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#### Interaction & Concurrency Course Unit (Lcc)

Universidade do Minho

## Reactive systems

Reactive system

system that computes by reacting to stimuli from its environment along its overall computation

 in contrast to sequential systems whose meaning is defined by the results of finite computations, the behaviour of reactive systems is mainly determined by interaction and mobility of non-terminating processes, evolving concurrently.

- observation  $\equiv$  interaction
- behaviour  $\equiv$  a structured record of interactions

### Reactive systems

Concurrency vs interaction

$$x := 0;$$
  
 $x := x + 1 | x := x + 2$ 

- both statements in parallel could read x before it is written
- which values can x take?
- which is the program outcome if exclusive access to memory and atomic execution of assignments is guaranteed?

#### Definition

A LTS over a set N of names is a tuple  $\langle S, N, \downarrow, \longrightarrow \rangle$  where

- *S* = {*s*<sub>0</sub>, *s*<sub>1</sub>, *s*<sub>2</sub>, ...} is a set of states
- $\downarrow \subseteq S$  is the set of terminating or final states

$$\downarrow s \equiv s \in \downarrow$$

•  $\longrightarrow \subseteq S \times N \times S$  is the transition relation, often given as an *N*-indexed family of binary relations

$$s \stackrel{a}{\longrightarrow} s' \equiv \langle s', a, s \rangle \in \longrightarrow$$

#### Morphism

A morphism relating two LTS over N,  $\langle S, N, \downarrow, \longrightarrow \rangle$  and  $\langle S', N, \downarrow', \longrightarrow' \rangle$ , is a function  $h: S \longrightarrow S'$  st

$$s \stackrel{a}{\longrightarrow} s' \Rightarrow hs \stackrel{a}{\longrightarrow}' hs'$$
  
 $s \downarrow \Rightarrow hs \downarrow'$ 

morphisms preserve transitions and termination

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#### System

Given a LTS  $(S, N, \downarrow, \longrightarrow)$ , each state  $s \in S$  determines a system over all states reachable from s and the corresponding restrictions of  $\longrightarrow$  and  $\downarrow$ .

### LTS classification

- deterministic
- non deterministic
- finite
- finitely branching
- image finite
- ...

# Reachability

#### Definition

The reachability relation,  $\longrightarrow^* \subseteq S \times N^* \times S$ , is defined inductively

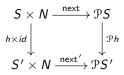
•  $s \xrightarrow{\epsilon} s$  for each  $s \in S$ , where  $\epsilon \in N^*$  denotes the empty word;

• if 
$$s \xrightarrow{a} s''$$
 and  $s'' \xrightarrow{\sigma} s'$  then  $s \xrightarrow{a\sigma} s'$ , for  $a \in N, \sigma \in N^*$ 

#### Reachable state $t \in S$ is reachable from $s \in S$ iff there is a word $\sigma \in N^*$ st $s \xrightarrow{\sigma}^* t$

Alternative characterization (coalgebraic)

A morphism  $h: \langle S, \text{next} \rangle \longrightarrow \langle S', \text{next}' \rangle$  is a function  $h: S \longrightarrow S'$  st the following diagram commutes



i.e.,

$$\mathcal{P}h \cdot \text{next} = \text{next}' \cdot (h \times id)$$

or, going pointwise,

$$\{h \ x \mid x \in \text{next} \langle s, a \rangle\} = \text{next}' \langle h \ s, a \rangle$$

Alternative characterization (coalgebraic) A morphism  $h: \langle S, next \rangle \longrightarrow \langle S', next' \rangle$ 

• preseves transitions:

$$s' \in \mathsf{next} \langle s, a \rangle \Rightarrow h \ s' \in \mathsf{next}' \langle h \ s, a \rangle$$

reflects transitions:

$$r' \in \mathsf{next}' \ \langle h \ s, a 
angle \Rightarrow \langle \exists \ s' \in S \ : \ s' \in \mathsf{next} \ \langle s, a 
angle : \ r' = h \ s' 
angle$$

(why?)

# Comparison

• Both definitions coincide at the object level:

$$\langle s, a, s' 
angle \in \mathcal{T} \; \equiv \; s' \in \mathsf{next} \; \langle s, a 
angle$$

• Wrt morphisms, the relational definition is more general, corresponding, in coalgebraic terms to

$$\mathcal{P}h \cdot \text{next} \subseteq \text{next}' \cdot (h \times id)$$

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## Automata

#### Back to old friends?

automaton behaviour  $\ \equiv \$  accepted language

Recall that finite automata recognize regular languages, i.e. generated by

• 
$$L_1 + L_2 \cong L_1 \cup L_2$$
 (union)

- $L_1 \cdot L_2 \cong \{ st \mid s \in L_1, t \in L_2 \}$  (concatenation)
- $L^* \cong \{\epsilon\} \cup L \cup (L \cdot L) \cup (L \cdot L \cdot L) \cup ...$  (iteration)

## Automata

There is a syntax to specify such languages:

 $E ::= \epsilon \mid a \mid E + E \mid EE \mid E^*$ 

where  $a \in \Sigma$ .

- which regular expression specifies {*a*, *bc*}?
- and {*ca*, *cb*}?

and an algebra of regular expressions:

$$(E_1 + E_2) + E_3 = E_1 + (E_2 + E_3)$$
$$(E_1 + E_2) E_3 = E_1 E_3 + E_2 E_3$$
$$E_1 (E_2 E_1)^* = (E_1 E_2)^* E_1$$

## Automata

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# After thoughts

... need more general models and theories:

- Several interaction points ( $\neq$  functions)
- Need to distinguish normal from anomalous termination (eg deadlock)
- Nondeterminisim should be taken seriously: the notion of equivalence based on accepted language is blind wrt nondeterminism
- Moreover: the reactive characters of systems entail that not only the generated language is important, but also the states traversed during an execution of the automata.