Computação Quântica
Problem 2 - 1 May 2020 - 15 May 2020

This exercise aims at improving your understanding of the quantum Fourier transform, a most relevant component in several quantum algorithms.

Recall the definition of QFT on K basis states $|0\rangle, |1\rangle \cdots |k-1\rangle$:

$$\mathsf{QFT}_K(|x\rangle) \;=\; \frac{1}{\sqrt{K}} \sum_{y=0}^{K-1} e^{2\pi i (\frac{x}{K}) y} |y\rangle$$

- Compute $\mathsf{QFT}_K(|00 \cdots 0\rangle)$.
- The following equality

  $$\mathsf{QFT}_K(|x_1 \cdots x_n\rangle) \;=\; \left(\frac{|0\rangle + e^{2\pi i (0.x_n)}|1\rangle}{\sqrt{2}}\right) \otimes \left(\frac{|0\rangle + e^{2\pi i (0.x_n x_{n-1})}|1\rangle}{\sqrt{2}}\right) \cdots \otimes \cdots \left(\frac{|0\rangle + e^{2\pi i (0.x_1 x_2 \cdots x_n)}|1\rangle}{\sqrt{2}}\right)$$

  was used in the lecture slides without proof. Verify it holds indeed.
- One can show, as we did in the lectures, that QFT is a unitary gate by building a unitary quantum circuit for its computation. Give an alternative, direct proof that the linear transformation defined above is unitary.
- Reproduce the circuit for QFT_2 and QFT_3, and compute the corresponding matrices. Give your calculation in detail.