Exercises 4 : Interaction and Concurrency

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Exercise I.6

Show that any process $\tau.(\tau.P + a.\mathbf{0})$ is a solution to equation $X = a.\,\mathbf{0} + \tau.X$.

Answer.

$$
\begin{array}{ll}
 & a.\,\mathbf{0} + \tau.\tau.(\tau.P + a.\mathbf{0}) \\
= & \quad \{\ \tau.\tau.P = P, \text{and} + \text{commutative}\} \\
 & a.\,\mathbf{0} + \tau.(a.\,\mathbf{0} + \tau.P) \\
= & \quad \{\ \text{law: } E + \tau.(E+F) = \tau.(E+F) \text{ from exercise 5. } \} \\
 & \tau.(a.\,\mathbf{0} + \tau.P)
\end{array}
$$

It remains to prove $E + \tau.(E+F) = \tau.(E+F)$. Indeed,

$$
\begin{array}{ll}
 & \tau.(E+F) \\
= & \quad \{\ \tau.X = X + \tau.X\} \\
 & E + F + \tau.(E+F) \\
= & \quad \{ + \text{ is idempotent (applied twice) } \} \\
 & E + E + F + \tau.(E+F) + \tau.(E+F) \\
= & \quad \{ \tau.X = X + \tau.X \ \} \\
 & E + \tau.(E+F) + \tau.(E+F) \\
= & \quad \{ + \text{ is idempotent } \} \\
 & E + \tau.(E+F)
\end{array}
$$

Equality $\tau.X = X + \tau.X$ can be established by definition: both sides have an initial τ transition that leads to X and $X + X$, respectively, which are bisimilar as $+$ is idempotent.