Exercises: Interaction and Concurrency

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1 Basics

The goal of the following exercise is to develop mathematical intuition, helpful in quantum computation.

1. Consider the following matrices:
   $Id = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$; $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$; $S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$;
   $H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$
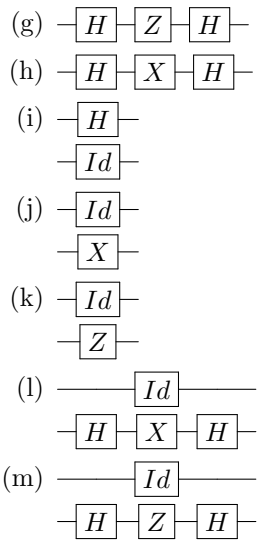
   Determine each of the following:

   (a) $Y \cdot S$

   (b) $S \cdot S$

   (c) $X \cdot X$

   (d) $H \cdot H$

   (e) $H \otimes Id$

   (f) $Id \otimes X$

   (g) $H \cdot Z \cdot H$

   (h) $H \cdot X \cdot H$

   (i) $Id \otimes (H \cdot X \cdot H)$
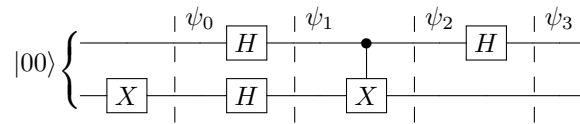
   (j) $Id \otimes (H \cdot Z \cdot H)$

2 Introduction to quantum computation

1. If possible associate each of the following circuits to the letters of exercise 1 section 1.

   (a) $-\boxed{Y}-\boxed{S}-$

   (b) $-\boxed{X}-\boxed{X}-$

   (c) $-\boxed{H}-\boxed{H}-$

   (d) $-\boxed{Id}-$

   (e) $-\boxed{X}-$

   (f) $-\boxed{Z}-$

(g) $-[H]-[Z]-[H]-$

(h) $-[H]-[X]-[H]-$

(i) $-[H]-$
$-[Id]-$

(j) $-[Id]-$
$-[X]-$

(k) $-[Id]-$
$-[Z]-$

(l) $-\quad[Id]\quad-$
$-[H]-[X]-[H]-$

(m) $-\quad[Id]\quad-$
$-[H]-[Z]-[H]-$

2. Recall dirac's notation T10.

$$|00\rangle \left\{ \begin{array}{c} \rule{0pt}{0pt} \end{array} \right.$$

Write each ψ in dirac's notation.