Interaction and Concurrency

Problem Set - 2
19 April 2021 - 3 May 2021

Please provide a complete, individual answer and quote suitably any reference used.

The modal connectives introduced in the lectures explore the structure of transitions in $\mathbb{P}$, i.e. binary relations $\xrightarrow{x} \subseteq \mathbb{P} \times \mathbb{P}$, for $x \in Act$, relating the validity of the formulas to sets of states reached through certain transitions.

It is possible, however, to define other transition relations in $\mathbb{P}$ that are computationally relevant. One of them is *observable transitions* $\stackrel{a}{\Longrightarrow} \subseteq \mathbb{P} \times \mathbb{P}$, labelled from $L = (Act - \{\tau\}) \cup \{\epsilon\}$. Remember that a $\stackrel{\epsilon}{\Longrightarrow}$-transition corresponds to zero or more transitions through an unobservable action τ.

Consider two new modal operators that express, respectively, the *possibility* and the *need* for a property to be valid after performing an arbitrary amount of unobservable behaviour.

$$
\begin{aligned}
E \models \langle\!\langle\rangle\!\rangle \phi \quad &\text{iff} \quad \exists_{F \in \{E' \mid E \stackrel{\epsilon}{\Longrightarrow} E'\}} \,.\, F \models \phi \\
E \models [\![\,]\!] \phi \quad &\text{iff} \quad \forall_{F \in \{E' \mid E \stackrel{\epsilon}{\Longrightarrow} E'\}} \,.\, F \models \phi
\end{aligned}
$$

By abbreviation we can now define the "observable versions " of $\langle K \rangle$ e $[K]$, for $K \subseteq L$. Thus,

$$
\begin{aligned}
\langle\!\langle K \rangle\!\rangle \phi &\stackrel{\text{abv}}{=} \langle\!\langle\rangle\!\rangle \; \langle K \rangle \; \langle\!\langle\rangle\!\rangle \phi \\
[\![K]\!] \phi &\stackrel{\text{abv}}{=} [\![\,]\!] \; [K] \; [\![\,]\!] \phi
\end{aligned}
$$

Another relevant operator is defined by

$$
E \models [\![\downarrow]\!] \phi \quad \text{iff} \quad E \downarrow \wedge \forall_{F \in \{E' \mid E \stackrel{\epsilon}{\Longrightarrow} E'\}} \,.\, F \models \phi
$$

where $E \downarrow$ means that process E is *convergent*, i.e. it does not commit to an infinite loop of internal actions (τ).

1. Explain in your own words the meaning of the three new logical operators just introduced. For each of them specify a property resorting to it and describes in your own words its intended meaning.

2. Explain the meaning of formulas $\langle\!\langle abac \rangle\!\rangle \mathtt{true}$ and $[\![-]\!]\mathtt{false}$. Illustrate their use through the specification of four different, non-bisimilar processes such that $\langle\!\langle abac \rangle\!\rangle \mathtt{true}$ holds in two of them and $[\![-]\!]\mathtt{false}$ in the other two.

3. In the logic you have studied in the lectures, formula

$$\langle - \rangle \mathtt{true} \wedge [-a]\mathtt{false}$$

expresses *inevitability*, i.e. the occurrence of action a is inevitable. Which of the formulas

   (a) $\langle\!\langle - \rangle\!\rangle \mathtt{true} \wedge [\![-a]\!]\mathtt{false}$

   (b) $[\![\;]\!] \langle\!\langle - \rangle\!\rangle \mathtt{true} \wedge [\![-a]\!]\mathtt{false}$

   if any, would express a similar property in the observational setting? Justify your answer. If none seems suitable, provide an alternative specification.

4. In the lectures, you have studied a close relationship between *bisimilarity* and *modal equivalence* for the logic then introduced. Discuss in some detail if and how a similar result holds relating *observational equivalence* and *modal equivalence* for the extended logic.