



Exercises 4 : Interaction and Concurrency

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Exercise 1

Formalise each of the following properties in Process Logic \mathcal{M} . Note they are formulated in a somehow ambiguous way, and can therefore be formalised in different ways.

1. The occurrence of a and b is impossible.
 2. The occurrence of a followed by b is impossible.
 3. Only the occurrence of a is possible.
 4. Once a occurred, b or c may occur.
 5. After a occurred followed by b , c may occur.
 6. Once a occurred, b or c may occur but not both.
 7. a cannot occur before b .
 8. There is only an initial transition labelled by a .
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Exercise 2

Consider the following processes and enumerate for each of them the properties they verify:

1. $E_1 \triangleq a.b.\mathbf{0}$
 2. $E_2 \triangleq a.c.\mathbf{0}$
 3. $E \triangleq E_1 + E_2$
 4. $F \triangleq a.(b.\mathbf{0} + c.\mathbf{0})$
 5. $G \triangleq E + F$
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Exercise 3

Consider the following specification of a CNC program:

$$\begin{aligned} Start &\triangleq fw.Go + stop.\mathbf{0} \\ Go &\triangleq fw.bk.bk.Start + right.left.bk.Start \end{aligned}$$

Formalise in \mathcal{M} the following properties:

1. After fw another fw is immediately possible
2. After fw followed by $right$, $left$ is possible but bk is not.
3. Action fw is the only one initially possible
4. The third action of process $Start$ is not fw .

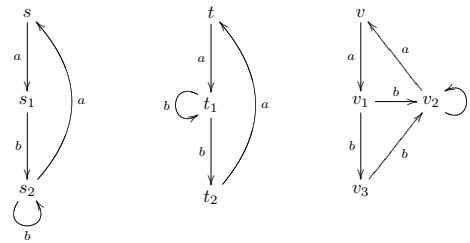
Exercise 4

Specify a LTS such that the following modal properties hold simultaneously in its initial state:

- $\langle a \rangle \langle b \rangle \langle c \rangle \text{true} \wedge \langle c \rangle \text{true}$
 - $\langle a \rangle \langle b \rangle ([a] \text{false} \wedge [c] \text{false} \wedge [b] \text{false})$
 - $\langle a \rangle \langle b \rangle (\langle a \rangle \text{true} \wedge [c] \text{false})$
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Exercise 5

Consider the following *Act*-labelled transition systems.



Show that states s, t and v are not bisimilar and determine the modal properties which distinguish between them.

Exercise 6

Let E be a process. A formula ϕ is said to be *characteristic* of E iff

$$\forall F \in \mathcal{P} . F \models \phi \text{ sse } F \sim E$$

Note that a process verifies the characteristic formula of E iff it is strongly bisimilar to E .

Determine the *characteristic* formula of process $x.0$.

Exercise 7

Consider processes $E \triangleq a.(b.0 + c.0)$ e $F \triangleq a.b.0 + a.c.0$. Propose a formula ϕ in \mathcal{M} valid in E but false in F .

Exercise 8

Consider processes below and write down a formula in \mathcal{M} valid in R but not in S .

$$E \triangleq b.c.\mathbf{0} + b.d.\mathbf{0} \tag{1}$$

$$F \triangleq E + b.(c.\mathbf{0} + d.\mathbf{0}) \tag{2}$$

$$R \triangleq a.E + a.F \tag{3}$$

$$S \triangleq a.F \tag{4}$$

Exercise 9

Define in \mathcal{M} , by abbreviation, a connective (K) , with $K \subseteq Act$, such that $E \models (K)\phi$ iff actions in K are the initial actions of E , all of them leading to states which validates ϕ .

Exercise 10

In general, parallel composite in process algebra fails to be idempotent.

1. Making $E \triangleq a.b.E$, formalise a property in \mathcal{M} to distinguish between E and $E \mid E$.
 2. In some cases idempotency holds. Build a bisimulation to witness equivalence $E \sim E \mid E$ when E is $E \triangleq \sum_{x \in K} x.E$, for any $K \subseteq Act - \{\tau\}$. Would this remain true for Act ?
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Exercise 11

Compute

1. $\| [a] [b] \langle c, d \rangle \text{true} \|$
2. $\| \langle a \rangle \langle - \rangle \text{true} \|$
3. $\| [a] \langle - \rangle \text{true} \wedge [b] [-] \text{false} \|$
4. $\| [a] \langle - \rangle \text{true} \vee [b] [-] \text{false} \|$