Exercises 5 : Interaction and Concurrency

Luís Soares Barbosa

Exercise 1

Formalise each of the following properties in Process Logic $\mathcal{M}$. Note they are formulated in a somehow ambiguous way, and can therefore be formalised in different ways.

1. The occurrence of a and b is impossible.
2. The occurrence of a followed by b is impossible.
3. Only the occurrence of a is possible.
4. Once a occurred, b or c may occur.
5. After a occurred followed by b, c may occur.
6. Once a occurred, b or c may occur but not both.
7. a cannot occur before b.
8. There is only an initial transition labelled by a.

Exercise 2

Consider the following processes and enumerate for each of them the properties they verify:

1. $E_1 \triangleq a.b.\mathbf{0}$
2. $E_2 \triangleq a.c.\mathbf{0}$
3. $E \triangleq E_1 + E_2$
4. $F \triangleq a.(b.\mathbf{0} + c.\mathbf{0})$
5. $G \triangleq E + F$

Exercise 3

Consider the following specification of a CNC program:

$$\begin{aligned} Start &\triangleq fw.Go + stop.\mathbf{0} \\ Go &\triangleq fw.bk.bk.Start + right.left.bk.Start \end{aligned}$$

Formalise in $\mathcal{M}$ the following properties:

1. After fw another fw is immediately possible
2. After fw followed by $right$, $left$ is possible but bk is not.
3. Action fw is the only one initially possible
4. The third action of process $Start$ is not fw.

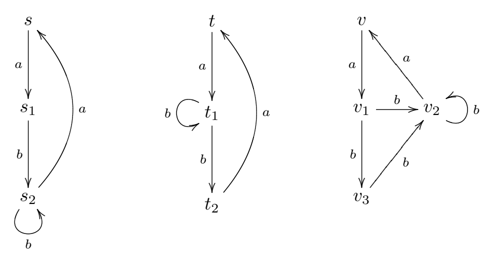
Exercise 4

Specify a LTS such that the following modal properties hold simultaneously in its initial state:

- $\langle a \rangle \langle b \rangle \langle c \rangle$ true $\wedge \langle c \rangle$ true
- $\langle a \rangle \langle b \rangle ([a]$ false $\wedge [c]$ false $\wedge [b]$ false$)$
- $\langle a \rangle \langle b \rangle (\langle a \rangle$ true $\wedge [c]$ false$)$

Exercise 5

Consider the following Act-labelled transition systems.



Show that states s, t and v are not bisimilar and determine the modal properties which distinguish between them.

Exercise 6

Let E be a process. A formula ϕ is said to be *characteristic* of E iff

$$\forall_{F \in \mathbb{P}} \,.\, F \models \phi \;\; sse \;\; F \sim E$$

Note that a process verifies the characteristic formula of E iff it is strongly bisimilar to E.

Determine the *characteristic* formula of process $x.\mathbf{0}$.

Exercise 7

Consider processes $E \triangleq a.(b.\mathbf{0} + c.\mathbf{0})$ e $F \triangleq a.b.\mathbf{0} + a.c.\mathbf{0}$. Propose a formula ϕ in $\mathcal{M}$ valid in E but false in F.

Exercise 8

Consider processes below and write down a formula in $\mathcal{M}$ valid in R but not in S.

$$E \triangleq b.c.\mathbf{0} + b.d.\mathbf{0} \tag{1}$$
$$F \triangleq E + b.(c.\mathbf{0} + d.\mathbf{0}) \tag{2}$$
$$R \triangleq a.E + a.F \tag{3}$$
$$S \triangleq a.F \tag{4}$$

Exercise 9

Define in $\mathcal{M}$, by abbreviation, a connective (K), with $K \subseteq Act$, such that $E \models (K)\phi$ iff actions in K are the initial actions of E, all of then leading to states which validates ϕ.

Exercise 10

In general, parallel composite in process algebra fails to be idempotent.

1. Making $E \triangleq a.b.E$, formalise a property in $\mathcal{M}$ to distinguish between E and$E \mid E$.
2. In some cases idempotency holds. Build a bissimulation to witness equivalence $E \sim E \mid E$ when E is $E \triangleq \sum_{x \in K} x.E$, for any $K \subseteq Act - \{\tau\}$. Would this remain true for Act?

Exercise 11

Compute

1. $\| [a] \, [b] \, \langle c, d \rangle \, \mathsf{true} \|$
2. $\| \langle a \rangle \, \langle - \rangle \, \mathsf{true} \|$
3. $\| [a] \, \langle - \rangle \, \mathsf{true} \wedge [b] \, [-] \, \mathsf{false} \|$
4. $\| [a] \, \langle - \rangle \, \mathsf{true} \vee [b] \, [-] \, \mathsf{false} \|$