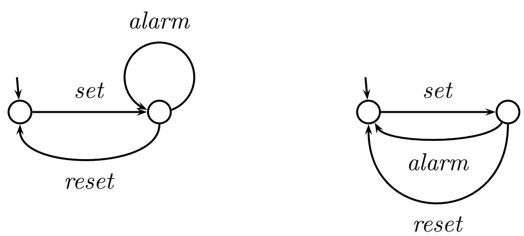
Exercises 1 Interaction and Concurrency

Luís Soares Barbosa

Exercise I.1 Consider two labelled transition systems representing two alternative behaviours of an alarm clock, as depicted below:



1. Describe each behaviour and distinguish between the two alarm clocks.
2. Describe one of these graphical specifications in the form of a labelled transition system conforming to the formal definition.
3. Add to alarm clock a new feature for specifying the number of alarms to go off.
4. Draw the behaviour of an alarm clock where it is always possible to do a set or a reset action.
5. Draw the behaviour of an alarm clock with unreliable buttons. When pressing the set button the alarm clock can be set, but this does not need to be the case. Similarly for the reset button. Pressing it can reset the alarm clock, but the clock can also stay in a state where an alarm is still possible.
6. Draw the behaviour of an alarm clock where the alarm sounds at most three times when no other action interferes.

Exercise I.2

1. Define a binary operator $|||$ which models the parallel execution of its two arguments. Its transitions come from merging the transitions of its arguments. It is assumed that there is no interference between them.
2. Discuss, starting with an example, whether this operator is associative.
3. Discuss, starting with an example, whether this operator is commutative.

Exercise I.3 Consider the following two programs:

$$P_1 \triangleq \texttt{x := 2*x} \qquad \text{and} \qquad P_2 \triangleq \texttt{x := 1+x}$$

1. Draw, for each program, a transition system whose states represent the values variable `x` may take. Assume `x = 4` as the initial state in both cases.
2. Compute and draw $P_1 \; ||| \; P_2$, where $|||$ was defined in the previous exercise.
3. Explain why, in the presence of interference (in this example through a shared variable), the interleaving operator may produce invalid states.

Exercise I.4 Suppose a labelled transition system is given by the following transition relation:

$$\{\langle 1,a,2\rangle,\langle 1,a,3\rangle,\langle 2,a,3\rangle,\langle 2,b,1\rangle,\langle 3,a,3\rangle,\langle 3,b,1\rangle,\langle 4,a,5\rangle,\langle 5,a,5\rangle,\langle 5,b,6\rangle,\langle 6,a,5\rangle,\langle 7,a,8\rangle,\langle 8,a,8\rangle,\langle 8,b,7\rangle\}$$

Prove or refute $1 \sim 4 \sim 6 \sim 7$.

Exercise I.5 Given two labelled transition systems $\langle S_A, \mathcal{N}, \longrightarrow_{\mathcal{A}}\rangle$ and $\langle S_B, \mathcal{N}, \longrightarrow_{\mathcal{B}}\rangle$, two states p and q are *equisimilar* iff

$$p \doteqdot q \;\equiv\; p \lesssim q \,\wedge\, q \lesssim p$$

1. Show that $\doteqdot$is an equivalence relation.
2. Compare this equivalence with bisimilarity $\sim$.

Exercise I.6 Suppose that the existential quantifiers in the definition of bisimulation were replaced by universal quantifiers. Which relation corresponds to bisimilarity in this new setting?

Exercise I.7 Show that bisimilarity is strictly included in equisimilarity, and that the latter is also strictly included on trace equivalence.

Exercise I.8 Discuss whether bisimilarity $\sim$

- is closed for union
- is closed for intersection

Exercise I.9 A relation R over the state space of a labelled transition system is a *word bisimulation* if, whenever $\langle p, q\rangle \in R$ and $s \in \mathcal{N}^*$, we have

$$p \xrightarrow{s} p' \;\Rightarrow\; \langle \exists\, q' \;:\; q' \in S_2 :\; q \xrightarrow{s} q' \,\wedge\, \langle p', q'\rangle \in R\rangle$$
$$q \xrightarrow{s} q' \;\Rightarrow\; \langle \exists\, p' \;:\; p' \in S_1 :\; p \xrightarrow{s} p' \,\wedge\, \langle p', q'\rangle \in R\rangle$$

1. Define formally relation $\xrightarrow{s}$, for $s \in \mathcal{N}^*$
2. Two states are *word bisimilar* iff they belong to a word bisimulation. Show that two states p and q are word bisimilar iff $p \sim q$.
