Interaction and Concurrency

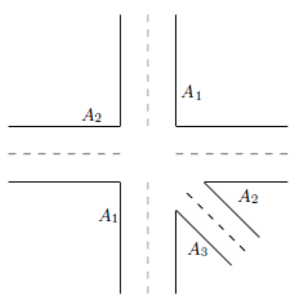
Problem Set - 1
23 March 2021 - 20 April 2021

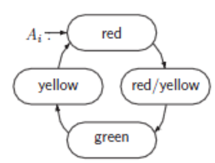
Please provide a complete, individual answer and quote suitably any reference (paper, book, software) used.

1. Make yourself familiar with MCRL2 (read the documentation and try examples). Complete the tutorial available from this course webpage.

2. Write a short note (maximum 3 pages) explaining carefully, for the layman, with a running example, how MCRL2 can be used. Extra bonus to answers not relying in (non modified) examples available from the tool documentation.

3. Consider the following junction controlled by three traffic lights (processes A_1, A_2 and A_1) whose individual behaviour is specified by the following labelled transition system:





   (a) Specify in MCRL2 the process traffic light. Please note you need to choose identifiers for all the relevant actions which are omitted in the graphical sketch.

   (b) Specify in MCRL2 a control process X to ensure that the green light is activates in sequence, and in infinite cycle respectiong the following order: A_1, A_2 and A_3.

   (c) Draw the transition system of the following process:

$$S \;\widehat{=}\; X \mid A_1 \mid A_2 \mid A_3$$

   (d) Apply once the expansion theorem to process S, Comment the result.

4. Consider the following specification of a *pipe*, as supported e.g. in UNIX:

$$U \rhd V \stackrel{\text{abv}}{=} (\{c/out\}U \mid \{c/in\}V)\backslash_{\{c\}}$$

under the assumption that, in both processes, actions $\overline{out}$ e in stand for, respectively, the output and input ports.

(a) Consider now the following processes only partially defined:

$$\begin{aligned} U_1 &\widehat{=} \overline{out}.T \\ V_1 &\widehat{=} in.R \\ U_2 &\widehat{=} \overline{out}.\overline{out}.\overline{out}.T \\ V_2 &\widehat{=} in.in.in.R \end{aligned}$$

Prove, by equational reasoning, or refute the following properties:

i. $U_1 \rhd V_1 \sim T \rhd R$

ii. $U_2 \rhd V_2 = U_1 \rhd V_1$

(b) Show or refute the associativity of $\rhd$ wrt process equality, *i.e.*, for all $P, T, V \in \mathbb{P}$,

$$(U \rhd V) \rhd T = U \rhd (V \rhd T)$$

(c) Show that $\mathbf{0} \rhd \mathbf{0} = \mathbf{0}$.

5. Consider a combinator $\circlearrowleft_n$ whose operational semantics is given by following rule

$$\frac{E \xrightarrow{a} E'}{\circlearrowleft_0 E \xrightarrow{a} E'} \qquad \frac{E \xrightarrow{a} E'}{\circlearrowleft_n E \xrightarrow{a} \circlearrowleft_{n-1} E} \qquad \text{for } n > 0$$

(a) Explain its purpose.

(b) Discuss whether, and for which values of m and n, one may have $\circlearrowleft_n (\circlearrowleft_m E) \sim \circlearrowleft_n E$.

(c) Show that $E \sim F$ implies $\circlearrowleft_n E \sim \circlearrowleft_n F$.

(d) Show, by a counter-example, that, whenever $\sim$ is replaced by $\approx$, the implication above fails.

(e) How could the operational semantics of this new combinator be changed so that the implication mentioned above holds? I.e. so that $E \approx F \Rightarrow \circlearrowleft_n E \approx \circlearrowleft_n F$?