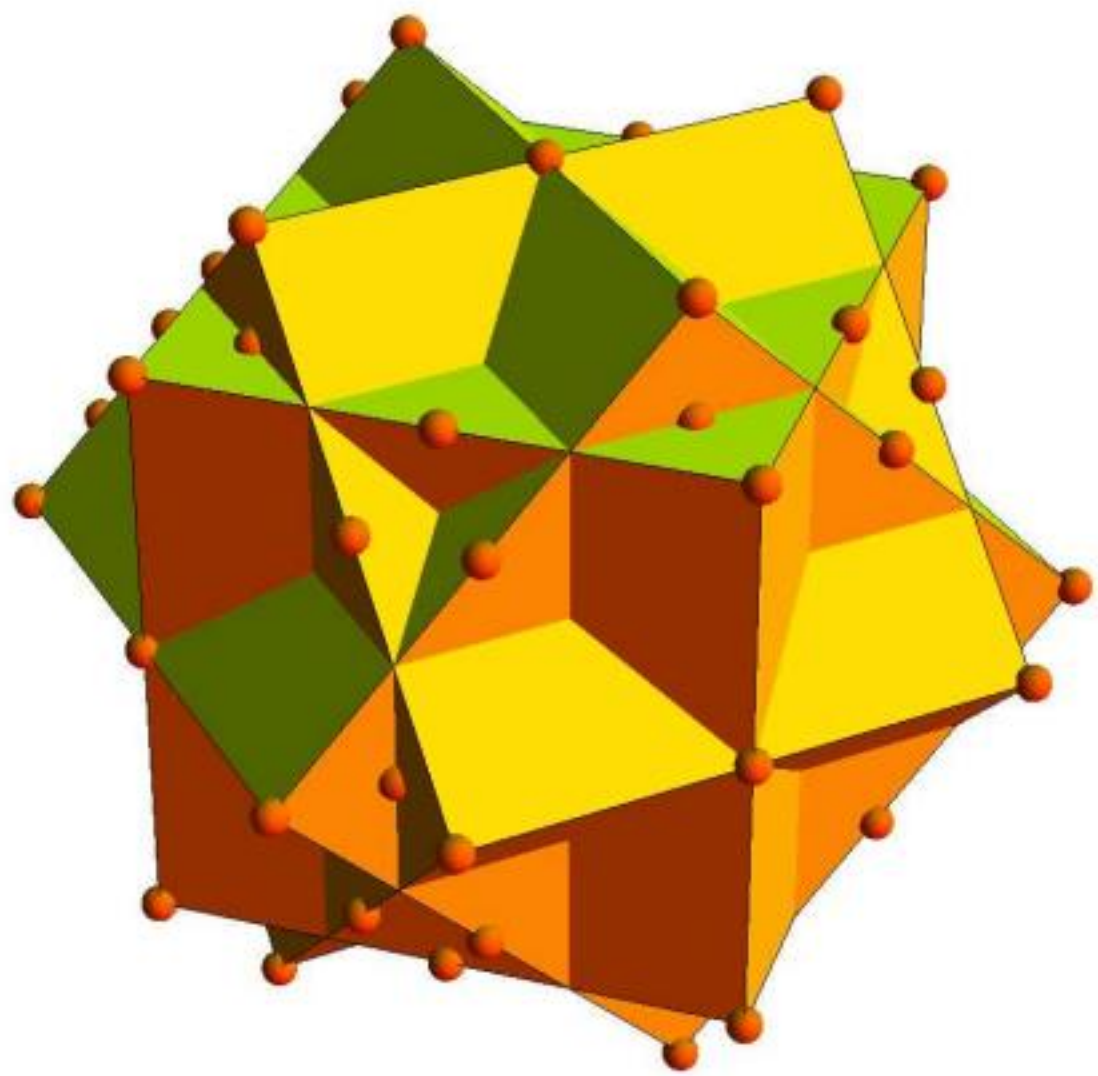
- EXCEPTION: third column:

$$\sigma_y \otimes \sigma_y = -\sigma_z \otimes \sigma_z \cdot \sigma_x \otimes \sigma_x$$

- So it's impossible to assign +1 or -1 values to each observable in a context-independent way. ⇒ QM is contextual.

Proof by Peres (1991) – Kochen and Specker flavour

- Consider 57 states in 3-dimensional Hilbert space, real amplitudes.
  - Orthogonal triads must be colored black, white, white.
  - Some of the triads above have vectors in common.
  - One can show that there's no possible coloring satisfying the orthogonality relations.



Contextuality is necessary for magic state distillation

Howard et al., Nature 310, 351 (2014)

- The Mermin square proof of quantum contextuality is state-independent – any state violates the non-contextuality hypothesis.
- For Hilbert space dimension d>2, all contextuality proofs are *state-dependent*.
- So what’s special about states revealing contextuality?

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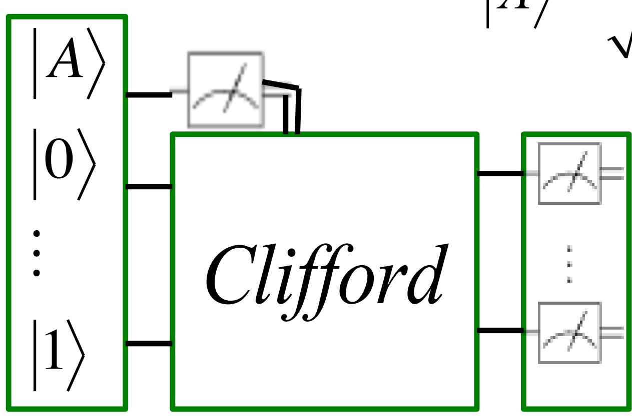
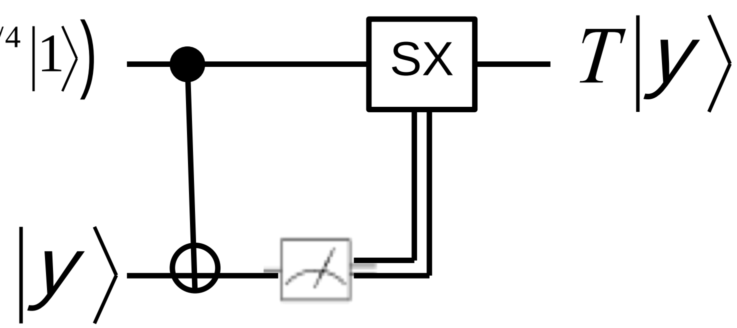
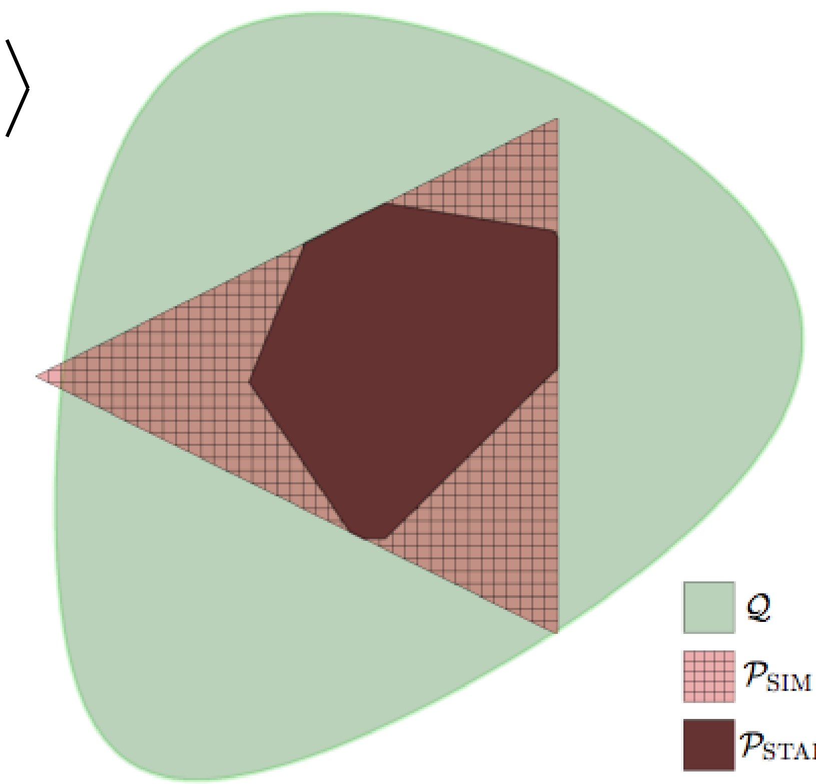
$$|A\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + e^{i\pi/4}|1\rangle\right)$$







from Howard et al., Nature 310, 351 (2014)

PSIM = simulable under stabilizer measurements
PSTAB = stabilizer states
Q = general quantum states

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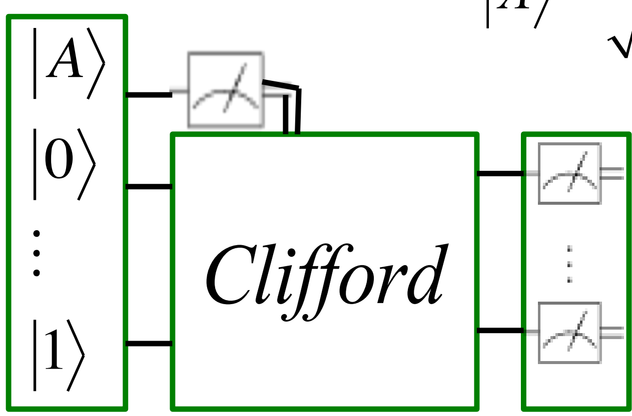

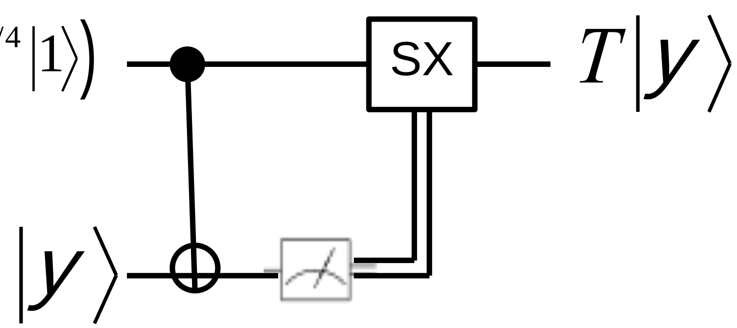

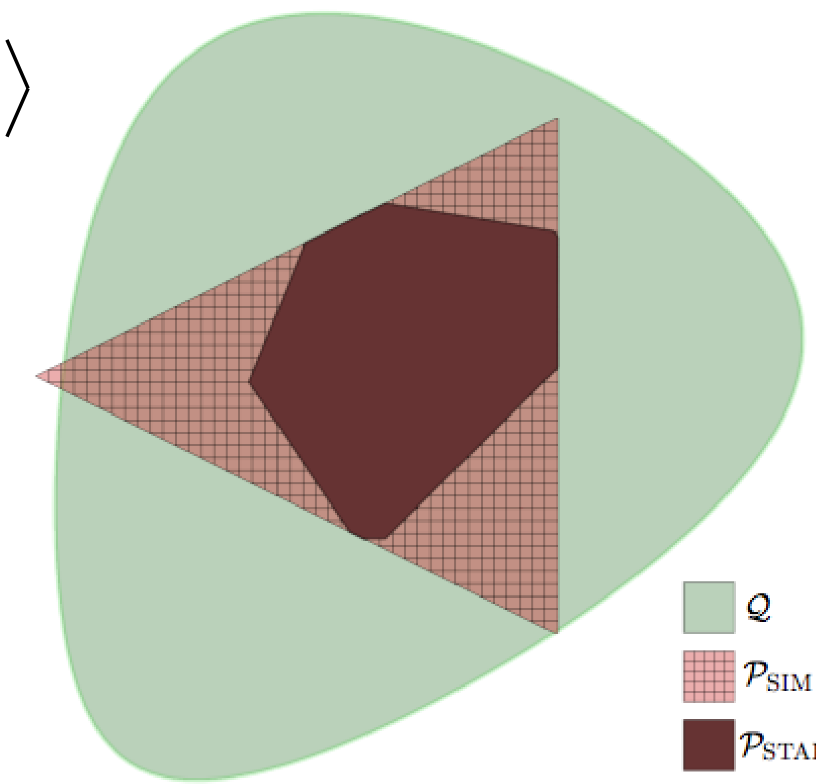

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- Result: any state out of PSIM violates a state-dependent non-contextuality inequality, using stabilizer measurements. States in PSIM are non-contextual.

→ contextuality is necessary for magic-state computation

Application: model for quantum spacetime

- MBQC can serve as a discrete toy model for quantum spacetime:

quantum space-time	**MBQC**
quantum substrate	graph states
events	measurements
principle establishing global space-time structure	determinism requirement for computations

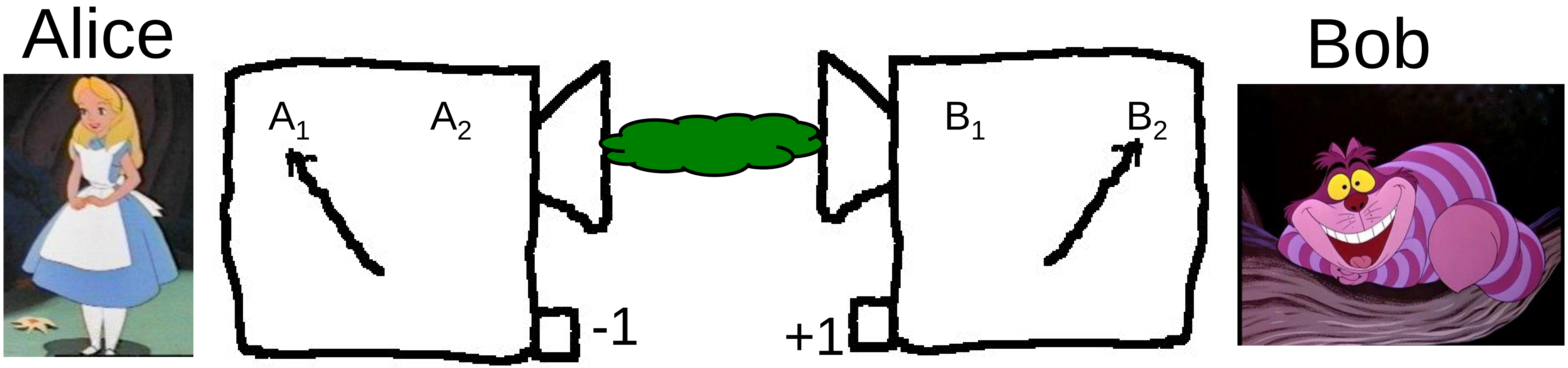
[Raussendorf *et al.*, arxiv:1108.5774]

- Even closed timelike curves (= time travel) have analogues in MBQC!

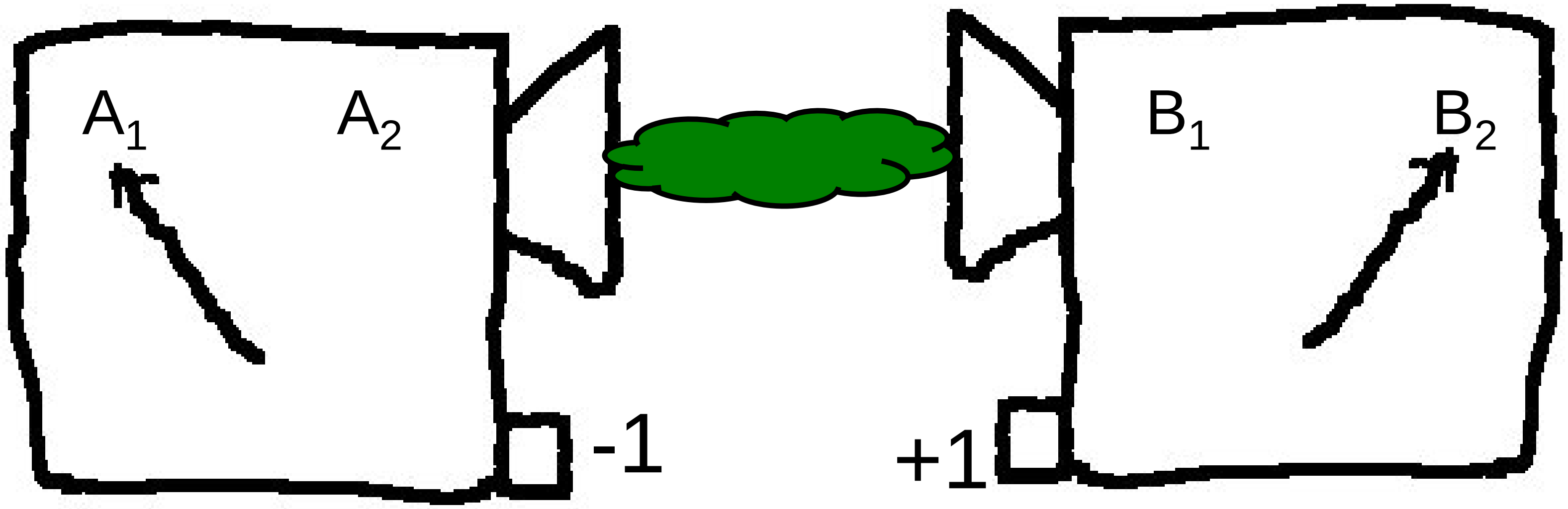
[Dias da Silva, Kashefi, Galvão PRA 83, 012316 (2011)]

Bell non-locality

- Bell inequalities (Bell 1964) are limits on the correlation of distant systems
- Example: **Clauser-Horn-Shimony-Holt (CHSH) inequality** (1969):
  - Alice e Bob measure dychotomic properties (results +1 or -1)
  - Each chooses randomly which property to measure:
    - Alice measures A_1 or A_2; result a_1 or a_2
    - Bob measures B_1 or B_2; result b_1 or b_2.



CHSH inequality



- Hypotheses:
  - Pre-determined value for experimental outcomes (realism)
  - Result of A doesn't depend on what B does (and vice-versa) (locality)

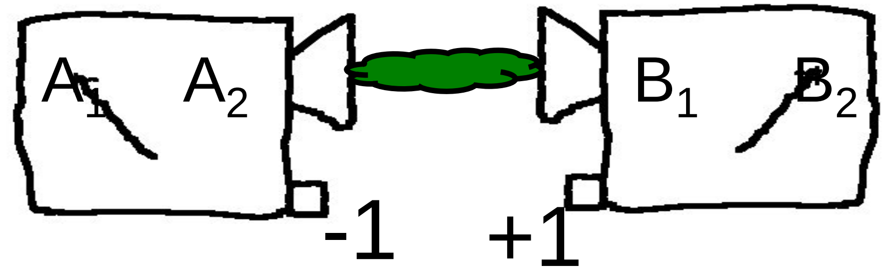
local realism

- CHSH inequality:

$$\left|\langle a_1 b_1\rangle + \langle a_2 b_1\rangle + \langle a_2 b_2\rangle - \langle a_1 b_2\rangle\right| \pounds 2$$

CHSH inequality

- Alice and Bob compare notes and jointly prepare spreadsheet:



a_1	a_2	b_1	b_2	a_1b_1	a_1b_2	a_2b_1	a_2b_2
+1		-1		-1			
	-1		+1				-1
	+1	+1				+1	
-1			+1		-1		

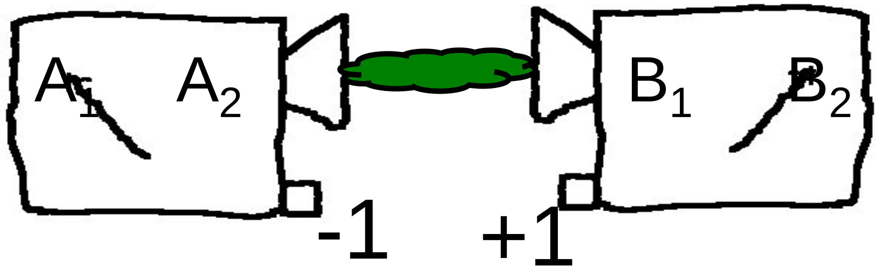
$$\langle a_1 b_1 \rangle \quad \langle a_1 b_2 \rangle \quad \langle a_2 b_1 \rangle \quad \langle a_2 b_2 \rangle$$

- If local realism holds, then:

$$\left| \langle a_1 b_1 \rangle + \langle a_2 b_1 \rangle + \langle a_2 b_2 \rangle - \langle a_1 b_2 \rangle \right| \leq 2$$

CHSH inequality

- Alice and Bob compare notes and jointly prepare spreadsheet:



a_1	a_2	b_1	b_2	a_1b_1	a_1b_2	a_2b_1	a_2b_2
+1		-1		-1			
	-1		+1				-1
	+1	+1				+1	
-1			+1		-1		

$\langle a_1 b_1 \rangle \quad \langle a_1 b_2 \rangle \quad \langle a_2 b_1 \rangle \quad \langle a_2 b_2 \rangle$

- If local realism holds, then:

$$\left| \langle a_1 b_1 \rangle + \langle a_2 b_1 \rangle + \langle a_2 b_2 \rangle - \langle a_1 b_2 \rangle \right| \leq 2$$

- But local measurements on particles in entangled state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A |\downarrow\rangle_B - |\downarrow\rangle_A |\uparrow\rangle_B \right)$

give $$\left| \langle a_1 b_1 \rangle + \langle a_2 b_1 \rangle + \langle a_2 b_2 \rangle - \langle a_1 b_2 \rangle \right| = 2\sqrt{2} > 2$$

QM violates local realism!