Time-critical reactive systems

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Architecture and Calculi Course Unit

Recall

- Timed automata: syntax
  - $\langle L, L_0, Act, C, Tr, Inv \rangle$
  - $g \ ::= \ x \sqsquare n \ \mid \ x - y \sqsquare n \ \mid \ g \wedge g \ \mid \ true$
- Composition $ta_1 \parallel_H ta_2$
- Timed automata: semantics
  - Timed Labelled Transition Systems
  - $\mathcal{T}(ta)$ with states $\langle$location, clock valuation$\rangle$
- Timelocks and zeno paths
- Equivalences
  - timed traces
  - (un)timed bisimulation
- Today: UPPAAL + verification

Note

- The elapse of time in timed automata only takes place in locations:
- ... actions take place instantaneously
- Thus, several actions may take place at a single time unit

Behaviours

- Paths in $\mathcal{T}(ta)$ are discrete representations of continuous-time behaviours in ta
- ... *i.e.* they indicate the states immediately before and after the execution of an action
- However, as interval delays may be realised in uncountably many different ways, different paths may represent the same behaviour

Behaviours

- Paths in $\mathcal{T}(ta)$ are discrete representations of continuous-time behaviours in ta
- ... *i.e.* they indicate the states immediately before and after the execution of an action
- However, as interval delays may be realised in uncountably many different ways, different paths may represent the same behaviour
- ... but not all paths correspond to valid (realistic) behaviours:

undesirable paths:

- time-convergent paths
- timelock paths
- zeno paths

Time-convergent paths

$$\langle l, \eta \rangle \xrightarrow{d_1} \langle l, \eta + d_1 \rangle \xrightarrow{d_2} \langle l, \eta + d_1 + d_2 \rangle \xrightarrow{d_3} \langle l, \eta + d_1 + d_2 + d_3 \rangle \xrightarrow{d_4} \cdots$$

such that

$$\forall_{i \in N}.\ d_i > 0 \ \wedge \ \sum_{i \in N} d_i = d$$

ie, the infinite sequence of delays converges toward d

- Time-convergent paths are conterintuitive; as their existence cannot be avoided, they are simply ignored in the semantics of Timed Automata
- Time-divergent paths are the ones in which time always progresses

Time-convergent paths

Definition

An infinite path fragment ρ is time-divergent if $\mathrm{ExecTime}(\rho) = \infty$
Otherwise is time-convergent.

where

$$\mathrm{ExecTime}(\rho) = \sum_{i=0..\infty} \mathrm{ExecTime}(\delta)$$

$$\mathrm{ExecTime}(\delta) = \begin{cases} 0 & \Leftarrow \delta \in Act \\ \delta & \Leftarrow \delta \in \mathcal{R}_0^+ \end{cases}$$

for ρ a path and δ a label in $\mathcal{T}(ta)$

Timelock paths

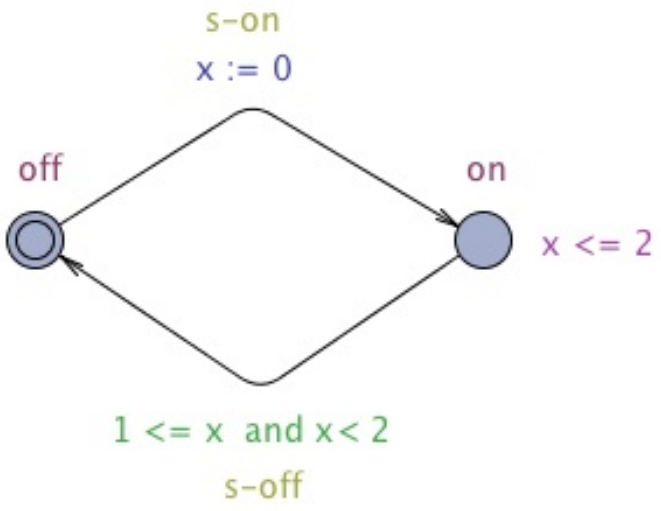
Definition

A path is timelock if it contains a state with a timelock, ie, a state from which there is not any time-divergent path

A timelock represents a situation that causes time progress to halt (e.g. when it is impossible to leave a location before its invariant becomes invalid)

- any teminal state ($\neq$ terminal location) in $\mathcal{T}(ta)$ contains a timelock
- ... but not all timelocks arise as terminal states in $\mathcal{T}(ta)$

Timelock paths

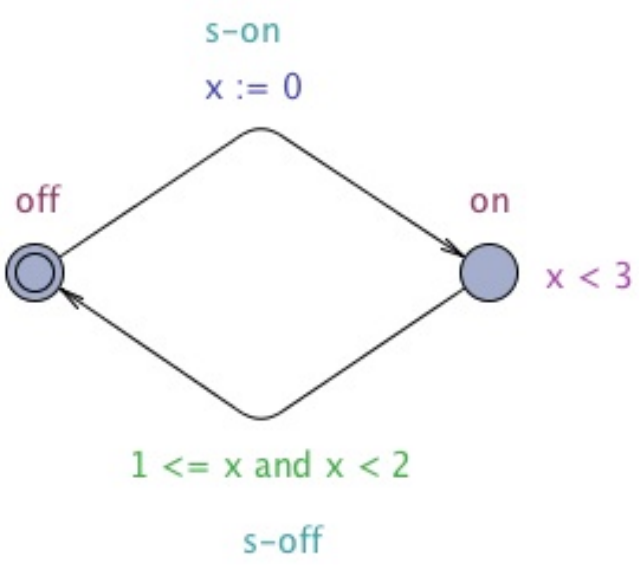


State $\langle on, 2 \rangle$ is reachable through path

$$\langle off, 0 \rangle \xrightarrow{s-on} \langle on, 0 \rangle \xrightarrow{2} \langle on, 2 \rangle$$

and is terminal

Timelock paths



State $\langle on, 2 \rangle$ is not terminal but has a convergent path:

$$\langle on, 2 \rangle \langle on, 2.9 \rangle \langle on, 2.99 \rangle \langle on, 2.999 \rangle \ldots$$

Zeno

In a Timed Automaton

- The elapse of time only takes place at locations
- Actions occur instantaneously: at a single time instant several actions may take place

... it may perform infinitely many actions in a finite time interval
(non realizable because it would require infinitely fast processors)

Zeno

In a Timed Automaton

- The elapse of time only takes place at locations
- Actions occur instantaneously: at a single time instant several actions may take place

> ... it may perform infinitely many actions in a finite time interval (non realizable because it would require infinitely fast processors)

Definition

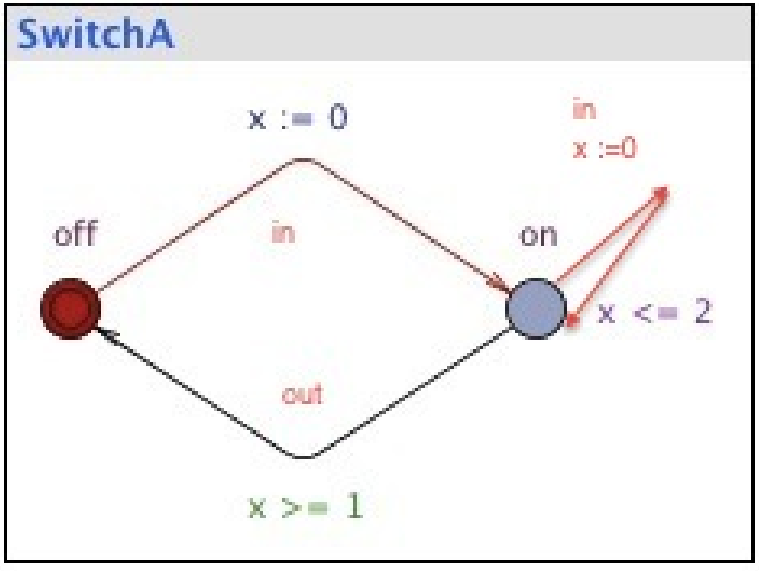
An infinite path fragment ρ is zeno if it is time-convergent and infinitely many actions occur along it

A timed automaton *ta* is non-zeno if there is not an initial zeno path in $\mathcal{T}(ta)$

Zeno

Example

Suppose the user can press the *in* button when the light is *on* in



In doing so clock x is reset to 0 and light stays *on* for more 2 time units (unless the button is pushed again ...)

Zeno

Example

Typical paths: The user presses *in* infinitely fast:

$$\langle off, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{in} \cdots$$

The user presses *in* faster and faster:

$$\langle off, 0\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{0.5} \langle on, 0.5\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{0.25} \langle on, 0.25\rangle \xrightarrow{in} \langle on, 0\rangle \xrightarrow{0.125} \cdots$$

Warning

Both

- timelocks
- zenoness

are modelling flaws and need to be avoided.

Example

In the example above, it is enough to impose a non zero minimal delay between successive button pushings.

Time-critical reactive systems (Timed-automata in UPPAAL)

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Uppaal

... an editor, simulator and model-checker for TA with extensions ...
Editor.

- Templates and instantiations
- Global and local declarations
- System definition

Simulator.

- Viewers: automata animator and message sequence chart
- Control (eg, trace management)
- Variable view: shows values of the integer variables and the clock constraints defining symbolic states

Verifier.

- (`see next session`)

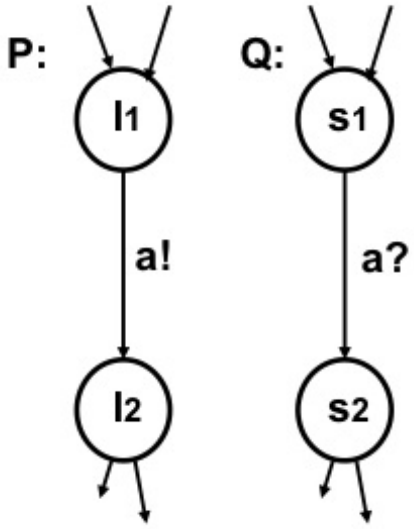
Extensions (modelling view)

- templates with parameters and an instantiation mechanism
- data expressions over bounded integer variables (eg, `int[2..45] x`) allowed in guards, assigments and invariants
- rich set of operators over integer and booleans, including bitwise operations, arrays, initializers ... in general a whole subset of C is available
- non-standard types of synchronization
- non-standard types of locations

Extension: broadcast synchronization

- A sender can synchronize with an arbitrary number of receivers
- Any receiver than can synchronize in the current state must do so
- Broadcast sending is never blocking (the send action can occur even with no receivers).

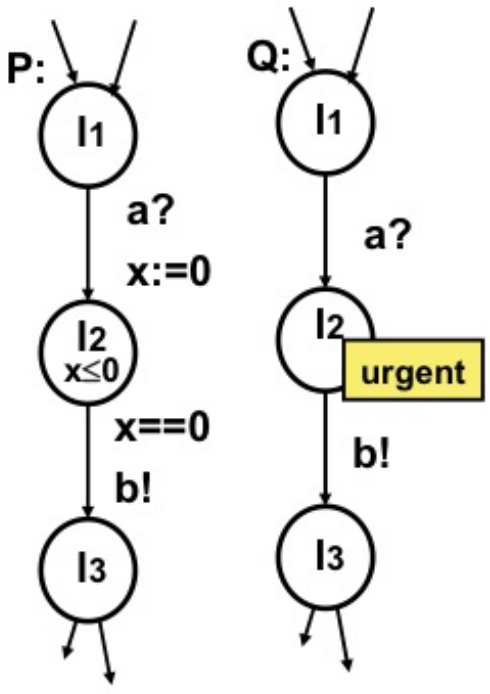
Extension: urgent synchronization



Channel a is declared `urgent chan a` if both edges are to be taken as soon as they are ready (simultaneously in locations ℓ_1 and s_1).
Note the problem can not be solved with invariants because locations ℓ_1 and s_1 can be reached at different moments

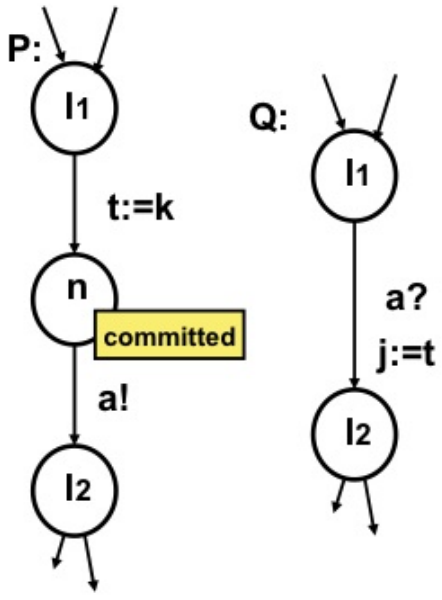
- No delay allowed if a synchronization transition on an urgent channel is enabled
- Edges using urgent channels for synchronization cannot have time constraints (ie, clock guards)

Extension: urgent location



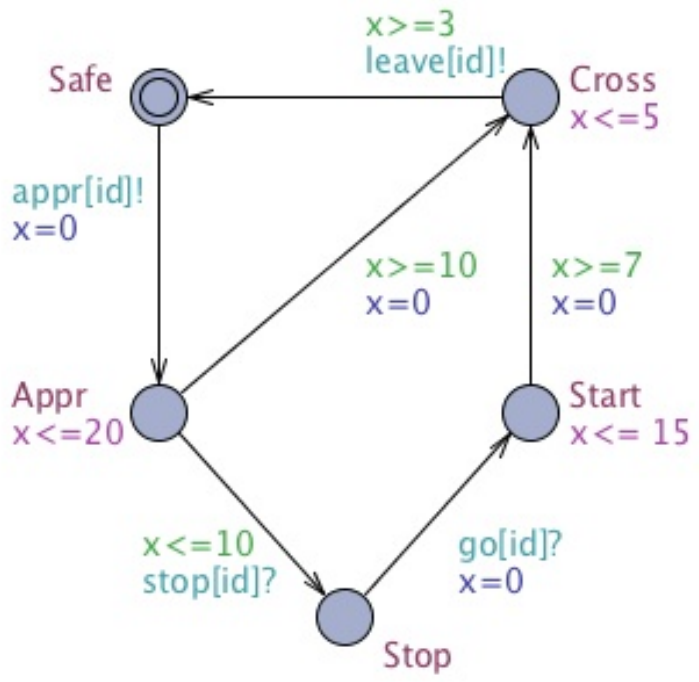
- Time does not progress but interleaving with normal location is allowed
- Both models are equivalent: no delay at an urgent location
- but the use of urgent location reduces the number of clocks in a model and simplifies analysis

Extension: committed location



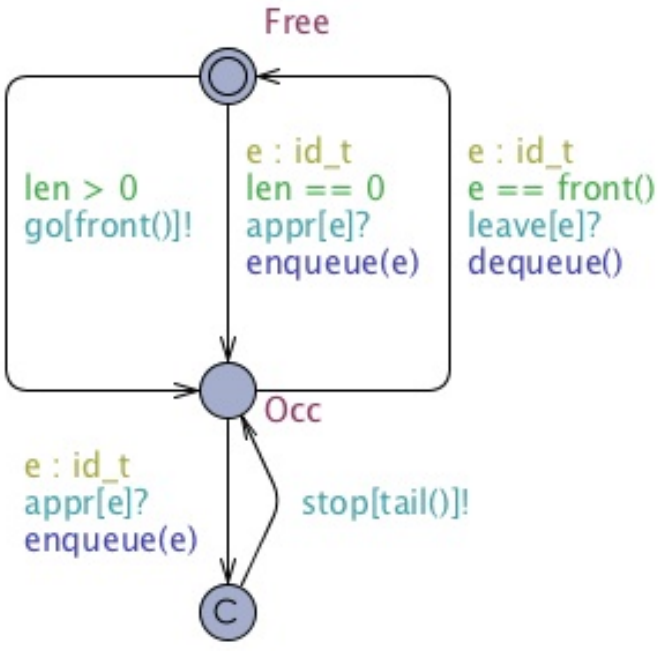
- delay is not allowed and the committed transition must be left in the next instant (or one of them if there are several), i.e., next transition must involve an outgoing edge of at least one of the committed locations
- Our aim is to pass the value k to variable j (via global variable t)
- Location n is committed to ensure that no other automata can assign j before the assignment $j := t$

The train gate example



- Events model approach/leave, order to stop/go
- A train can not be stopped or restart instantly
- After approaching it has 10m to receive a stop.
- After that it takes further 10 time units to reach the bridge
- After restarting takes 7 to 15m to reach the cross and 3-5 to cross

The train gate example



- Note the use of parameters and the select clause on transitions
- Programming ...

Time-critical reactive systems (Verification)

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Properties: expression and satisfaction

The satisfaction problem

Given a timed automata, ta, and a property, ϕ, show that

$$\mathcal{T}(ta) \models \phi$$

Properties: expression and satisfaction

The satisfaction problem

Given a timed automata, ta, and a property, ϕ, show that

$$\mathcal{T}(ta) \models \phi$$

- in which logic language shall ϕ be specified?
- how is $\models$ defined?

Expressing properties: UPPAAL

UPPAAL variant of CTL

- state formulae: describes individual states in $\mathcal{T}(ta)$
- path formulae: describes properties of paths in $\mathcal{T}(ta)$

Expressing properties: UPPAAL

State formulae

Any expression which can be evaluated to a boolean value for a state (typically involving the clock constraints used for guards and invariants and similar constraints over integer variables):

```
x >= 8, i == 8 and x < 2, ...
```

Additionally,

- $ta.\ell$ which tests current location: $(\ell, \eta) \models ta.\ell$
  provided (ℓ, η) is a state in $\mathcal{T}(ta)$
- `deadlock`: $(\ell, \eta) \models \forall_{d \in \mathcal{R}_0^+}$. there is no transition from $\langle \ell, \eta + d \rangle$

Expressing properties: UPPAAL

Path formulae

$$\Pi ::= A\Box\,\Psi \mid A\Diamond\,\Psi \mid E\Box\,\Psi \mid E\Diamond\,\Psi \mid \Phi \rightsquigarrow \Psi$$

$$\Psi ::= ta.\ell \mid g_c \mid g_d \mid \texttt{not}\ \Psi \mid \Psi\ \texttt{or}\ \Psi \mid \Psi\ \texttt{and}\ \Psi \mid \Psi\ \texttt{imply}\ \Psi$$

where

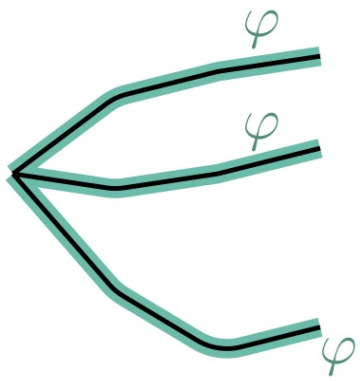
- A, E quantify (universally and existentially, resp.) over paths
- $\Box$, $\Diamond$ quantify (universally and existentially, resp.) over states in a path

also notice that

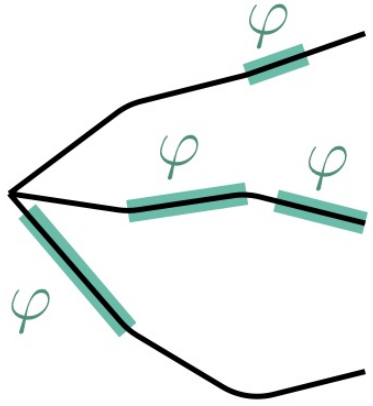
$$\Phi \rightsquigarrow \Psi \overset{\text{abv}}{=} A\Box\,(\Phi \Rightarrow A\Diamond\,\Psi)$$

Expressing properties: UPPAAL

$A\Box\,\varphi$

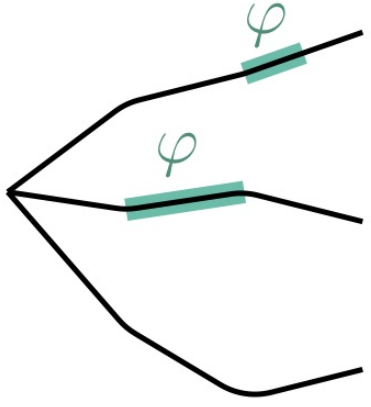


$A\Diamond\,\varphi$

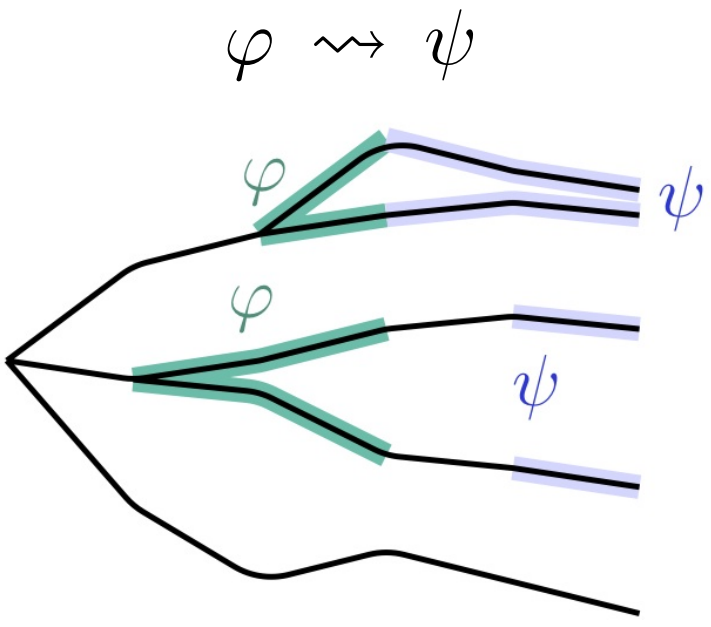


$E\Box\,\varphi$

$E\Diamond\,\varphi$



Expressing properties: UPPAAL



Example

If a message is sent, it will eventually be received –
`send(m)` $\rightsquigarrow$ `received(m)`

Reachability properties

$E\Diamond\,\phi$

Is there a path starting at the initial state, such that a state formula ϕ is eventually satisfied?

- Often used to perform sanity checks on a model:
  - is it possible for a sender to send a message?
  - can a message possibly be received?
  - ...
- Do not by themselves guarantee the correctness of the protocol (i.e. that any message is eventually delivered), but they validate the basic behavior of the model.

Safety properties

$A\Box\,\phi$ and $E\Box\,\phi$

Something bad will never happen
or something bad will possibly never happen

Examples

- In a nuclear power plant the temperature of the core is always (invariantly) under a certain threshold.
- In a game a safe state is one in which we can still win, ie, will possibly not loose.

In Uppaal these properties are formulated positively: something good is invariantly true.

Liveness properties

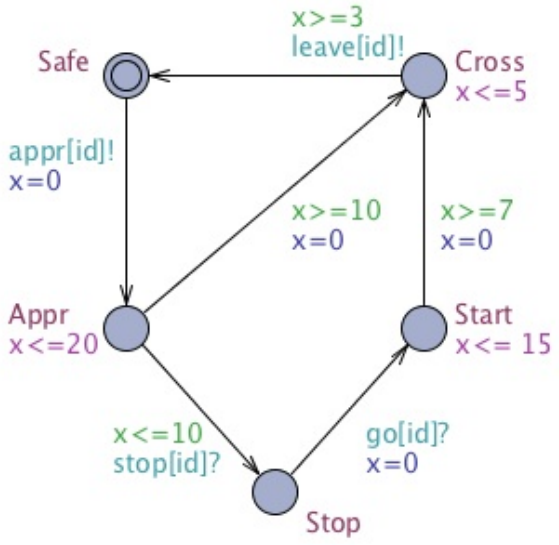
$A\Diamond\,\phi$ and $\phi \rightsquigarrow \psi$

Something good will *eventually happen*
or if *something* happens, then *something else* will eventually happen!

Examples

- When pressing the on button, then eventually the television should turn on.
- In a communication protocol, any message that has been sent should eventually be received.

The train gate example

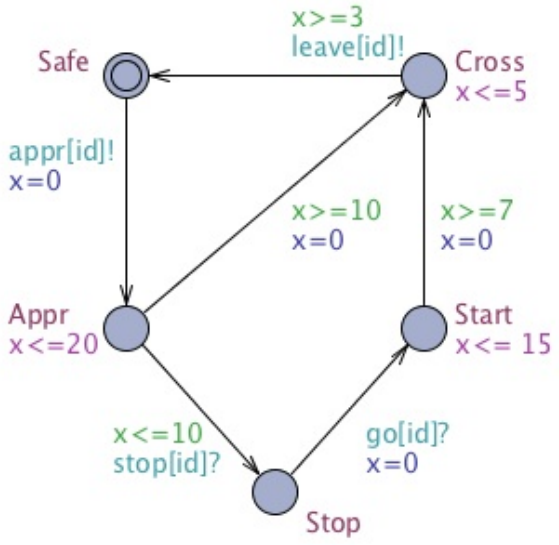


(Train 0 can reach the cross)

(Train 0 can be crossing bridge while Train 1 is waiting to cross)

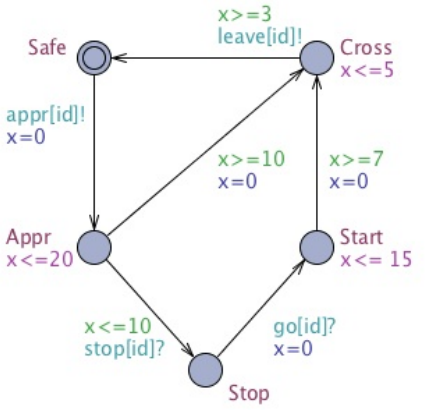
(Train 0 can cross bridge while the other trains are waiting to cross)

The train gate example



- `E<> Train(0).Cross`
  (Train 0 can reach the cross)
- `E<> Train(0).Cross and Train(1).Stop`
  (Train 0 can be crossing bridge while Train 1 is waiting to cross)
- `E<> Train(0).Cross and`
  `(forall (i:id-t) i != 0 imply Train(i).Stop)`
  (Train 0 can cross bridge while the other trains are waiting to cross)

The train gate example



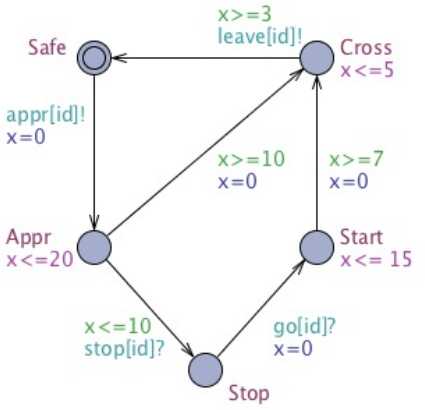
There can never be N elements in the queue

There is never more than one train crossing the bridge

Whenever a train approaches the bridge, it will eventually cross

The system is deadlock-free

The train gate example



- `A[] Gate.list[N] == 0`
  There can never be N elements in the queue
- `A[] forall (i:id-t) forall (j:id-t) Train(i).Cross && Train(j).Cross imply i == j`
  There is never more than one train crossing the bridge
- `Train(1).Appr -> Train(1).Cross`
  Whenever a train approaches the bridge, it will eventually cross
- `A[] not deadlock`
  The system is deadlock-free

Mutual exclusion

Properties

- mutual exclusion: no two processes are in their critical sections at the same time
- deadlock freedom: if some process is trying to access its critical section, then eventually some process (not necessarily the same) will be in its critical section; similarly for exiting the critical section

Mutual exclusion

The Problem

- Dijkstra's original asynchronous algorithm (1965) requires, for n processes to be controlled, $\mathcal{O}(n)$ read-write registers and $\mathcal{O}(n)$ operations.
- This result is a theoretical limit (proved by Lynch and Shavit in 1992) which compromises scalability.

Mutual exclusion

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- This result is a theoretical limit (proved by Lynch and Shavit in 1992) which compromises scalability.

but it can be overcome by introducing specific timing constraints

Two *timed* algorithms:

- Fisher's protocol (included in the UPPAAL distribution)
- Lamport's protocol

Fisher's algorithm

The algorithm

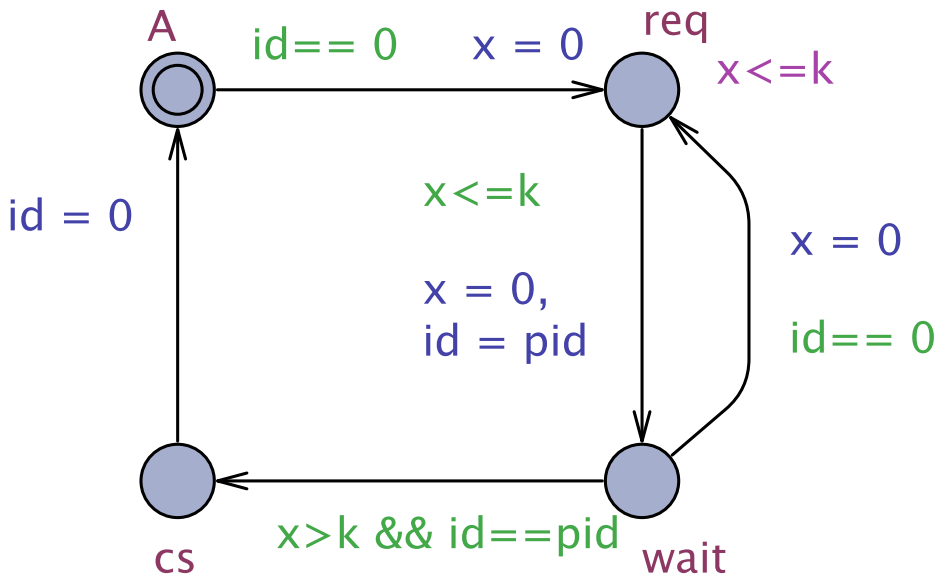
```
repeat
        repeat
                await id = 0
                id := i
                delay(k)
        until id = i
        (critical section)
        id := 0
forever
```

Fisher's algorithm

Comments

- One shared read/write register (the variable *id*)
- Behaviour depends crucially on the value for k — the time delay
- Constant k should be larger than the longest time that a process may take to perform a step while trying to get access to its critical section
- This choice guarantees that whenever process i finds $id = i$ on testing the loop guard it can enter safely ist critical section: all other processes are out of the loop or with their index in *id* overwritten by i.

Fisher's algorithm in UPPAAL



- Each process uses a local clock x to guarantee that the upper bound between between its successive steps, while trying to access the critical section, is k (cf. invariant in state *req*).
- Invariant in state *req* establishes k as such an upper bound
- Guard in transition from *wait* to *cs* ensures the correct delay before entering the critical section

Fisher's algorithm in UPPAAL

Properties

```
% P(1) requests access => it will eventually wait
P(1).req → P(1).wait
% the algorithm is deadlock−free
A[] not deadlock
% mutual exclusion invariant
A[] forall (i:int[1,6]) forall (j:int[1,6])
   P(i).cs && P(j).cs imply i == j
```

- The algorithm is deadlock-free
- It ensures mutual exclusion if the correct timing constraints.
- ... but it is critically sensible to small violations of such constraints: for example, replacing $x > k$ by $x \geq k$ in the transition leading to *cs* compromises both mutual exclusion and liveness.

Lamport's algorithm

The algorithm

start : $a := i$

if $b \neq 0$ then goto start

$b := i$

if $a \neq i$ then delay(k)

else if $b \neq i$ then goto start

(critical section)

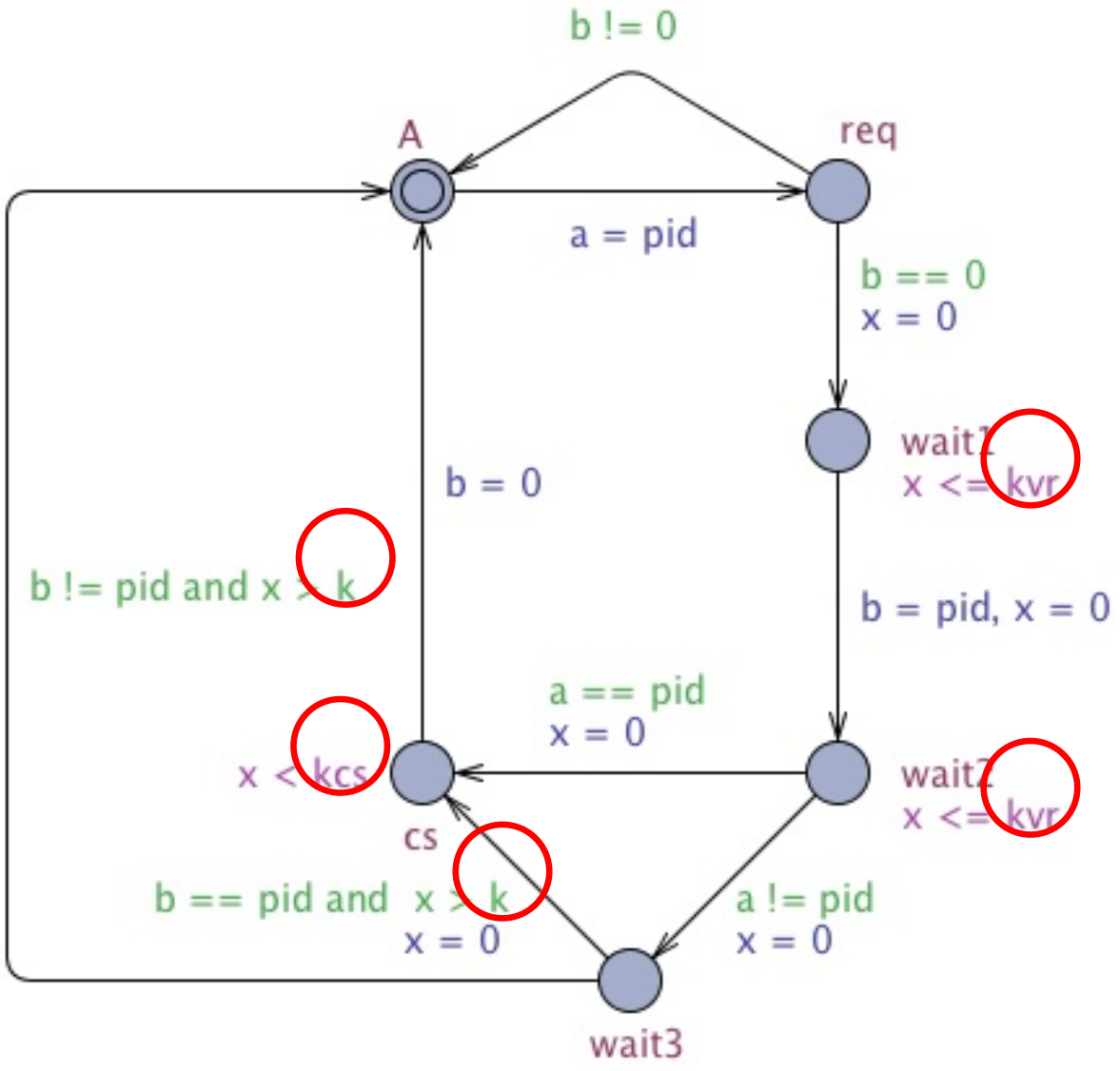
$b := 0$

Lamport's algorithm

Comments

- Two shared read/write registers (variables a and b)
- Avoids forced waiting when no other processes are requiring access to their critical sections

Lamport's algorithm in UPPAAL



Lamport's algorithm

Model time constants:

k — time delay

kvr — max bound for register access

kcs — max bound for permanence in critical section

Typically $\boxed{k \geq kvr + kcs}$

Experiments

	k	kvr	kcs	verified?
Mutual Exclusion	4	1	1	Yes
Mutual Exclusion	2	1	1	Yes
Mutual Exclusion	1	1	1	No
No deadlock	4	1	1	Yes
No deadlock	2	1	1	Yes
No deadlock	1	1	1	Yes